# Queuing Markets (lectures 6+)

## 1 Where we are

- Last week, we discussed Walrasian-style markets:

  centralized markets for a single commodity good

- We look at two examples:

  U.S. stock markets, which use continuous limit-order books to process trades in real time,

  and electricity auctions, which aggregate firm bids into a supply function and use it to meet demand in real time

- In the first case, we saw that continuous-time trading creates an arms race to be a little bit faster than everyone else,

  to take advantage of arbitrage opportunities across markets,

  and that this can affect liquidity in a market;

  some smart guys proposed frequent batch auctions instead of continuous-time trading as a possible solution

- In the second case, we saw that firms exercise some market power in their bids –

  they bid above marginal cost when selling, and below marginal cost when buying –

  and that this has moderate, but not enormous, effects on productive efficiency

- Today, we move on to a different type of market:

  in fact, markets that people may not typically even think of as markets

- Any questions before we start?

## 2 When is a market not a market?

- There are certain things that certain people don't believe should be allocated based on price or willingness-to-pay
  - a meme that started as "No one should die because they cannot afford health care" was shared over a million times on Facebook a few years ago[1]
  - it's a nice sentiment – but taken literally, it suggests that medical care should not be something that's allocated based on peoples' willingness to pay a market-clearing price
  - there are lots of things that people believe shouldn't be freely bought and sold – and thus, things that shouldn't be allocated based on price
  - This is often described as there "not being a market for" something

  - I googled the phrase "there shouldn't be a market for" and got a bunch of hits
  - Some were statements of surprise that something was demanded at all – that there shouldn't be a market for other peoples' old Christmas photos, or for Tampa Bay backup QB Mike Glennon, who the Bucs were thinking of trading
  - But others were philosophical statements that certain things should not be for sale: valuable antiquities, human organs, citizenship, ivory, illegal arms, fossils, certain wild birds and fish, pets, certain drugs, human eggs and embryos, cloned foods
- Many of these things, it's illegal to buy and sell, of course
- And many people think this means there's "no market for" these things
- But that's not exactly true:

  we're thinking of markets as whatever determines the allocation of things,

  and these things do still end up getting allocated somehow
- I mentioned on day one: a friend used to TA a class on the world food economy, and told me the professor liked to say,

  "All markets clear; in this market, starvation is a form of market clearing."

[1] https://www.theatlantic.com/technology/archive/2014/01/no-one-should-die-because-they-cannot-afford-healthcare-one-memes-story/282920/

- When people say there shouldn't be a market for something,

  they usually mean that price shouldn't determine who gets it

- And when we say economists is pro-market,

  it often means that many of us think that price is typically a good way to determine who gets what

- But if price isn't what clears the market, what does determine who gets it?

- This week, we'll consider one answer:

  willingness to wait in line

# 3 Queuing Examples

## 3.1 Kryzewskiville

- Here's a picture of Kryzewskiville ("Shu-shev-ski-ville")[2]
- This is students waiting in line for several weeks,

  to get tickets to the Duke-UNC basketball game

- College basketball is insanely popular at Duke,

  but the basketball stadium only seats about 9,000 people
- If tickets were sold openly, the market clearing price would be very high,

  and students wouldn't get to go to the games
- So a certain number of seats – between 2000 and 3000 – are reserved for students

- Prior to 1986, students just got in line before the game to get in
- The line would form several hours before the game – especially the annual game with UNC –

  and once in a while, a group of students would come the night before with sleeping bags,

  to sleep out at the front of the line and get the best seats
- In 1986, a group of students showed up two days before the game and set up tents at the front of the line
- Ohter students found out, other groups of friends went and grabbed tents,

  and by the time doors opened, there were 75 tents set up in the line
- Since then, "tenting" before games – before the UNC game each year, and sometimes before another game as well – has become a tradition,

  with its own set of complicated rules

[2] Here's a good source: https://en.wikipedia.org/wiki/Krzyzewskiville

- This year, the Duke-UNC basketball game at Duke is February 20
- Tenting started on January 12
- So, six weeks before the game, groups of students set up tents in front of the basketball stadium
- Tent groups are up to 12 students;

  for the first two weeks, there need to be 2 students in the tent during the day, and 10 at night
- For the next two weeks, groups need to have 1 person in the tent during the day, and 6 each night;

  for the two weeks leading up to the game, 1 during the day and just 2 at night
- Tent groups have to register, and there are lots of additional rules, but the basic idea is,

  if you want to get into the Duke-UNC basketball game, you need to wait in line, at least part of the time, for six weeks
- (Just out of curiosity, since the game is in eight days,

  I checked Stubhub yesterday morning,

  and you could buy tickets starting about $1650 per seat.)

