

Economics 101
Spring 2017
Answers to Homework #4
Due Thursday, April 6, 2017

Directions:

- The homework will be collected in a box **before** the lecture.
- Please place **your name, TA name** and **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time.
- **Show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

<https://www.bls.gov/cpi/cpid1612.pdf>

Part I (Consumer Theory).

1. Joseph works 30 hours a week at a wage rate of \$8 per hour. He purchases only two goods with the income he earns: food and books. If he only buys books he finds that he can purchase 24 books per week and if he purchases two books he finds that he has enough money to purchase 11 units of food. You are also told that Joseph's utility function for these two goods is given by the equation $U = B \cdot F$ where U is his level of utility, B is the number of books, and F is the number of units of food. His marginal utility from books is given by the equation $MU_B = F$ and his marginal utility from food is given by the equation $MU_F = B$. Given this information:

- a. What is Joseph's weekly income? Show how you found this answer: do not just provide a number.
- b. What is the price of a book? Explain your reasoning.
- c. What is the price of a unit of food? Explain your reasoning.
- d. Draw a graph of Joseph's budget line (BL1) measuring books (B) on the vertical axis and food (F) on the horizontal axis.
- e. Write an equation in slope-intercept form for Joseph's budget line (BL1) measuring books (B) as the y variable.
- f. Given the above information how many books and how many units of food should Joseph purchase in order to maximize his utility? Show how you calculated this combination and verify

that Joseph can afford this combination and that this combination exhausts all of his income. How much utility does Joseph get when he purchases this combination of books and food?

g. Suppose that Joseph continues to work for 30 hours a week but that his wage rate rises to \$12 an hour. The price of food and books does not change from their original levels. Draw a graph that represents both BL1 and BL2 for Joseph given this information. Provide an equation in slope-intercept form for BL2 given this information. Measure books as the y variable.

h. Given the above information about BL2 how many books and how many units of food should Joseph purchase in order to maximize his utility? Show how you calculated this combination and verify that Joseph can afford this combination and that this combination exhausts all of his income. How much utility does Joseph get when he purchases this combination of books and food?

i. When you compare the optimal consumption of food for Joseph with BL1 and the optimal consumption of food for Joseph with BL2, how much of the change in food consumption is due to the substitution effect and how much is due to the income effect? Be specific in your answers and explain your answer fully and completely.

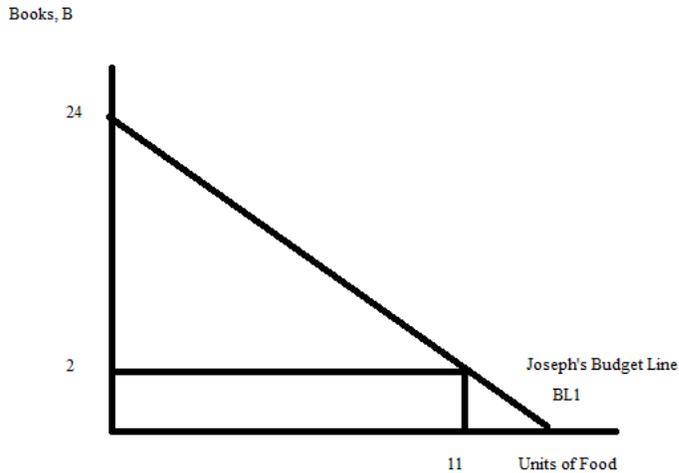
Answers:

a. Joseph works 30 hours per week for \$8 per hour: his weekly income = (number of hours of work per week)(wage per hour) = (30 hours of work per week)(\$8 per hour of work) = \$240 per week in income.

b. Since Joseph has \$240 of income per week and he can afford a maximum of 24 books with this income, this implies that the price of a book must be \$10. We can see this using the formula:
Price of a Book = (Amount of Income Spent on Books)/(Number of Books Purchased)
Price of a Book = \$240/24 Books = \$10 per book

c. We are told that Joseph can afford 2 books and 11 units of food per week. We also know that his weekly income is \$240. The cost of 2 books is \$20 and so when Joseph purchases two books that means that he has \$220 of income left to spend on food. If he can purchase 11 units of food with this income then it must be that the price of a unit of food is \$20.

d.



e. BL1: $B = 24 - 2F$

We can get the slope of the budget line as follows: $(24 - 2)/(0-11) = (22)/(-11) = -2$. Or, you might find the x-intercept: $\text{Income/Price of a Unit of Food} = \$240/\$20 \text{ per unit of food} = 12 \text{ units of food}$. The x-intercept remember is just like the individual saying I am going to use all my income to only buy the good measured on the X-axis. With the two intercepts it is easy to see that the slope is -2.

f. We know that Joseph's budget line can be written as $B = 24 - 2F$. We also know that when Joseph maximizes his utility the addition to his utility from the last dollar he spends on books must equal the addition to his utility from the last dollar he spends on food. That is,

$$MU_B / P_B = MU_F / P_F$$

We can substitute the information we know about the marginal utilities and the prices of these two goods into this equation to get:

$$F/10 = B/20$$

$$\text{or, } 20F = 10B \text{ which can be simplified to } B = 2F$$

Let's use this equation ($B = 2F$) and the budget line equation to find the optimal combination of (F, B) for Joseph:

$$B = 24 - 2F$$

$$2F = 24 - 2F$$

$$4F = 24$$

$$F = 6 \text{ units of food}$$

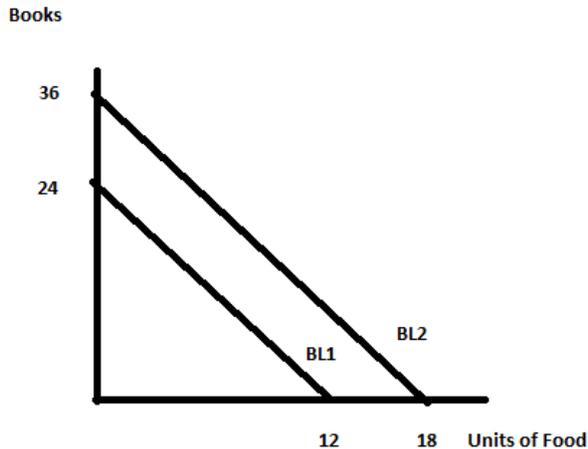
If $F = 6$ units of good, then $B = 2F = 2(6) = 12$ books

The optimal consumption bundle for Joseph is $(F, B) = (6, 12)$ given his tastes and preferences, his income and the prices of the two goods. 6 units of food cost Joseph \$120 and 12 books cost Joseph \$120 for a total cost of \$240 which is exactly Joseph's income.

To find his utility we use this combination and the equation $U = B * F$.

$$U = B * F = 12 * 6 = 72 \text{ utils}$$

g. BL2: $B = 36 - 2F$



h. We know that Joseph's budget line, BL2, can be written as $B = 36 - 2F$. We also know that when Joseph maximizes his utility the addition to his utility from the last dollar he spends on books must equal the addition to his utility from the last dollar he spends on food. That is,

$$MU_B / P_B = MU_F / P_F$$

We can substitute the information we know about the marginal utilities and the prices of these two goods into this equation to get:

$$F/10 = B/20$$

$$\text{or, } 20F = 10B \text{ which can be simplified to } B = 2F$$

Let's use this equation ($B = 2F$) and the budget line equation to find the optimal combination of (F, B) for Joseph:

$$B = 36 - 2F$$

$$2F = 36 - 2F$$

$$4F = 36$$

$$F = 9 \text{ units of food}$$

If $F = 9$ units of good, then $B = 2F = 2(9) = 18$ books

The optimal consumption bundle for Joseph is $(F, B) = (9, 18)$ given his tastes and preferences, his income and the prices of the two goods. 9 units of food cost Joseph \$180 and 18 books cost Joseph \$180 for a total cost of \$360 which is exactly Joseph's income with BL2.

