

Economics 101

Fall 2012

Answers to Homework #1

Due 9/18/12

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered**, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. **Please show your work; otherwise you will not receive full credit.** Good luck!

1. The population of Gotham was 100,000 in 2012.

- a. In 2013, the population is 110,000. What is the percentage change in the population from 2012 to 2013?

$$\% \Delta = \frac{\text{Pop}(2013) - \text{Pop}(2012)}{\text{Pop}(2012)} \times 100 = \frac{110,000 - 100,000}{100,000} \times 100 = 10\%$$

- b. Suppose the population increased by 30% from 2012 to 2013. What is the population of Gotham in 2013?

$$\begin{aligned}\text{Pop}(2013) &= \text{Pop}(2012) + \frac{30}{100} \times \text{Pop}(2012) = 100,000 + \frac{30}{100} \times 100,000 \\ &= 130,000\end{aligned}$$

- c. Suppose the population of Gotham has grown by 25% from 2011 to 2012. What was the population in 2011?

$$\begin{aligned}\text{Pop}(2012) &= \text{Pop}(2011) + \frac{25}{100} \times \text{Pop}(2011) \\ \text{Pop}(2012) &= \frac{125}{100} \times \text{Pop}(2011) \\ \text{Pop}(2011) &= \frac{100}{125} \times \text{Pop}(2012) = \frac{100}{125} \times 100,000 = 80,000\end{aligned}$$

2. Answer the following questions:

- a. (4, 3) and (3, 4) are two points on the same line. What is the equation of this line?

$$\begin{aligned}y &= mx + b \\ m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{3 - 4} = -1 \\ y &= -x + b \rightarrow b = y + x = 4 + 3 = 7 \\ y &= -x + 7\end{aligned}$$

- b. Suppose the equation of a line is given by $y = 10 + 2x$. Also suppose a point on this same line has coordinates (6, z). What is z?

$$z = 10 + 2 \times 6 = 22$$

- c. The point (5, 15) is on a line whose equation is given by $y = k + 3x$. What is k? If you know that a point on this line has coordinates (p, 30), what is p?

$$y = k + 3x \rightarrow k = y - 3x = 15 - 3 \times 5 = 0$$

$$y = 3x$$

$$30 = 3 \times p \rightarrow p = 10$$

- d. What are the coordinates of the intersection point of the lines given by $y = 20 - 6x$ and $y = 4x - 20$?

To find the intersection point, simply set the two equations equal:

$$20 - 6x = -20 + 4x$$

$$10x = 40$$

$$x = 4$$

$$y = 20 - 6 \times 4 = -4$$

The intersection point has coordinates (4, -4).

- e. Consider the two different temperature scales, Fahrenheit (F) and Celsius (C). They are related by the following equation $F = 32 + 1.8C$. Suppose the temperature increases from 212 F to 221 F. What is the percentage change in the temperature in Celsius?

First, transform the relationship equation by moving C to the left side and everything else to the right side:

$$F = 32 + 1.8C \rightarrow C = (5/9)F - (160/9)$$

Now, transform both temperatures from F to C:

$$C_1 = (5/9)*212 - (160/9) = 100$$

$$C_2 = (5/9)*221 - (160/9) = 105$$

Then, find the percentage change:

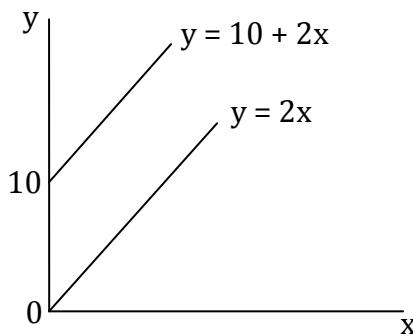
$$\% \Delta = [(C_2 - C_1)/C_1] \times 100 = [(105 - 100)/100] \times 100 = 5\%$$

3. Consider the line given by the equation $y = 10 + 2x$.

- a. Suppose you want to shift each point on this line down by 10 units. What is the equation of the new line? Does it intersect the original line? Draw the original and the new line on one graph.