- So, here's a good – seats to a basketball game –

  with fixed supply and a lot of demand,

  but rather than letting the market-clearing price determine who gets a seat,

  at least part of the market clears based on who is willing to wait in line the longest

- Kryzzewskiville is a fun example, but we'll come back later to why it's atypical in some ways
- But there are other situations where people line up for goods that are over-demanded

### 3.2 Other examples

- Before the collapse of the Soviet Union, food shortages were common,

  and in the late 1980s and early 1990s, just before things changed,

  people in the U.S. started to see pictures of Soviet citizens waiting in enormous lines to get bread and other basic foods

- That is, rather than letting prices rise to equalize supply and demand,

  prices were kept low enough that everyone still wanted things,

  and then people had to wait in line to get them

- Less depressing than standing in line for Soviet bread,

  here are pictures of people waiting in line outside of Apple stores to get a new version of the iPhone on the day it's released

  (One from Tianjin, China, and two from New York)

- And going back a generation, here are pictures of people waiting in line – sometimes for more than a day – to see the new-release Star Wars movies

- In each case, the regular price was being charged, even though some people were obviously much more eager than others to buy the good,

  so those willing to wait the longest in line ended up getting it

- Kids in the Rotunda at the Overture Center

- Finally, here's a 2012 Wisconsin State Journal article

  when Obama was running for reelection and visited Madison a few days before the election

- The article starts out with a story of six friends who got in line at 4 a.m., twelve hours before the speech, and talks about the long wait to hear the speech –

  people getting rained on at noon,

  and officials limiting entry to people in line by 2 p.m., still almost two hours before the speech

- In this case, the price was zero, there was excess demand –

  and again, those willing to wait in line were the ones who got to hear Obama speak

## 4 So...

- lots of examples where price is either set at zero,

  or kept below the price that would equate demand and supply;

  and the good ends up going to whoever is willing to spend the most time waiting in line

- we can describe this as **"rationing by queuing"**

- When economists are described as being "pro-market,"

  this often comes down to the observation that most economists think price is a pretty good way to determine allocations

- And the reason for this is typically that when you don't use price,

  the alternative is often worse

- So, let's see why

- We're going to try a few different ways to model a rationing-by-queuing setting,

  starting with the simplest,

  then building up in complexity

## 5 Queuing Model 1

- Let's think about a free concert, with people lining up ahead of time to get in

- Obviously, for the allocation to even be a question,

  there need to be more people who would like to go

  than the venue can hold

## 5.1 working with a continuum of music fans

- Somewhat paradoxically, it ends up being easier to analyze this situation with a slightly more complicated model
- In particular, rather than using game theory and thinking about a venue that can hold a finite number of people,

  things are simpler if we think of an infinite number of people,

  competing to get into a venue that can hold a smaller, but still infinite, subset of them
- Specifically, we will think of a continuum of people –

  picture a part of the real line,

  but where each real number is a very tiny person
- We can think of, say, all the potential concertgoers as being laid out on the interval $[0, 2]$

  (with density 1, so there's a measure 1 of concertgoers)
- And the seats at the venue are laid out on the interval $[0, 1]$

- Working this way makes each concertgoer infinitessimal –

  it means each individual can't influence the overall distribution of what people are doing,

  which makes things easier to analyze
- (In fact, this is what we're used to doing with a demand curve –

  if we think about the demand for a good like cars, or refrigerators, which are desired in discrete quantities,

  we still draw a market demand curve as being smooth,

  and we assume people are price-takers – they assume that they can't influence the market-clearing price by misrepresenting their willingness to pay
- Here, working with a continuum of concertgoers has a similar effect –

  each person can choose when they get in line to try to see the show,

  but can't affect what time they would have to show up in order to get in

## 5.2 basic model

- So, our basic model
- A continuum of concertgoers with measure 2,

  and a continuum of concert seats of measure 1
- So if we think of $x \in [0, 2]$ being the people,

  and we let $f(x)$ be 1 if person $x$ goes to the concert and 0 if he or she doesn't,

  then $\int_0^2 f(x)dx$ is the measure of people who go to the concert,

  and feasibility is the constraint that

$$\int_0^2 f(x)dx \quad \leq \quad 1$$

- We'll assume the concert starts at time $T$, and at that point, we just let in the people who have been waiting in line the longest, up until the venue is full