To find his utility we use this combination and the equation $U = B * F$.

$$U = B * F = 18 * 9 = 162 \text{ utils}$$

i. BL1 and BL2 are parallel budget lines and this tells us that the ratio of the two prices has not changed. Recall that the slope of the budget line is given as $-P_x/P_y$, or, in this case, as the negative of the price of food divided by the price of books (-2). The only thing that has changed between BL1 and BL2 is Joseph's income so the change in food consumption of 3 units (going from 6 units of food to 9 units of food) is all due to the income effect. The substitution effect is 0 units of food while the income effect is 3 units of food.

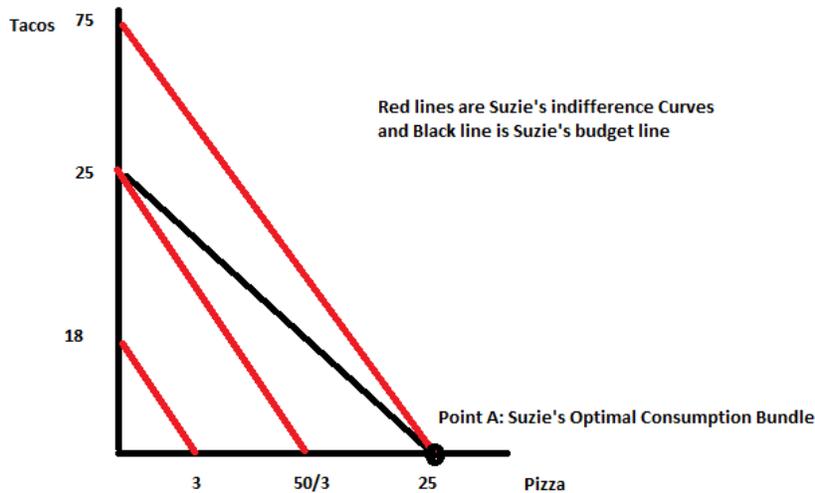
2. Consider Suzie who has income of \$200 per week that she spends on two goods: tacos and pizza. If she spends all of her income on pizza she can purchase 25 pizzas per week and if she

buys only tacos she can purchase 50 tacos. You are also told that from Suzie's perspective she gets the equivalent amount of utility from 3 tacos as she does from 1 pizza. That is, she considers three tacos to be a perfect substitute for 1 pizza.

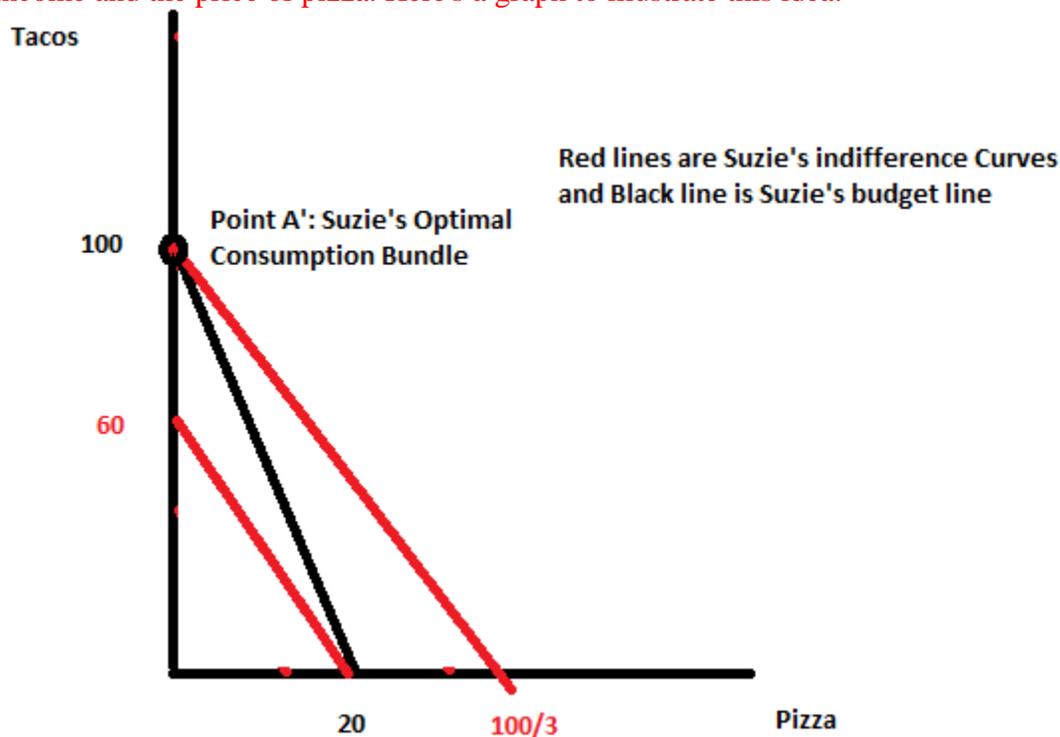
- a. Given the above information, what is the price of a pizza? Explain how you found your answer.
- b. Given the above information, what is the price of a taco? Explain how you found your answer.
- c. Write the equation for Suzie's budget line in slope-intercept form where tacos (T) is measured as the y-variable and pizza (P) is measured as the x-variable.
- d. Given the above information what is the optimal consumption bundle (P, T) for Suzie to consume given her tastes and preferences, her income and the prices of the two goods. Explain your answer and then provide a graph to illustrate the budget line, the indifference curve map, and the consumer optimization point for Suzie.
- e. Suppose that Suzie still has the same tastes and preferences and her income is still \$200 per week but that the price of pizza changes to \$10 per pizza while the price of tacos changes to \$2 per taco. Determine Suzie's optimal consumption bundle (P, T) given this information. Provide an explanation for how you got your answer and a graph depicting your answer as well (should include Suzie's budget line, her indifference curves, and the consumer optimization point for Suzie).