Shifts up and down are shifts along the y-axis. Thus, what we want to do is decrease the y-coordinate of each point on the line by 10 units. This is the same as

$y_{\text{new}} = 10 + 2x - 10 \rightarrow y_{\text{new}} = 2x$. The two lines have identical slopes, so they are parallel, i.e., they do not intersect.

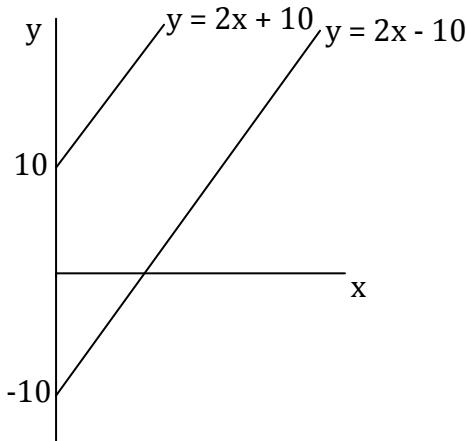


- b. Suppose you want to shift each point on this line to the right by 10 units. What is the equation of the new line? Does it intersect the original line? Draw the original and the new line on one graph.

Shifts to the right and left are shifts along the x-axis. So, to shift right, we want to increase the x-coordinate of each point on the line by 10 units. Thus, we need to first transform the equation so that x is on the left side and everything else is on the right side; then, increase each x-coordinate by 10; and finally, transform the equation back to having y on the left side. The procedure is the following:

$$\begin{aligned}y &= 10 + 2x \\2x &= y - 10 \\x &= 0.5y - 5 \\x_{\text{new}} &= 0.5y - 5 + 10 \\x_{\text{new}} &= 0.5y + 5 \\0.5y &= x_{\text{new}} - 5 \\y_{\text{new}} &= 2x_{\text{new}} - 10\end{aligned}$$

Again, the old and the new line have the same slope, so they do not intersect.



4. Suppose two friends—Jon and Sam—live in a deserted fortress in the bitter cold North. They need to produce swords and axes in order to defend themselves from wild animals. Jon has to work for 2 hours to produce a sword and for 5 hours to produce an axe. Sam needs 10 hours to produce a sword, but only 1 hour to produce an axe. Every day, each friend devotes 20 hours to the production of the two weapons. Use this information to answer the following questions.

- a. How many hours does Jon need to produce 5 swords? How many hours does Sam need to produce 5 swords?

Jon needs 2 hours to produce 1 sword, so he needs $2 \times 5 = 10$ hours to produce 5 swords. Sam needs 10 hours to produce 1 sword, so he needs $10 \times 5 = 50$ hours to produce 5 swords.

- b. How many axes can Jon produce in 1 hour? How many axes can Jon produce in 10 hours?

Jon can produce 1 axe in 5 hours, so he can produce $1/5$ axe in 1 hour and $10/5 = 2$ axes in 10 hours.

- c. Assuming Jon only works on swords, how many swords can he produce in 20 hours?
 Assuming Jon only works on axes, how many axes can he produce in 20 hours?
 If Jon only works on swords and given that he needs 2 hours to produce 1 sword, then he can produce $20/2 = 10$ swords in 20 hours.
 If Jon only works on axes and given that he needs 5 hours to produce 1 axe, then he can produce $20/5 = 4$ axes in 20 hours.
- d. Assuming Sam only works on swords, how many swords can he produce in 20 hours?
 Assuming Sam only works on axes, how many axes can he produce in 20 hours?
 If Sam only works on swords and given that he needs 10 hours to produce 1 sword, then he can produce $20/10 = 2$ swords in 20 hours.
 If Sam only works on axes and given that he needs 1 hour to produce 1 axe, then he can produce $20/1 = 20$ axes in 20 hours.
- e. Fill out the following table:

	Opportunity cost of 1 sword (in terms of axes)	Opportunity cost of 1 axe (in terms of swords)
Jon	2/5 axes	5/2 swords
Sam	10 axes	1/10 swords