- For the first model, all the people have identical preferences
- Each one gets a value $V$ if they get to attend the concert
- And each one pays a cost of $c$ per unit of time they have to wait in line
- So if they get in line at time $T - t$, that is, $t$ hours before the concert,

  and they get in to see the show,

  they get a payoff of $V - ct$
- If they wait in line but don't get in, they get payoff $-ct$,

  and if they never wait in line and don't see the show, they get 0

- Finally, let's assume that people can always tell how many people are already in line –

  at every point in time, whether or not I'm waiting, I can tell how long the line is, relative to the venue's capacity,

  so I know whether I'd get in if I got in line

### 5.3 equilibrium

- What is an equilibrium?
- Well, it's a decision for each concertgoer of what to do –

  whether to try to go to the concert, and what time to show up and get in line –

  such that each one is maximizing his own payoff, given what everyone else is doing
- or, a decision for each person,

  such that given what everyone else is doing,

  nobody can improve their payoff by changing strategies

- What does equilibrium look like here?
- Let's work it out in steps.

- **Claim 1.** In any equilibrium, nobody gets a payoff less than 0
  - If someone did, they could deviate and stay home

- **Claim 1'.** In any equilibrium, nobody gets in line more than $\frac{V}{c}$ before the concert starts
  - if you did, your cost of waiting would be more than $V$,

    so even if you got in, you'd get negative payoff
- **Claim 2.** In any equilibrium, some people get 0 payoff
  - This seems simple: there aren't enough seats for everyone to see the show,

    and those who don't get in must get a payoff of 0 or less[3]

[3] This seems to get more complicated, if you allow for randomness: if some people play randomly, for example, get in line at one time with probability $\frac{1}{2}$ and another time with probability $\frac{1}{2}$, then I could have a strategy that gives me a positive expected payoff, even though there's some chance I don't get into the show, so you could imagine this could be the case for everyone. But it turns out, this is impossible in equilibrium – there can't be an equilibrium of this game where every player gets positive expected payoff.

- **Claim 3.** At any time less than $\frac{V}{c}$ before the show starts, there must be a measure 1 of people already in line
  - If not, someone who was expecting a payoff of 0 could get in line right then, and earn positive payoff

- **Corollary.** In any equilibrium, a measure 1 of people get in line at time $T - \frac{V}{c}$, and nobody else gets in line after them.

- So that's the equilibrium outcome:

  everyone who's going to the show shows up $\frac{V}{c}$ before the show starts,

  gets benefit $V$ from seeing the concert,

  and pays $c\frac{V}{c} = V$ in cost from waiting in line
- So the concert is free, and nobody gets any utility from it
- This is called **full rent dissipation**[4]
  - "rents" are the excess value people get from getting into a free concert $(V - 0)$, and the point here is that these rents are fully eaten up by competition to get in

[4]For a more formal definition and discussion of rent dissipation, see, for example, Gordon Tullock (2004), "Rent Dissipation," in Rowley and Schneider (eds), *The Encyclopedia of Public Choice*, Springer (link)

## 5.4 What do we learn?

- Basically, since people can't "pay" with money, they pay with time
- So what's the problem?
- If tickets just cost $V$, nobody would get surplus from going to the show,

  but at least that money would go somewhere –

  the venue, or the band, would make some money,

  or could use it to buy something valuable for everyone (not just the people who attend the concert)
- But if people pay time instead of money,

  that expenditure is wasted –

  nobody benefits from people waiting in line.
- Economists like to call this "competing by burning money"
- Since there's excess demand, the concert needs to become less desireable for the market to clear;

  this is done by pairing the free concert with a wasteful activity (waiting in line)

- How could we improve on the situation?
- Well, right now, everyone's getting payoff 0,
  so almost any change would be a Pareto improvement
- Things that would be better:
  - Charging concertgoers $V$ to get in, and then sending every person a check for $\frac{V}{2}$
  - Giving tickets out randomly, without requiring people to stand in line
  - Either of these would be a Pareto improvement
- What about making it less costly to wait in line?
  - What if we installed an awning over the place where people lined up,
    so they wouldn't get rained on,
    and set up free wifi, so people were less inconvenienced by waiting in line?
  - This wouldn't help at all!
  - If we reduced $c$, the cost per time of waiting in line, by half,
    people would just have to wait twice as long in order for the market to clear,
    so this would do nothing!
- What about making the concert better?
  - Actually, if we increased $V$, this wouldn't accomplish anything,
    because the added value would just get competed away through longer wait times

## 6 Why is this a dumb model?

- (brainstorm)

- (what if different customers have different eagerness to attend concert?)
- (what if different customers have different cost of waiting in line?)
- (what if the venue has better seats and worse seats?)