Answers:

- a. We are told that Suzie can purchase 25 pizzas per week with her \$200 in income if she only buys pizzas. So, if we divide the \$200 in income by the 25 pizzas we get that each pizza costs \$8.
- b. We are told that Suzie can purchase 50 tacos per week with her \$200 in income if she only buys tacos. So, if we divide the \$200 in income by the 50 tacos we get that each taco costs \$4.
- c. $\text{Income} = (\text{Price of Pizza})(\text{Number of Pizzas}) + (\text{Price of Tacos})(\text{Number of Tacos})$
 $200 = 8P + 4T$
 $4T = 200 - 8P$
 $T = 50 - 2P$
- d. Suzie will maximize her utility when she consumes 25 pizzas and 0 tacos. She chooses this combination (P, T) = (25, 0) because pizzas and tacos are perfect substitutes for one another and 1 pizza costs less than 3 tacos (\$8 versus \$12). To get the same satisfaction as 25 pizzas provides Suzie would need to consume 75 tacos and she cannot afford this many tacos. Here's a graph to illustrate this idea: the BL is drawn in black and the indifference curves are drawn in red.



e. Suzie will maximize her utility when she consumes 100 tacos and 0 pizzas. She chooses this combination $(P, T) = (0, 100)$ because pizzas and tacos are perfect substitutes for one another and 3 tacos cost less than 1 pizza (\$6 versus \$10). To get the same satisfaction as 100 tacos provides Suzie would need to consume $100/3$ pizzas and she cannot afford this many pizzas given her income and the price of pizza. Here's a graph to illustrate this idea:



3. Consider Pablo who has \$100 in income that he spends on good X and good Y. If he spends all of his income on good X he finds that he can purchase 20 units of good X and if he spends all of his income on good Y he finds that he can purchase 10 units of good Y. Pablo's utility is given

by the following equation where U is his total utility, X is the number of units of good X and Y is the number of units of good Y :

$$U = XY$$

You are also told that Pablo's marginal utility for good X and for good Y can be expressed as follows:

$$MU_X = Y$$

$$MU_Y = X$$

- a. Given the above information find the price of good X and good Y .
- b. Given this information find the combination of good X and good Y , (X, Y) , that maximizes Pablo's utility given his income, the prices of the two goods, and his tastes and preferences. Show your work and explain your answer. Let's call this optimal bundle, bundle A . Bundle A sits on indifference curve 1 and on budget line 1.
- c. Suppose that the price of good X increases by 100% from its original level. Everything else stays the same. Given this new information, find the combination of good X and good Y (X', Y') , that maximizes Pablo's utility given his income, the prices of the two good (using the new price for good X), and his tastes and preferences. Show your work and explain your answer. Let's call this optimal bundle, bundle B . Bundle B sits on indifference curve 2 and on budget line 2.
- d. Suppose that Pablo's demand curve is linear. Use the information from (b) and (c) to write the equation for Pablo's demand curve. Show your work and explain in words what you are doing.
- e. When the price of good X increases, Pablo responds by decreasing his consumption of good X due to the substitution and the income effects. Pablo wants you to figure out how much of this change in consumption is due to the substitution effect and how much is due to the income effect. To do this, you will need to find bundle C where Pablo faces the new price for good X (the price in (c)) while having the same utility as he had at point A . Here are some questions to guide you on this work.
 - i. What was Pablo's utility at point A ? Show your work.
 - ii. What will be Pablo's utility at point C where point C corresponds to the consumption bundle (X'', Y'') ?
 - iii. What is the consumer optimization relationship at point C where point C corresponds to the consumption bundle (X'', Y'') ?
 - iv. Use the information in (ii) and (iii) to find the values of X'' and Y'' . These values are the coordinates for bundle C . Provide a check that the utility level at bundle A equals the utility level at bundle C .
 - v. Approximately (to the nearest dollar amount) how much income does Pablo need to purchase bundle C ?

vi. How much of the change in consumption of good X that occurs when Pablo moves from point A to point B is due to the substitution effect?

vii. How much of the change in consumption of good X that occurs when Pablo moves from point A to point B is due to the income effect?

Answers:

a. The price of good X is \$5 per unit of good X since if we divide Pablo's income by the number of units of good X he can purchase if he only buys good X we get this price. The price of good Y is \$10 per unit of good Y using the same reasoning.

b. We know that Pablo will maximize his utility when he satisfies the following condition:

$$MU_X/P_X = MU_Y/P_Y$$

We can substitute the information we have about both his marginal utilities as well as the prices of the two goods to get:

$$Y/5 = X/10$$

$$10Y = 5X \text{ or } 2Y = X$$

With this optimality condition you are basically stating that the slope of the indifference curve (MU_X/MU_Y) must be equal to the slope of the budget line (P_X/P_Y) in absolute value terms (both the indifference curve and the budget line are assumed to be downward sloping so both would have negative slopes).

We also know that Pablo's budget line can be written as:

$$100 = P_X * X + P_Y * Y$$

$$100 = 5X + 10Y$$

Let's use these two equations to find the optimal bundle:

$$100 = 5X + 10Y$$

$$2Y = X$$

So,

$$100 = 5(2Y) + 10Y$$

$$100 = 20Y$$

$$5 = Y$$

And, since $X = 2Y$, X must be equal to 10 units. So, the optimal bundle, bundle A, is (X, Y) = (10, 5).

c. If the price of good X increases by 100%, then the new price of good X must now be \$10 per unit instead of the original \$5 per unit. This price change will alter the equation for the budget line:

$$\text{Equation for Budget Line 2: } 100 = P_X' * X' + P_Y * Y'$$

$$100 = 10X' + 10Y'$$

The optimality condition also changes:

$$Y'/10 = X'/10$$

$$10Y' = 10X' \text{ or } Y' = X'$$

Let's use these two equations to find the optimal bundle:

$$100 = 10X' + 10Y'$$

$$Y' = X'$$

So,

$$100 = 10(Y') + 10Y'$$

$$100 = 20Y'$$

$$5 = Y'$$

And, since $X' = Y'$, X' must be equal to 5 units. So, the optimal bundle, bundle B, is $(X', Y') = (5, 5)$.

d. From (b) we know that when the price of good X is \$5 per unit that Pablo maximizes his utility by consuming 10 units of good X and 5 units of good Y. For purposes of finding Pablo's demand curve for good X we need $(Q_x, P_x) = (10 \text{ units of good X}, \$5 \text{ per unit of good X})$. From (c) we know that when the price of good X is \$10 per unit that Pablo maximizes his utility by consuming 5 units of good X and 5 units of good Y. For purposes of finding Pablo's demand curve for good X we need $(Q_{x'}, P_{x'}) = (5 \text{ units of good X}, \$10 \text{ per unit of good X})$. We now have two points on Pablo's linear demand curve: $(Q_x, P_x) = (10, 5)$ and $(5, 10)$. Use these two points to write his demand curve:

$$y = mx + b \text{ (general slope intercept form)}$$

$$P = mQ + b$$

$$\text{Calculate the slope, } m: m = \text{rise/run} = (5 - 10)/(10 - 5) = -5/5 = -1$$

$$P = -Q + b$$

Use one of the known points to find the value of b, the y-intercept:

$$10 = -5 + b$$

$$b = 15$$

The equation for Pablo's demand curve can be written as: $P = 15 - Q$

Verify that the other point lies on this line: $5 = 15 - 10$both points are on this line!

e.

i. Utility at point A: $U = XY$

$$U = (10)(5) = 50 \text{ utils}$$

ii. Utility at point C: $U = X''Y'' = XY = 50 \text{ utils}$

iii. This will be the same consumer optimization condition we had in (c). That is,

$$Y''/10 = X''/10$$

$$10Y'' = 10X'' \text{ or } Y'' = X''$$

$$\text{iv. } X''*Y'' = 50$$

$$X''*X'' = 50$$

$$X'' = 5\sqrt{2} \approx 7.07$$

Since $X'' = Y''$ this implies that bundle C, $(X'', Y'') = (5\sqrt{2}, 5\sqrt{2}) \approx (7.07, 7.07)$

Utility at bundle A = 50 utils

$$\text{Utility at bundle C} = X''*Y'' = (5\sqrt{2})(5\sqrt{2}) = (7.07)(7.07) = 49.98 \text{ utils} \approx 50 \text{ utils}$$

v. Bundle C = $(5\sqrt{2}, 5\sqrt{2})$

Price of X = \$10

Price of Y = \$10

Income needed to buy bundle C = (Price of X)(X") + (Price of Y)(Y")

Income needed to buy bundle C = $10(5\sqrt{2}) + 10(5\sqrt{2}) = 100(1.414) \approx \141

vi. Substitution effect with respect to good X is the change in the consumption of good X that occurs when Pablo moves from Point A to Point C: Substitution Effect = $10 - 7.07 = 2.93$ units of good X.

vii. Income effect with respect to good X is the change in the consumption of good X that occurs when Pablo moves from Point C to Point B: Income Effect = $7.07 - 5 = 2.07$ units of good X.