For Jon: From part d, we know that in 20 hours, Jon can either produce 10 swords or 4 axes. In other words, the opportunity cost of 10 swords is 4 axes, because in order to produce 10 swords, Jon has to give up 4 axes. Now, cross-multiply to get the opportunity cost of 1 sword in terms of axes:

$$10 \text{ swords} = 4 \text{ axes}$$

$$1 \text{ sword} = ?$$

$$\Rightarrow ? = (1 \times 4)/10 = 4/10 = 2/5 \text{ axes}$$

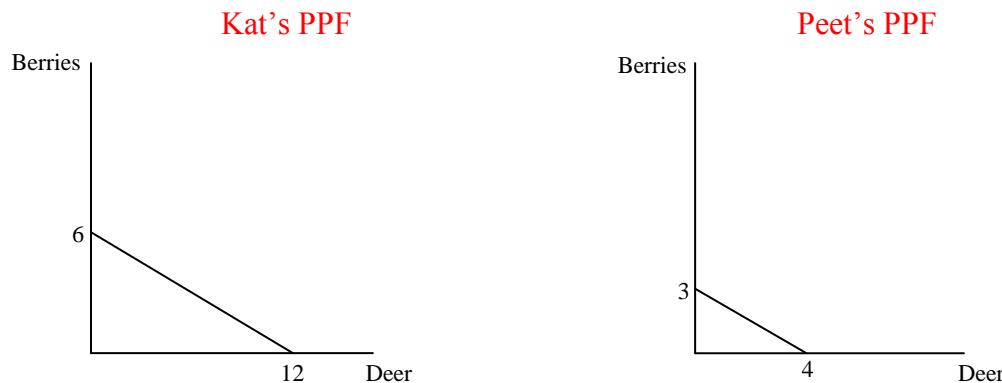
Similarly, the opportunity cost of 4 axes is 10 swords and cross-multiplying gives that the opportunity cost of 1 axe is $10/4$ or $5/2$ swords.

The calculation for Sam is analogous.

- f. Who has the absolute advantage in producing swords? Who has the absolute advantage in producing axes?
 Since in the allotted time Jon can produce 10 swords and Sam can produce 2 swords, Jon has the absolute advantage in the production of swords.
 Since in the allotted time Jon can produce 4 axes and Sam can produce 20 axes, Sam has the absolute advantage in the production of axes.
- g. Who has the comparative advantage in producing swords? Who has the comparative advantage in producing axes?
 Since Jon has a smaller opportunity cost for producing swords in terms of axes ($2/5 < 10$), Jon has the comparative advantage in producing swords.
 Since Sam has a smaller opportunity cost for producing axes in terms of swords ($1/10 < 5/2$), Sam has the comparative advantage in producing axes.
5. Another pair of friends—Kat and Peet—live in an isolated forest. They can only feed themselves by hunting deer and gathering berries. The following table shows how many hours each friend needs to catch one deer or gather a pound of berries. Assume Kat and Peet each have 12 hours every day to hunt and gather.

	Hours of labor needed to catch one deer	Hours of labor needed to gather one pound of berries
Kat	1 hour of labor	2 hours of labor
Peet	3 hours of labor	4 hours of labor

- a. Using the above information, draw the production possibilities frontier (PPF) of each friend on two separate graphs. Make sure to label your axes, putting deer on the x-axis and berries on the y-axis.



For Kat: The PPF gives the boundary of possible production bundles. Thus, the intercepts of the PPF with the two axes show the maximum amount of each good that can be produced if Kat devotes her time only to the production of this good. So, if she only hunts and given that she can hunt 1 deer in 1 hour, in 12 hours Kat can catch 12 deer. Similarly, given that she can gather 1 pound of berries in 2 hours, in 12 hours Kat can gather $12/2 = 6$ pounds of berries.

The calculation for Peet is analogous.

- b. Who has the absolute advantage in hunting deer? Who has the absolute advantage in gathering berries?

Kat has the absolute advantage in the production of both goods. In their allotted time, Kat can catch more deer ($12 > 4$) and gather more berries ($6 > 3$) than Peet.

- c. Who has the comparative advantage in hunting deer? Who has the comparative advantage in gathering berries?

