Part I: Real vs. Nominal

1) Suppose the nominal prices over time for the following goods in some fictional city are given by the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Price per Cup of Diet Drinks</th>
<th>Price per Pizza</th>
<th>Price per TV</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>$1.75</td>
<td>$10.00</td>
<td>$165</td>
</tr>
<tr>
<td>2015</td>
<td>$1.50</td>
<td>$10.50</td>
<td>$170</td>
</tr>
<tr>
<td>2016</td>
<td>$1.65</td>
<td>$11.00</td>
<td>$155</td>
</tr>
<tr>
<td>2017</td>
<td>$2.00</td>
<td>$11.50</td>
<td>$150</td>
</tr>
</tbody>
</table>

Suppose a typical consumer basket throughout the year consists of 200 cups of diet drinks, 25 Pizzas, and 1 TV.

a. Using the above information to calculate the cost of the market basket for each of the years and present your calculations in the table below:
<table>
<thead>
<tr>
<th>Year</th>
<th>Cost of Market Basket</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>$765</td>
</tr>
<tr>
<td>2015</td>
<td>$732.5</td>
</tr>
<tr>
<td>2016</td>
<td>$760</td>
</tr>
<tr>
<td>2017</td>
<td>$837.5</td>
</tr>
</tbody>
</table>

Solution:
Cost of Market Basket in Year $n = (\text{Price of diet drink in Year } n) \times (200 \text{ cups of diet drinks}) + (\text{Price of pizza in Year } n) \times (25 \text{ Pizzas}) + (\text{Price of TV in Year } n) \times (1 \text{ TV})$

<table>
<thead>
<tr>
<th>Year</th>
<th>CPI</th>
<th>Inflation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>2015</td>
<td>95.75</td>
<td>-4.25%</td>
</tr>
<tr>
<td>2016</td>
<td>99.35</td>
<td>3.76%</td>
</tr>
<tr>
<td>2017</td>
<td>109.48</td>
<td>10.20%</td>
</tr>
</tbody>
</table>

b. Let 2014 be the base year, calculate the CPI for each year using a 100-point scale. Then, for 2015 to 2017, calculate the annual inflation rate. Calculate your answers to two places past the decimal.

Solution:
CPI for year $n = \left( \frac{\text{Price of basket in year } n}{\text{Price of basket in base year}} \right) \times 100$

\% Inflation = \left( \frac{\text{CPI this year} - \text{CPI last year}}{\text{CPI last year}} \right)
c. 2014 is still the base year. Calculate the real price of pizza in each year. Again, calculate your answers to two places past the decimal.

<table>
<thead>
<tr>
<th>Year</th>
<th>Real price of pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td></td>
</tr>
<tr>
<td>2017</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
Real price = (Nominal Price / CPI) * 100

<table>
<thead>
<tr>
<th>Year</th>
<th>Real price of pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>$10.00</td>
</tr>
<tr>
<td>2015</td>
<td>$10.97</td>
</tr>
<tr>
<td>2016</td>
<td>$11.07</td>
</tr>
<tr>
<td>2017</td>
<td>$10.50</td>
</tr>
</tbody>
</table>

Part II: Elasticity

2) This set of questions focuses on elasticity.

a. Suppose Howard’s demand for hamburgers decreases from 8 hamburgers to 5 hamburgers when the price increases from $4 per hamburger to $6 per hamburger. What is his price elasticity of demand for pizza? Use the standard formula for the percentage change to calculate this value.

Solution:
Regular percentage elasticity of demand formula is
\[ e = \left| \frac{\% \text{ change in demand}}{\% \text{ change in price}} \right| = \left| \frac{((5 - 8) / 8)}{((6 - 4) / 4)} \right| = 3/4 \]

b. Although the price of a hamburger remains at $6 per hamburger, Howard’s demand for hamburgers increases from 5 hamburgers to 8 hamburgers when his hourly wage rises from $20 to $36. What is his income elasticity of demand for hamburgers? Use the standard formula for the percentage change to calculate this income elasticity. Are hamburgers normal or inferior goods for Howard?