Part II: CPI, Real vs. Nominal, and Inflation

4. In this problem you will go through the process of calculating the CPI for a simple example. For purposes of calculating the CPI you are told that the market basket consists of 10 apples, 20 oranges, and 5 books. You are provided the following information:

| Year | Price per Apple | Price per Orange | Price per Book | Cost of Market Basket |
|------|--|------------------|----------------|--|
| 1 | \$1 | \$2 | \$5 | V |
| 2 | \$1 | X | \$6 | 100 |
| 3 | Y | \$4 | \$5 | U = Cost of Market Basket in Year 3 is 25% greater than in Year 2 |
| 4 | S = Price in Year 4 is 50% less than in Year 3 | \$3 | Z | T = Cost of Market Basket in Year 4 is 100% greater than in Year 3 |

a. Fill in the missing cells in the above table and show the work you did to get each answer.

b. Using the table you completed in (a), calculate the CPI based on the given market basket for these four years. Use Year 1 as the base year and a 100 point scale. Put your results in the following table. Round your answers to the nearest tenth.

| Year | CPI with base year Year 1 and a 100 point scale |
|------|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

c. Calculate the annual rate of inflation based upon the CPI figures you calculated in (b) and enter your results in the following table. Round your answer to the nearest whole percentage.

| Year | Annual Rate of Inflation based on CPI with Year 1 as base year |
|------|--|
| 2 | |
| 3 | |
| 4 | |

d. Using the table you completed in (a), calculate the CPI based on the given market basket for these four years. Use Year 2 as the base year and a 100 point scale. Put your results in the following table:

| Year | CPI with base year Year 2 and a 100 point scale |
|------|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

e. Calculate the annual rate of inflation based upon the CPI figures you calculated in (d) and enter your results in the following table. Round your answer to the nearest whole percentage. Did you get the same annual rate of inflation (with possibly a bit of rounding error) as you did in (c)?

| Year | Annual Rate of Inflation based on CPI with Year 2 as base year |
|------|--|
| 2 | |
| 3 | |
| 4 | |

Answers:

a. $V = \text{Cost of Market Basket in Year 1} = (\text{Price per apple in Year 1})(\text{Number of apples in market basket}) + (\text{Price per orange in Year 1})(\text{Number of oranges in market basket}) + (\text{Price per book in Year 1})(\text{Number of books in market basket}) = (\$1)(10) + (\$2)(20) + (\$5)(5) = \$75$

$X = \text{Price per orange in year 2}$

To find the value of X, write the equation for the cost of the market basket:

$\text{Cost of Market Basket in Year 2} = (\text{Price per apple in Year 2})(\text{Number of apples in market basket}) + (\text{Price per orange in Year 2})(\text{Number of oranges in market basket}) + (\text{Price per book in Year 2})(\text{Number of books in market basket}) = (\$1)(10) + (\$X)(20) + (\$6)(5) = 100$

$20X = 60$

$X = \$3 \text{ per orange}$

$U = \text{Cost of market basket in year 3} = (1.25)(\text{Cost of market basket in year 2}) = 1.25(100) = 125$

$Y = \text{Price per apple in year 3}$

$U = (\text{Price per apple in Year 3})(\text{Number of apples in market basket}) + (\text{Price per orange in Year 3})(\text{Number of oranges in market basket}) + (\text{Price per book in Year 3})(\text{Number of books in market basket}) = (\$Y)(10) + (\$4)(20) + (\$5)(5) = 125$

$125 = 10Y + 80 + 25$

$10Y = 20$

$Y = \$2 \text{ per apple}$

$T = \text{Cost of market basket in year 4} = 2(125) = 250$

S = Price per apple in year 4 is 50% less than the price per apple in year 3

Price of apple in year 3 = \$2

50% of \$2 is \$1

S = price of apple in year 4 = \$1

Z = price per book in year 4

Cost of market basket in year 4 = (Price per apple in Year 4)(Number of apples in market basket) + (Price per orange in Year 4)(Number of oranges in market basket) + (Price per book in Year 4)(Number of books in market basket) = (\$1)(10) + (\$3)(20) + (\$Z)(5) = 250

$250 = 10 + 60 + 5Z$

$5Z = 180$

$Z = 36$

Z = \$36 per book

Here's the completed table:

| Year | Price per Apple | Price per Orange | Price per Book | Cost of Market Basket |
|------|--|------------------|----------------|--|
| 1 | \$1 | \$2 | \$5 | V = 75 |
| 2 | \$1 | X | \$6 | 100 |
| 3 | Y = \$2 | \$4 | \$5 | U = Cost of Market Basket in Year 3 is 25% greater than in Year 2 = 125 |
| 4 | S = Price in Year 4 is 50% less than in Year 3 = \$1 | \$3 | Z = \$36 | T = Cost of Market Basket in Year 4 is 100% greater than in Year 3 = 250 |

b. To find the CPI for year n use the following formula:

CPI for year n = [(Cost of market basket in year n)/(Cost of Market basket in base year)]*(scale factor)

| Year | CPI with base year Year 1 and a 100 point scale |
|------|---|
| 1 | $(75/75)*100 = 100$ |
| 2 | $(100/75)*100 = 133.3$ |
| 3 | $(125/75)*100 = 166.7$ |
| 4 | $(250/75)*100 = 333.3$ |

c. To calculate the annual rate of inflation based on the CPI figures use the following formula:

Annual rate of inflation in year n = [(CPI in year n) – (CPI in year (n-1))]/[(CPI in year (n-1))] * 100%

Note: this is just an application of the standard percentage change formula:

Percentage Change = [(New Value – Initial Value)/(Initial Value)] * 100%

| Year | Annual Rate of Inflation based on CPI with Year 1 as base year |
|------|--|
| 2 | $[(133.3 - 100)/100]*100\% = 33\%$ |
| 3 | $[(166.7 - 133.3)/133.3]*100\% = 25\%$ |
| 4 | $[(333.3 - 166.7)/(166.7)]*100\% = 100\%$ |

d. To find the CPI for this question you can use the same formula as in (b) and just change the Cost of the market basket in the base year to reflect a change in the base year. Or, you can find it by rescaling. If you rescale, you would use the following formula:

CPI in year n with base year Year 2 = [(CPI in year n with base year Year 1)/(CPI in year 2 using Year 1 as the base Year)]*(scale factor)