- We're going to consider two of these complications
- What if customers have different value from attending the concert,
  and what if the venue has better and worse seats?
- Different costs of waiting in line is not that different from different value for the concert,
  except that it's easier to get an efficient outcome through random allocation,
  so I think different $V$ is more "interesting" than different $c$
  (although I might ask you to consider different $c$ on a future homework)
- For the two extensions – heterogeneous preferences for the concert,
  and heterogeneous goods (better and worse seats),
  we'll see what happens if we add either one to the model,
  and then see what happens if we add both

## 6.1 Model 2: heterogeneous concertgoers

- Suppose that concertgoes have different preferences
- We've already thought about associating each consumer with a point in the interval $[0, 2]$
- So now imagine each consumer $x \in [0, 2]$ gets benefit $V + \beta x$ from attending the concert
- And everyone has the same waiting costs, so payoffs for consumer $x$ are $V + \beta x - ct$ if she arrives at time $T - t$ and gets in,

  and 0 if she doesn't get in (and doesn't wait in line)

- What happens in this model?
- Not surprisingly, the consumers with higher $x$ are more willing to wait in line,

  so the consumers with $x < 1$ don't get in, and therefore must get 0 payoff in any equilibrium

- So now consider some consumer just below 1, i.e., $x = 1 - \epsilon$
- He would get value $V + \beta(1 - \epsilon)$ from getting into the show,

  which means if he got in line after time $T - \frac{V+\beta(1-\epsilon)}{c}$ and got into the show,

  he'd get strictly positive payoff

- This can't happen – in equilibrium, he has to get 0 –

  so this means by time $T - \frac{V+\beta(1-\epsilon)}{c}$, the line has to be full

  (measure 1 of consumers must already be in line)

- Since this is true for any $\epsilon$, this means that at any time after $T - \frac{V+\beta}{c}$, the line must be full

- What about before that?
- Well, nobody with $x < 1$ is willing to get in line before $T - \frac{V+\beta}{c}$,

  which means that everyone knows that if they show up any time at or before $T - \frac{V+\beta}{c}$,

  they'll get into the show;

  since all seats are the same, there's no reason to arrive before that

- So the equilibrium is,

  everyone with $x \geq 1$ gets in line at time $T - \frac{V+\beta}{c}$ and gets into the show,

  and everyone with $x < 1$ never gets in line

- So for anyone with $x > 1$,

  they get in line at time $T - \frac{V+\beta}{c}$,

  meaning they're in line for $\frac{V+\beta}{c}$ length of time,

  or they pay a waiting cost of $V + \beta$

- So $V + \beta$ is essentially the market-clearing "price" –

  everyone who goes to the show, "pays" $V + \beta$ to get in,

  and they get surplus of $V + \beta x - (V + \beta) = \beta(x - 1)$

- So the people who go to the show, do get some surplus

- But the equilibrium is still kind of lousy,

  because instead of the $V + \beta$ they all pay being money that someone gets to collect and do something with,

  it just gets set on fire

- Once again, charging the market-clearing price of $V + \beta$ (in money, rather than wait times),

  and then doing anything productive with the money

  (such as giving $\frac{V+\beta}{2}$ to everyone, or buying cocaine for the band)

  would be a Pareto-improvement

- Once again, decreasing $c$, or increasing $V$, would not help,

  as the "market-clearing" wait time would increase to compensate

- Interestingly, increasing $\beta$ *would* increase surplus –

  since the market-clearing wait time is based on the marginal consumer,

  something that increases concertgoers' enjoyment of the concert *relative to* the marginal guy ($x = 1$) would not get competed away;

  on the other hand, something that compressed preferences (say, increasing $V$ but decreasing $\beta$) would make people worse off

## 6.2 Model 3: heterogeneous seats

- what if some seats are better than others?
- let's go back (for now) to homogeneous tastes –

  every concertgoer has exactly the same preferences –

  but now suppose the venue has some better and some worse seats
- We've already thought about the seats being represented as the interval $[0, 1]$;

  now let $\theta \in [0, 1]$ (uniformly distributed) be the quality of a seat,

  and suppose that whoever sits in it gets value $V + \theta$ from attending the concert
- So any consumer gets payoff $V + \theta - ct$ from waiting in line for time $t$, attending the show, and sitting in seat $\theta$
- What happens in equilibrium?