Solution:
Income elasticity of demand = (% change in demand / % change in income)
\[ = \frac{((8 - 5) / 5)}{((36 - 20) / 20)} \]
\[ = 3/4 \]
Since the income elasticity of demand is a positive number this tells us that hamburgers are a normal good for Howard: when Howard’s income increases, his demand for hamburgers also increases.

c. John, a seller of hamburgers, sells hamburgers to Howard for $6 per hamburger. John has observed that Howard’s demand for hamburgers decreases from 8 hamburgers to 5 hamburgers when the price of sandwiches decreases from $5 per sandwich to $4 per sandwich. What is his cross-price elasticity of hamburgers for sandwiches? Use the arc elasticity formula concept when calculating this cross-price elasticity. Based upon your value for the cross-price elasticity of demand of hamburgers for sandwiches, are these two goods substitutes or complements? Explain your answer.

Solution:
The arc elasticity of demand formula is: 
\[ e = \frac{(Q_2 - Q_1)}{(Q_1 + Q_2)} \div \frac{(P_2 - P_1)}{(P_1 + P_2)} \]
Then we get:
Cross-price elasticity of demand = \[ \frac{(5 - 8)}{(5 + 8)} \div \frac{(4 - 5)}{(5 + 4)} \]
\[ = \frac{-3}{13} \div \frac{-1}{9} \]
\[ = \frac{27}{13} \]
Since the cross-price elasticity of demand of hamburgers for sandwiches is positive this tells us that these two goods are substitutes: when the price of sandwiches falls, Howard substitutes away from hamburgers: the quantity of hamburgers he demands at every price decreases relative to his initial demand.

d. Suppose at $6 per hamburger, John can supply an infinite quantity of hamburgers but he will supply none at a price below $6. What do you know about his supply when price rises above $6? What is John’s price elasticity of supply?

Solution:
At any price above $6 per hamburger, the quantity that John supplies is extremely large. Then we know John’s price elasticity of supply equals $\infty$, which means he has a perfectly elastic supply. When even a tiny increase or reduction in the price will lead to very large changes in the quantity he supplied, so that the price elasticity of supply is infinite.

e. When the price of hamburgers is $8 per hamburger, John can sell 45 hamburgers in a day. We know when the price of hamburgers decreases to $6 per hamburger, John’s total revenue remains unchanged. Given this information and assuming that John’s demand curve for hamburgers is linear, what do we know about the price elasticity of demand for his hamburgers? Use the standard formula for the percentage change to calculate this elasticity value. Given this information write an equation for this demand curve in slope-intercept form. Once you find the demand curve provide an explanation for why John’s total revenue is not changing as the price of hamburgers falls from $8 per hamburger to $6 per hamburger.
Solution:
Since John’s total revenue remains unchanged, we know the demand for his hamburger at P = 6 must equals 45*8/6 = 60 hamburgers.
Regular percentage elasticity of demand formula is:
e = \left| \frac{\% \text{ change in demand}}{\% \text{ change in price}} \right| = \left| \frac{(60 - 45)}{45} / \frac{(6 - 8)}{8} \right| = 4/3
We now know two points on the demand curve that John faces: (Q, P) = (45, 8) and (60, 6).
From these two points, we can calculate the slope of the demand curve: slope = (8 - 6)/(45 - 60) = -2/15. Then use the standard y-intercept form: y = mx + b to find the demand curve. Thus,
Y = mX + b
P = (-2/15) Q + b
Plug in one of our known points to find the value of b:
6 = (-2/15) (60) + b
14 = b
Equation for the demand curve: P = 14 – (2/15) Q
When the price of hamburgers falls from $8 per hamburger to $6 per hamburger John’s revenue does not fall because we are moving symmetrically around the unit elastic point on this linear demand curve. With a linear demand curve, we know that total revenue is maximized at the midpoint: in this example, the midpoint is at (Q, P) = (52.5, 7) and John’s maximum total revenue is $367.50. Our two known points are equally distant from this midpoint.

Part III: Consumer Theory

3) The goal of this problem is to help you understand the idea of the income effect and the substitution effect.

Suppose a Wisconsin resident Alice only consumes two goods: bread (B) and rice (R). Her budget constraint and utility are given by the following formula:

Y = P_B B + P_R R
U = BR

Where Y is the amount of income that Alice has, P_B is the price of bread, P_R is the price of rice, B and R are the quantity that Alice consumes for bread and rice respectively. U is her utility from consuming these two goods.

a. Initially, Alice has Y = $40, and the prices of bread and rice are $4 and $1 respectively. Graph Alice’s budget constraint (BL1), with bread on the horizontal axis and rice on the vertical axis.

Solution: The budget constraint is 40 = 4B + R
b. At the initial price levels, Alice’s optimal consumption bundle is (B = 5, R = 20). Mark this point on your graph. What is Alice’s utility level from consuming this consumption bundle?