So, for example in (b) the CPI for year 1 had a value of 100:

CPI for year 1 using Year 2 as the base year = $[(100)/(133.1)]*(100) = 75$ (approximately)

New CPI figures using the first method:

| Year | CPI with base year Year 2 and a 100 point scale |
|------|---|
| 1 | $(75/100)*100 = 75$ |
| 2 | $(100/100)*100 = 100$ |
| 3 | $(125/100)*100 = 125$ |
| 4 | $(250/100)*100 = 250$ |

New CPI figures using the rescaling method:

| Year | CPI with base year Year 2 and a 100 point scale |
|------|---|
| 1 | $(100/133.3)*100 = 75$ |
| 2 | $(133.3/133.3)*100 = 100$ |
| 3 | $(166.7/133.3)*100 = 125$ |
| 4 | $(333.3/133.3)*100 = 250$ |

e. Here's the work to prove that the results in (c) and (e) are the same:

| Year | Annual Rate of Inflation based on CPI with Year 2 as base year |
|------|--|
| 2 | $[(100 - 75)/75]*100\% = 33\%$ |
| 3 | $[(125 - 100)/100]*100\% = 25\%$ |
| 4 | $[(250 - 125)/125]*100\% = 100\%$ |

5. After four years of studying, Marci is scheduled to graduate from the UW-Madison in May! She is excited and she is also pleased that she has four job offers all starting July 1. For the sake of this problem let's assume that the work is the same at all four jobs and that she has no locational preference. Her only criteria for selecting a job is that it pays the highest real wage. Marci paid attention during her Economics 101 class and knows that she should be able to figure out the real wage for each of these locations if she has the CPI for the location. She plans to use the December 2016 CPI index number for each location (since the 2017 figures will not yet be available). She plans to utilize the government as the source of this data. She knows that the

Bureau of Labor Statistics (BLS) provides this information and she challenges herself to find it on her own! [Don't cheat here: you try to find it as well!....But, if you can't find it, the web address is buried in this document!....I can at least get you to hunt through this document!...But, you will learn more if you go to the web and check out the BLS!] You will want to look for the "CPI Detailed Report" and once you find it, you will want to go to table 16 and then use the data for the Dec. 2016 "All Item" index number for the particular city. Plan to use the CPI figure that includes three places past the decimal for your calculations (fine to use a calculator on this set of questions!).

Here are Marci's offers:

| Location of Job Offer | Nominal Salary | CPI for Dec. 2016 from BLS website | Real Salary in 1982-84 (base year for CPI index) dollars |
|---|----------------|------------------------------------|--|
| Atlanta, Georgia | \$45,000 | | |
| Los Angeles-Riverside-Orange County, CA | \$61,000 | | |
| Miami-Fort Lauderdale, FL | \$63,000 | | |
| Seattle-Tacoma-Bremerton, WA | \$58,000 | | |

- Complete the above table. You will get the CPI numbers from the website and then you will use the formula provided in class to calculate the real salary for each of these locations.
- Based solely on real income, which offer should Marci take?
- Given the data you collected in (a), what would each offer need to be in order for all the offers to have the equivalent purchasing power as the Seattle-Tacoma-Bremerton, WA offer as measured in 2016 dollars? Show how you computed these new nominal income salaries. Report the nearest whole number for your salary figures.
- You just did a lot of work for answer (c), can you now provide a rationale for why Marci might want to know how to do this?

Answers:

a.

| Location of Job Offer | Nominal Salary | CPI for Dec. 2016 from BLS website | Real Salary in 1982-84 (base year for CPI index) dollars |
|-----------------------|----------------|------------------------------------|--|
| Atlanta, Georgia | \$45,000 | 226.739 (based on | $= (45,000/226.739)*100 = \19846.61 |

| | | | |
|---|----------|--|---------------------------------------|
| | | 1982-84 being base year) | |
| Los Angeles-Riverside-Orange County, CA | \$61,000 | 250.189 (based on 1982-84 being base year) | $= (61,000/250.189)*100 = \24381.57 |
| Miami-Fort Lauderdale, FL | \$63,000 | 253.629 (based on 1982-84 being base year) | $= (63,000/253.629)*100 = \24839.43 |
| Seattle-Tacoma-Bremerton, WA | \$58,000 | 256.821 (based on 1982-84 being base year) | $= (58,000/256.821)*100 = \22583.82 |

b. The job offer with the greatest real income is the offer in Miami-Fort Lauderdale, FL.

c. We want the real salary for each offer to equal \$22583.82 in 1982-84 dollars. You might think you have to rescale your CPI figures, but that won't be necessary here. We know the general formula:

$$\text{Real value} = [(\text{Nominal Value})/(\text{Inflation Index})]*(\text{Scale Factor})$$

We know the Inflation Index for each location from the data we gathered. We also know the real value that we want (\$22,583.82) and we know that the scale factor is 100. So, here are the calculations:

$$\text{For Atlanta, GA: } 22583.82 = [(\text{Nominal Salary Needed})/226.739]*100$$

$$\text{Nominal Salary Needed} = (22583.82 * 226.739)/100 = \$52,206$$

$$\text{For Los Angeles-Riverside-Orange County, CA: } 22583.82 = [(\text{Nominal Salary Needed})/250.189]*100$$

$$\text{Nominal Salary Needed} = 22583.82 * 250.189/100 = \$56,502$$

$$\text{For Miami-Fort Lauderdale, FL: } 22583.82 = [(\text{Nominal Salary Needed})/253.629]*100$$

$$\text{Nominal Salary Needed} = (22583.82 * 253.629)/100 = \$57,279$$

d. Marci is much more likely to care not only about her real income but also where she lives. She has four job offers in four different locations with different costs of living. If she can do this calculation, then she can go back to the various employers and make a good argument for why the offer should be adjusted so that she gets equivalent purchasing power as her best offer but in her favorite location.

Part III: Production and Cost

6. For this problem I want you to use EXCEL or a similar spreadsheet program. If you have never used a spreadsheet program, it is not that hard and it is a very valuable skill to have for personal use as well as professional use.

Imagine that you produce widgets and you know that you need to use labor (L) and capital (K) to produce your widgets. Your capital is a fixed input while your labor is a variable input. You also know that your production of widget is described by the following production function:

$$q = \sqrt{K}\sqrt{L}$$

q is the number of widgets, K is the number of units of capital, and L is the number of units of labor.

a. Given the above information, fill in the missing values in the table below. Actually make your own spreadsheet, insert the necessary formulas and compute the values for each missing cell. For all computations except fixed cost (FC) and variable cost (VC) carry your answer to two places past the decimal. For FC and VC calculate the round to the closest whole number. Here are a couple of hints in doing this work:

i. You should be able to fill in the first three columns from the provided information.

ii. You will need to have a strong command of the various definitions we discuss in class:

$$ATC = TC/q$$

$$AFC = FC/q$$

$$AVC = VC/q$$

$$VC = (\text{Price of the variable input})(\text{Number of units of the variable input})$$

$$FC = (\text{Price of the fixed input})(\text{Number of units of the fixed input})$$

$$TC = FC + VC$$

$$MC = (\text{Change in total cost})/(\text{Change in output})$$

$$MPI = (\text{Change in output})/(\text{Change in labor})$$

iii. You will need to think and consider this a logic puzzle: easy to do if you just stop and apply the various pieces of information you have.