- **Claim 1.** In any equilibrium, someone gets payoff 0.
  - Again, half the potential concertgoers can't get in, so they get 0 or worse, so they get 0
- **Claim 2.** In any equilibrium, everyone gets payoff 0.
  - If there was someone getting positive payoff by arriving at some particular time $t'$, anyone who was expecting a payoff of 0 could imitate them, arrive at $t'$, and expect a positive payoff,

    so it wouldn't be an equilibrium
- **Claim 3.** In any equilibrium, at any time $T - \frac{V+\theta}{c}$ ($\theta \in [0, 1]$), there are already a measure $1 - \theta$ of people in line.
  - Whoever gets seat $\theta$ gets surplus of $V + \theta$, so they must pay wait cost of $V + \theta$ to get 0 payoff;

    so they must have gotten in line $\frac{V+\theta}{c}$ before the concert
- So the equilibrium is,

  people start lining up at time $T - \frac{V+1}{c}$,

  and the line fills up gradually until time $T - \frac{V}{c}$,

  such that at every moment in time in between, if you arrive right then,

  you get a seat good enough to exactly compensate you for your wait time
- And in equilibrium, we once again get full rent dissipation

### 6.3 Model 4: both

- What if we combine these two complications –

  there are better and worse seats, and more and less eager fans?

- Let $\theta$ be the quality of a seat, and let $\theta$ vary uniformly over $[0, 1]$

- And let $x$ denote how eager a fan someone is;

  continue to suppose a measure 2 of consumers with $x$ uniform over $[0, 2]$

- When a customer with "type" $x$ gets a seat with quality $\theta$ and has been waiting in line since time $T - t$, his payoff is

$$V + \beta x \theta - ct$$

  where $V$ is everyone's value from the concert, and $\beta$ and $c$ are parameters (constants) we can mess with later.

- (Note that "more eager" concertgoers attach more value to a better seat, not to just getting into the concert at all;

  this ends up being the "more interesting" case, since the question is how to allocate each seat, not just who to let into the concert.)

- What does equilibrium look like?

- I claim the equilibrium is as follows:

  - Customers with types $x < 1$ don't show up
  - Customers with types $x \geq 1$ each show up $\frac{1}{c}\left(V + \frac{\beta}{2}(x^2 - 1)\right)$ before the concert starts

- Today, we'll show that this is indeed an equilibrium

- Thursday, I'll show you how I found it –

  that is, how to solve models like this

- Why is this an equilibrium?
- Equilibrium means that for each consumer $x$,

  if all the other consumers are following this strategy,

  it's a best-response for $x$ to follow this strategy as well

- So suppose I'm consumer $x$, and all my opponents are following this strategy
- This means that nobody gets in line before $T - \frac{V + \frac{3\beta}{2}}{c}$,

  and the line is "full" at time $T - \frac{V}{c}$,

  and the line fills gradually in between those two times, as more and more of the other consumers show up one by one

- Now, as consumer $x$, there are three things I could do:
  - show up before $T - \frac{V + \frac{3\beta}{2}}{c}$ – this can't be optimal, because there's nobody in line, and if I waited a little longer, I could still get the best seat but not have to wait as long
  - show up after $T - \frac{V}{c}$ – this is pointless, since I won't get in
  - show up at some time in between – in this case, for any time $t$ I arrive, there's some other consumer type $x'$ whose equilibrium strategy is to show up just then
- So instead of thinking of a time $t$ to show up, we can think about me choosing a consumer type $x'$ to adopt their equilibrium strategy
- If we think of the problem that way, then by playing the equilibrium strategy of $x'$, I get allocated the seat $\theta = x' - 1$,

  since arrival times are in order of decreasing types, so higher types get better seats

- So if my true type (preferences) are $x$, but I adopt the strategy of type $x'$, my payoff is

$$u(x, x') \quad = \quad V + \beta x(x' - 1) - c\frac{V + \frac{\beta}{2}((x')^2 - 1)}{c} \quad = \quad \beta x(x' - 1) - \frac{\beta}{2}((x')^2 - 1)$$

- Now, $x$ is outside of my control, but $x'$ is my choice,

  so we can think of me choosing $x'$ to maximize this

- And taking the derivative,

$$\frac{\partial u}{\partial x'} = \beta x - \beta x'$$

- So we get literally the easiest FOC in the world –

  if I'm choosing whose equilibrium strategy to adopt, I maximize by adopting my own,

  that is, setting $x' = x$,

  that is, I optimize by playing my own equilibrium strategy,

  so this strategy is an equilibrium.

**What's next?**

- Well, I didn't tell you how I found this equilibrium,

  I just claimed it was an equilibrium and then showed I was right

- Next time, I'll show you how I found it –

  that is, how we solve models like this