Alice’s utility level from consuming this bundle (B, R) = (5, 20) is equal to 

\[ U = BR = (5 \times 20) = 100. \]

c. Now, due to a special bread sale, the price of bread drops to $1. What is Alice’s new budget constraint (BL2)? Add this to your graph from part (a).

Solution: The budget constraint is 

\[ 40 = B + R \]

d. With this bread sale, Alice’s consumption bundle is (B = 20, R = 20). Add this point to your graph as well. What is Alice’s utility level from consuming this consumption bundle?
Alice’s utility level from consuming this bundle \((B, R) = (20, 20)\) is equal to \(U = BR = (20 \times 20) = 400\).

f. Twice this month Alice was late to work and this has resulted in her income decreasing this month. The bread sale is still on, but her new optimal consumption bundle is \((B = 10, R = 10)\). Given this information and holding everything else constant, what is her new monthly income? Add this new budget constraint, BL3, to your graph from part (a). What is Alice’s utility level from consuming this consumption bundle?

Solution: Her income now is \(P_B B + P_R R = 10 \times 1 + 10 \times 1 = 20\)

Alice’s utility level from consuming this bundle \((B, R) = (10, 10)\) is equal to \(U = BR = (10 \times 10) = 100\). Note: that Alice gets the same level of utility from the first bundle as she does from the third bundle: these two bundles must sit on the same indifference curve.
What is the income effect and the substitution effect if the price of bread decreases from $4 to $1 given Alice’s initial income is $40?

Solution: Notice that the consumption bundle in (f) and the consumption bundle in (b) give Alice the same level of utility. We know the change from (b) to (f) should be the substitution effect, so the substitution effect is B increases from 5 units of bread to 10 units of bread: the substitution effect is 5 units of bread. The income effect is measured as the change in bread consumption from 10 units of bread to 20 units of bread: the income effect is 10 units of bread.

4) With the same information as above, a Wisconsin resident Alice only consumes two goods: bread (B) and rice (R). Her budget constraint and utility are given by the formula:

\[ Y = P_B B + P_R R \quad U = BR \]

Given her current consumption bundle (B, R), her marginal utility from consuming bread is given by \( MU_B = R \) and her marginal utility from consuming rice is given by \( MU_R = B \).

a. Suppose Alice’s income is \( Y = $40 \), and the prices for bread and rice are \( P_B = $4, P_R = $1 \). What is her optimal consumption bundle? (You have to derive it instead of using the result of Question 3(a)). What is the value of Alice’s utility at this consumer optimization point? Show how you found this value.
Solution: If (B, R) is the optimal consumption bundle for Alice, then it must satisfy that \( \frac{MU_B}{MU_R} = \frac{P_B}{P_R} \), and \( Y = P_B B + P_R R \), so we have \( 40 = 4B + R \) and \( \frac{R}{B} = \frac{4}{1} \). Then we can solve that \( B = 5 \), \( R = 20 \), so her optimal consumption bundle is \( (B, R) = (5, 20) \). Her utility is \( U = BR = (5 \times 20) = 100 \).

b. Fill out all the missing information in the table

<table>
<thead>
<tr>
<th>Quantity of Bread</th>
<th>Quantity of Rice</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>100</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Quantity of Bread</th>
<th>Quantity of Rice</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>25</td>
<td>4</td>
<td>100</td>
</tr>
</tbody>
</table>
c. Graph the optimal consumption bundle in (a), with bread on the horizontal axis and rice on the vertical axis. Also graph the indifference curve, IC1, for $U = 100$ using the values you found in part (b).

Solution: see the graph in (d).

d. Draw the indifference curve, IC2, for $U = 400$ in this same graph. You might find it helpful to construct a similar table to part (b) with $U = 400$.

e. Now suppose the price level is $P_B = 1$, $P_R = 1$ and Alice’s income is now equal to $20$. Suppose Alice wants to achieve a utility level of $U = 100$. Given this information and holding everything else constant, what should be her optimal consumption bundle?

Solution: Again, if $(B, R)$ is the optimal consumption bundle for Alice, then it must satisfy that \[ \frac{MU_B}{MU_R} = \frac{P_B}{P_R}, \] which is equivalent to \[ \frac{R}{B} = \frac{1}{1}. \] Since she wants to achieve $U = 100$. Then we have $BR = 100$. The solution is $B = 10$, $R = 10$. Therefore, her optimal consumption bundle will be $(10, 10)$. 