| K | L | q | FC | VC | TC | AFC | AVC | ATC | MC | MPI |
|-----|-----|---|----|----|----|------|------|-----|-----|-----|
| 100 | 0 | | | | | --- | --- | --- | --- | --- |
| | 10 | | | | | | | | | |
| | 20 | | | | | | | | | |
| | 30 | | | | | | 1.10 | | | |
| | 40 | | | | | | | | | |
| | 50 | | | | | | | | | |
| | 60 | | | | | | | | | |
| | 70 | | | | | | | | | |
| | 80 | | | | | | | | | |
| | 90 | | | | | 1.05 | | | | |
| | 100 | | | | | | | | | |
| | 110 | | | | | | | | | |
| | 120 | | | | | | | | | |
| | 130 | | | | | | | | | |
| | 140 | | | | | | | | | |

- b. What is the price of a unit of capital? What is the price of a unit of labor? Explain how you found these prices.
- c. Describe what happens to the Marginal Product of Labor (MPI) as output increases. Use the information that you calculated in the table.
- d. Examine the results in your table. Describe what happens to AFC as output increases. Explain why this is happening.
- e. Examine the results in your table. Describe what happens to ATC as output increases. In your answer include a reference and description of the spreading effect as well as the diminishing returns effect (you may need to consult your textbook on this: make sure that you do not write a copy of your textbook's material here: that would be academic misconduct-plagiarism!).
- f. Suppose this firm is a perfectly competitive firm. Furthermore, suppose that the market price for this good is equal to \$4. What is the profit maximizing level of output for this firm to produce given this information? What will be the firm's level of profits? Show how you found this answer.

Answers:

We know that $K = 100$ for the entire column, since K is a fixed input set at 100 units. We know that we can find q by using the aggregate production function provided. Once we have the q column filled in, we know that $AFC = 1.05$ when $q = 94.87$. This implies that $FC = 99.61$ for this level of output, or rounding to the nearest whole number, $FC = \$100$. (You are using the formula $AFC = FC/q$ and rearranging this to be $FC = AFC * q$.) This implies that the price of a unit of capital is \$1.

Similar reasoning can be used with regard to the AVC information in the table: we know that $AVC = 1.10$ when $q = 54.77$. This implies that $VC = 60.25$ for this level of output, or rounding to the nearest whole number, $VC = \$60$. (You are using the formula $AVC = VC/q$ and rearranging this to be $VC = AVC * q$.) This implies that the price of a unit of labor is \$2.

| K | L | q | FC | VC | TC | AFC | AVC | ATC | MC | MPI |
|-----|-----|--------|-----|-----|-----|------|------|------|------|------|
| 100 | 0 | 0.00 | 100 | 0 | 100 | --- | --- | --- | --- | --- |
| 100 | 10 | 31.62 | 100 | 20 | 120 | 3.16 | 0.63 | 3.79 | 0.63 | 3.16 |
| 100 | 20 | 44.72 | 100 | 40 | 140 | 2.24 | 0.89 | 3.13 | 1.53 | 1.31 |
| 100 | 30 | 54.77 | 100 | 60 | 160 | 1.83 | 1.10 | 2.92 | 1.99 | 1.01 |
| 100 | 40 | 63.25 | 100 | 80 | 180 | 1.58 | 1.26 | 2.85 | 2.36 | 0.85 |
| 100 | 50 | 70.71 | 100 | 100 | 200 | 1.41 | 1.41 | 2.83 | 2.68 | 0.75 |
| 100 | 60 | 77.46 | 100 | 120 | 220 | 1.29 | 1.55 | 2.84 | 2.96 | 0.67 |
| 100 | 70 | 83.67 | 100 | 140 | 240 | 1.20 | 1.67 | 2.87 | 3.22 | 0.62 |
| 100 | 80 | 89.44 | 100 | 160 | 260 | 1.12 | 1.79 | 2.91 | 3.46 | 0.58 |
| 100 | 90 | 94.87 | 100 | 180 | 280 | 1.05 | 1.90 | 2.95 | 3.69 | 0.54 |
| 100 | 100 | 100.00 | 100 | 200 | 300 | 1.00 | 2.00 | 3.00 | 3.90 | 0.51 |
| 100 | 110 | 104.88 | 100 | 220 | 320 | 0.95 | 2.10 | 3.05 | 4.19 | 0.48 |

| | | | | | | | | | | |
|-----|-----|--------|-----|-----|-----|------|------|------|------|------|
| 100 | 120 | 109.54 | 100 | 240 | 340 | 0.91 | 2.19 | 3.10 | 4.29 | 0.47 |
| 100 | 130 | 114.02 | 100 | 260 | 360 | 0.88 | 2.28 | 3.16 | 4.47 | 0.45 |
| 100 | 140 | 118.32 | 100 | 280 | 380 | 0.85 | 2.37 | 3.21 | 4.65 | 0.43 |

b. (Repeated from the answer to (a): Once we have the q column filled in, we know that $AFC = 1.05$ when $q = 94.87$. This implies that $FC = 99.61$ for this level of output, or rounding to the nearest whole number, $FC = \$100$. (You are using the formula $AFC = FC/q$ and rearranging this to be $FC = AFC \cdot q$.) This implies that the price of a unit of capital is \$1.

Similar reasoning can be used with regard to the AVC information in the table: we know that $AVC = 1.10$ when $q = 54.77$. This implies that $VC = 60.25$ for this level of output, or rounding to the nearest whole number, $VC = \$60$. (You are using the formula $AVC = VC/q$ and rearranging this to be $VC = AVC \cdot q$.) This implies that the price of a unit of labor is \$2.

c. As output increases, the marginal product of labor decreases. This is due to the fact that you are adding additional units of labor to a fixed amount of capital. As you add labor to this fixed capital eventually output will increase but at a diminishing rate. In our table, output increases at a diminishing rate immediately.

d. As output increases, AFC decreases. This is because fixed costs do not change and since AFC is just FC/q , as q gets bigger and FC stays constant, the value of FC/q must be getting smaller. This is the spreading effect: you are spreading the cost of those fixed costs over bigger and bigger quantities, so this means that your average fixed cost of producing greater amounts of the product must fall.

e. As output increases the AFC is decreasing due to the spreading effect (see last answer). At the same time the variable costs are increasing due to the hiring of more labor. But, as you add more labor the MPL is decreasing: each successive unit of labor is less productive than the prior units of labor. This diminishing returns effect eventually overwhelms the spreading effect resulting in the average total cost rising as output increases. This means that the ATC curve will have a U-shape.

f. If the market price is \$4 for the good and the firm is a perfectly competitive firm, this means that the marginal revenue from selling one more unit of the good is \$4. The firm will want to produce where $MR = MC$. From the completed table we see that we have $MC = 3.9$ when $q = 100$ and $MC = 4.19$ when $q = 104.88$. So, let's calculate profit at these two marginal costs to see which one gives the firm greater profits. (An enterprising student might go back to their Excel file and add in the skipped levels of labor to see if they could get a MC figure that was even closer to 4. I will do that after going through the first pass at the answer.)