To answer this question, you need to calculate the opportunity costs of both friends for both goods. Using the algorithm from the previous exercise, we know that in 12 hours Kat can either hunt 12 deer or gather 6 pounds of berries. Thus, her opportunity cost for 1 deer is $6/12 = 1/2$ pounds of berries and her opportunity cost for 1 pound of berries is $12/6 = 2$ deer. Similarly, we know that Peet can either hunt 4 deer or gather 3 pounds of berries. So, his opportunity cost for 1 deer is $3/4$ pounds of berries and his opportunity cost for 1 pound of berries is $4/3$ deer.

Kat has a lower opportunity cost for deer in terms of berries than Peet ($1/2 < 3/4$), so Kat has the comparative advantage in hunting deer. Peet has a lower opportunity cost for berries in terms of deer than Kat ($4/3 < 2$), so Peet has the comparative advantage in gathering berries.

- d. What is the acceptable range of trading prices for 1 deer in terms of pounds of berries? Would both sides agree to trade if the price is 1 pound of berries for 1 deer?

Given the comparative advantages established above, it makes sense that Kat would specialize in deer and trade her deer for berries and that Peet would specialize in berries and trade his berries for deer.

For Kat the opportunity cost of 1 deer is $1/2$ pounds of berries. For Peet the opportunity cost of 1 deer is $3/4$ pounds of berries. Thus, Kat would not trade 1 deer for less than $1/2$ pound of berries (otherwise, she can just give up hunting that 1 deer and gather berries instead) and Peet would not give more than $3/4$ pounds of berries for 1 deer (otherwise, he can simply hunt the deer himself). So, the acceptable range of prices for 1 deer in terms of pounds of berries is $(1/2, 3/4)$.

Given that acceptable range, Peet would not agree to the trade, since $1/2 < 3/4 < 1$.

- e. What is the acceptable range of trading prices for 1 pound of berries in terms of deer?

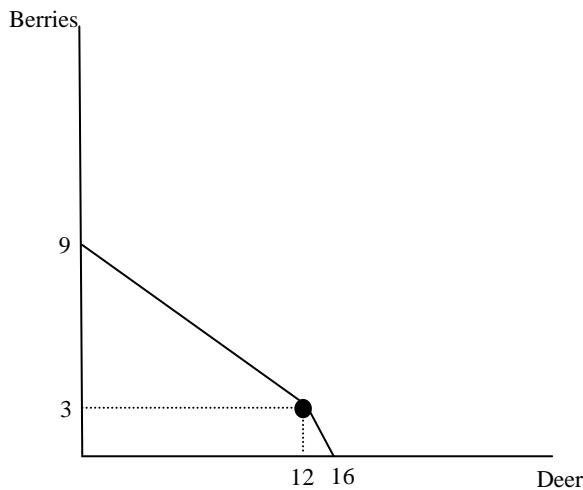
Would both sides agree to trade if the price is $1/2$ deer for 1 pound of berries?

For Kat the opportunity cost of 1 pound of berries is 2 deer. For Peet the opportunity cost of 1 pound of berries is $4/3$ deer. Thus, Kat would not give more than 2 deer for a pound of berries and Peet would not trade his berries for less than $4/3$ deer. So, the acceptable range of trading prices for berries in terms of deer is $(4/3, 2)$.

Given that acceptable range, Peet would not agree to a trade if the price is $1/2$ deer for 1 pound of berries, since $1/2 < 4/3$.

- f. On a third graph, draw the joint PPF of Kat and Peet—that is, their PPF if they decide to specialize and combine their production.

Kat and Peet's Combined PPF



If they both only hunt deer, they can hunt a total of $12 + 4 = 16$ deer. If they decide to also gather berries, the first person to start gathering berries is Peet, since he has the comparative advantage in berries. Then, if Peet is only gathering berries and Kat is only hunting deer, they can have a total of 12 deer (Kat's maximum) and 3 pounds of berries (Peet's maximum). If they both only gather berries, they can gather a total of $6 + 3 = 9$ pounds of berries.