$MR = 4$, $MC = 3.9$ and $q = 100$

$TR = P \cdot q = 4(100) = \400

$TC = \$300$

Profit for firm = $400 - 300 = \$100$

$MR = 4$, $MC = 4.1$ and $q = 104.88$

$TR = P \cdot q = 4(104.88) = \419.52
 $TC = 320$
 Profit for firm = $419.52 - 320 = \$99.52$

The firm is better off producing $q = 100$ units than 104.88 units.

Here's the expanded table:

| K | L | q | FC | VC | TC | AFC | AVC | ATC | MC | MPI |
|-----|-----|--------|-----|-----|-----|------|------|------|------|------|
| 100 | 90 | 94.87 | 100 | 180 | 280 | 1.05 | 1.90 | 2.95 | 3.69 | 0.54 |
| 100 | 100 | 100.00 | 100 | 200 | 300 | 1.00 | 2.00 | 3.00 | 3.90 | 0.51 |
| 100 | 101 | 100.50 | 100 | 202 | 302 | 1.00 | 2.01 | 3.01 | 4.01 | 0.50 |
| 100 | 102 | 101.00 | 100 | 204 | 304 | 0.99 | 2.02 | 3.01 | 4.03 | 0.50 |
| 100 | 103 | 101.49 | 100 | 206 | 306 | 0.99 | 2.03 | 3.02 | 4.05 | 0.49 |
| 100 | 104 | 101.98 | 100 | 208 | 308 | 0.98 | 2.04 | 3.02 | 4.07 | 0.49 |
| 100 | 105 | 102.47 | 100 | 210 | 310 | 0.98 | 2.05 | 3.03 | 4.09 | 0.49 |
| 100 | 106 | 102.96 | 100 | 212 | 312 | 0.97 | 2.06 | 3.03 | 4.11 | 0.49 |
| 100 | 107 | 103.44 | 100 | 214 | 314 | 0.97 | 2.07 | 3.04 | 4.13 | 0.48 |
| 100 | 108 | 103.92 | 100 | 216 | 316 | 0.96 | 2.08 | 3.04 | 4.15 | 0.48 |
| 100 | 109 | 104.40 | 100 | 218 | 318 | 0.96 | 2.09 | 3.05 | 4.17 | 0.48 |
| 100 | 110 | 104.88 | 100 | 220 | 320 | 0.95 | 2.10 | 3.05 | 4.19 | 0.48 |
| 100 | 120 | 109.54 | 100 | 240 | 340 | 0.91 | 2.19 | 3.10 | 4.29 | 0.47 |

Now, we can consider two possibilities:

$MR = 4$, $MC = 3.9$ and $q = 100$

This possibility we already analyzed and found that the firm's profits were \$100.

Or, $MR = 4$, $MC = 4.01$ (so very close to 4 and closer than 3.9) and $q = 100.5$

$TR = P \cdot q = 4(100.5) = \402

$TC = 302$

Profit for firm = \$100

We could keep doing this by considering smaller units of labor like labor equal to 100.1, 100.2, ...and see where the profit is absolutely maximized. That will occur when $MR = MC$. (I actually expanded my Excel file and saw that the firm's profits were bigger at 100.15 units of output where labor is 100.3 units. That is the pleasure of Excel: you can easily consider expanding your "thought experiments".)

Part IV: Perfect Competition in Short-run and Long-run

7. Consider the market for widgets. This is a perfectly competitive market where all the firms are identical. Each firm has identical cost curves where q is the quantity of widgets:

$$TC \text{ for each firm: } TC = q^2 + 9q + 9$$

$$MC \text{ for each firm: } MC = 2q + 9$$

The market demand curve is given by the following equation where P is the price per widget and Q is the total number of widgets produced in the market.

$$\text{Market Demand: } P = 559 - Q$$

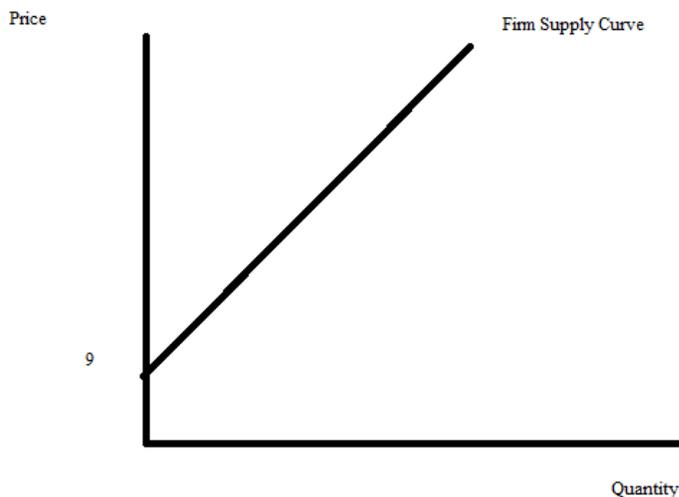
Initially there are twenty firms in the market.

- Find the short run market supply curve given the above information.
- Given the short run market supply curve you found in (a) and the market demand curve, find the short run market equilibrium quantity and the market equilibrium price. Then calculate the quantity produced by the representative firm. Then, compute the representative firm's profit in the short run. Finally make a prediction about what will happen in the long run. Explain the logic behind your prediction. Show how you found all of these answers.
- Given the market demand curve and cost curves, find the long run market equilibrium price, the level of production by the representative firm, and the market equilibrium quantity. Then calculate the total number of firms in the industry in the long run. Show your work for each of these steps. (It is okay to have a "partial firm" in your answer!)

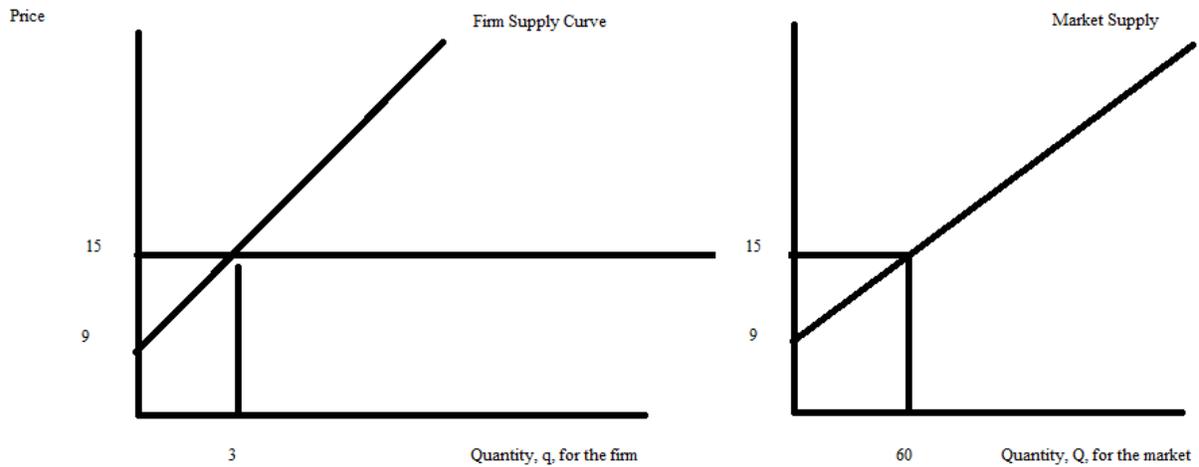
Answers:

a. $P = (1/10)Q + 9$

To see this start by drawing a sketch of the individual firm supply curve:



We have one point $(q, P) = (0, 9)$ on this curve. But we need a second point. So consider another q : for example, if $q = 3$ then $MC = 15$. So the point $(q, P) = (3, 15)$ sits on this line. Then think about twenty firms, all identical. If one firm is willing to supply 3 units at a price of \$15 per unit, then twenty firms will be willing to supply 60 units (3 units per firm * 20 firms) at a price of \$15 per unit. Here is the graph representing this idea:



We now have two points on the market supply curve: $(Q, P) = (0, 9)$ and $(60, 15)$. The slope of this market supply curve is $1/10$, and the y-intercept is 9. So, the equation for the short run market supply curve is $P = (1/10)Q + 9$.

Alternatively, you can rewrite the individual firm supply curve in x-intercept form:

$$P = 2q + 9$$

$$2q = P - 9$$

$$q = (1/2)P - 4.5$$

The short run market supply curve can be found by recognizing that there are twenty firms in the market:

$$Q = (20)[(1/2)P - 4.5]$$

$$Q = 10P - 90$$

$$10P = Q + 90$$

$$P = (1/10)Q + 9$$

b. We have the market supply and market demand curves:

$$\text{Market demand: } P = 559 - Q$$

$$\text{Market supply: } P = (1/10)Q + 9$$

Set these two equations equal to one another and solve for the solution:

$$559 - Q = (1/10)Q + 9$$

$$550 = (11/10)Q$$

$$Q = 500 \text{ widgets}$$

$$P = 559 - 500 = \$59 \text{ per widget or}$$

$$P = (1/10)(500) + 9 = \$59 \text{ per widget}$$

$$\text{Equilibrium quantity and price in the market} = (Q, P) = (500, \$59)$$

Each representative firm will produce where $MR = MC$.

Since this is a perfectly competitive market the equilibrium price in the market is also the firm's

$$MR: MR = 59$$

$$\text{So, } MR = MC$$

$$59 = 2q + 9$$

$$2q = 50$$

$$q = 25 \text{ widgets per firm}$$

You can also check that 20 firms each making 25 widgets results in a total of 500 widgets being produced.

$$\text{Profit for the firm} = \text{TR} - \text{TC}$$

$$\text{TR} = P \cdot q = (\$59 \text{ per widget})(25 \text{ widgets}) = \$1475$$

$$\text{TC} = q^2 + 9q + 9 = (25)(25) + 9(25) + 9 = 625 + 225 + 9 = \$859$$

$$\text{Profit for the firm} = 1475 - 859 = \$616$$

Since the firms are earning positive economic profit in the short run we can predict that new firms will enter the market in the long run so that profits are driven to zero.

c. In the long run the firm must make zero economic profit and that implies that $MC = ATC$. So, let's start by finding the equations for these two cost curves:

$$ATC = \text{TC}/q = q + 9 + (9/q)$$

$$MC = 2q + 9$$

Set these two equations equal to one another:

$$q + 9 + (9/q) = 2q + 9$$

$$9q = q$$

$$q \cdot q = 9$$

$$q = 3 \text{ widgets}$$

So the long run breakeven quantity is 3 widgets per firm. What price will the firm's sell the widget for? We know that $MC = MR = P$ for perfectly competitive firms, so we can find the price by substituting $q = 3$ into the MC curve. $MC = 2(3) + 9 = \$15$ per widget. So the long run equilibrium price for widgets will be \$15 per widget.

At this price, how many widgets are demanded in the market?

$$P = 559 - Q$$

$$15 = 559 - Q$$

$$Q = 544 \text{ widgets}$$

How many firms will there be in the market in the long run?

$$Q/q = \text{number of firms in the market}$$

$$544 \text{ widgets} / 3 \text{ widgets} = 181.33 \text{ firms}$$

8. Consider the market for coffee shops in Madison. The city has so many coffee shops that that market can be seen as perfectly competitive. Each coffee shop can serve at most 90 cups of coffee per day. Assume for mathematical simplicity that the marginal cost of serving a cup of coffee is constant and equal to \$2. The market demand for coffee in a day is given by the equation:

$$Q = 1000 - 5P$$

a. How much is a cup of coffee in this perfectly competitive market?

- b. How many cups of coffee will be sold in Madison every day?
- c. How many coffee shops will be in Madison?

Answers:

a. At the perfectly competitive equilibrium, $P = MC$. Hence, since $MC = \$2$ per cup of coffee, the price of a cup of coffee must be $P = \$2$.

b. Since $P = \$2$ by (a), plugging it into the demand function,
 $Q = 1000 - 5 * 2 = 990$ cups of coffee per day

c. Since each coffee shop can serve at most 90 cups of coffee in a day, the number of coffee shops needed is:

$$Q/100 = 990/90 = 11 \text{ coffee shops}$$

Part V: Monopoly

9. Polly owns a monopoly in her small town. She knows that the market demand curve for her product is given by the following equation where P is the price per unit of the good and Q is the number of units of the good:

$$\text{Market Demand: } P = 330 - 2Q$$

Polly also knows her total cost equation and her marginal cost equation:

$$\text{Total Cost: } TC = (1/16)Q^2 + 5$$

$$\text{Marginal Cost: } MC = (1/8)Q$$

- a. What is the profit maximizing quantity and price for Polly given the above information? Show how you found your answer.
- b. What is the value of Polly's profits? Show how you found your answer.
- c. What is the value of consumer surplus when Polly acts as a monopolist in this market? Show how you found your answer.
- d. What is the value of producer surplus when Polly acts as a monopolist in this market? Show how you found your answer.
- e. What is the deadweight loss due to this firm acting as a monopolist? Show how you found your answer.
- f. Draw a graph that illustrates this monopolist's situation. In your graph include the monopolist's demand curve, marginal revenue curve, marginal cost curve. Label the monopolist's price and the monopolist's quantity in your graph. Shade and label the areas that correspond to consumer surplus, producer surplus, and deadweight loss for the monopolist.

Answers:

a. To find the profit maximizing price we need to set $MR = MC$ to first find the profit maximizing quantity and then use this quantity and the demand curve to find the profit maximizing price.

$$MR = 330 - 4Q$$

$$MC = (1/8)Q$$

$$MR = MC$$

$$330 - 4Q = (1/8)Q$$

$$330 = (33/8)Q$$

$$Q = 80 \text{ units}$$

$$P = 330 - 2Q = 330 - 2(80) = 330 - 160 = \$170 \text{ per unit}$$

b. Profits = Total Revenue - Total Cost

$$\text{Total Revenue} = P * Q = (\$170 \text{ per unit})(80 \text{ units}) = \$13,600$$

$$\text{Total Cost} = (1/16)Q^2 + 5 = (1/16)(80)(80) + 5 = 5(80) + 5 = \$405$$

$$\text{Profits} = \$13,600 - \$405 = \$13,195$$

c. Consumer Surplus = $CS = (1/2)(330 - 170)(80) = (1/2)(160)(80) = \6400

d. Producer Surplus = $PS = (1/2)(10)(80) + (170 - 10)(80) = 400 + 12,800 = \$13,200$

e. Deadweight Loss = $DWL = (1/2)(170 - 10)(155.29 - 80) = \6023.20

f.

