Auctions
(lectures 19-??)

1 Plan for today

- Midterm grades posted, HW3 as well
- I'll return bluebooks after lecture
- (Exam and solutions are on Canvas)

- Today, we move on from matching markets,
  and begin talking about auctions

- Any questions before we start?

2 Auctions

- Next few lectures will be on auctions
- Auctions go back surprisingly far in history –

  there are early examples from Roman times

  (women being sold to be wives, spoils of war, debtors' assets)
- Auctions became common again in Europe in the 1700s –

  the famous auctionhouses Sotheby's and Christy's, which still exist today,

  were both founded in London in the mid-1700s,

  and I have a paper on auctions in a financial market in Amsterdam from about 1750 onward,

  although there are examples from the 1600s as well
- Auctions are used in lots of settings today –

  the U.S. treasury uses auctions to issue new debt;

  the government uses auctions for procurement, which we'll talk about in a bit;

  we already talked about electricity markets earlier in the semester;

  auctions are commonly used to sell used cars, art, wine,

  contents of abandoned storage units on the show Storage Wars,

  half the sales on eBay, and a trillion other things
- Today, we'll talk about the most commonly used auction formats,

  and start setting up a basic model of bidding behavior
- We'll spend a couple of days getting deeper into the theory and seeing some pretty cool theoretical results,

  then double back to look closer at some real-world examples of markets where auctions play a central role

3 First Price Auctions and Equivalents

3.1 The Sealed-Bid (First Price) Auction

- Easiest to understand: everyone writes down a bid, seals it in an envelope, and turns it in to the auctioneer
- Once all the bids are in, they're opened, and the highest bidder wins, and pays their bid
- (The significance of "sealed bid" is that none of the bids are opened until they're all in,
  that is, until nobody can raise their bid,
  so everyone is functionally bidding at the same time,
  in the dark about what everyone else is biding;
  the significance of "first price" is that the actual payment is equal to the highest price bid)
- (One example: government sells rights to offshore oil drilling this way)
- So, three bidders, submit bids of \$1,000, \$1,200, and \$1,400 –
  the third bidder wins, and pays \$1,400

3.2 The Dutch Auction

- Made famous during Dutch "tulip craze" of the 1600s
- A Dutch auction is done out loud, with the bidders all in a room together
- The auctioneer begins at a high price – high enough that nobody will want to pay it –
  and then calls out lower and lower prices,
  and the first person to bid wins at the current price
- So, the auctioneer starts at \$3,000,
  then starts calling out prices – \$2,900, \$2,800, \$2,700...
- When he says \$2,300, someone yells, "mine!"
  and that ends the auction –
  the person who bid, pays \$2,300

3.3 Whoa, Weird, They're The Same!

- These sound like very different auctions, and they are –
  one can be done with bidders in different locations, sending in bids by mail;
  one is done in person, and over in a few minutes;
  but what's surprising is that if we consider them strategically,
  they're basically identical
- In both a sealed-bid first-price auction and a Dutch auction,
  we can think of each bidder choosing a number
- In the sealed-bid auction, this number is their bid;
  in the Dutch auction, it's the price at which they'll bid,
  assuming nobody else has bid yet
- In both auctions, nobody knows anyone else's number when they choose theirs;
  and in both auctions, whoever picked the highest number wins,
  and that's how much they pay
- So while the auctions are different in how they're actually run,
  they're the same from a strategic point of view

4 The Ascending ("English") Auction

- What most people picture when they think of an auction
  is an open-outcry ascending auction –
  an auctioneer keeps calling out higher prices,
  and waiting for someone to bid that,
  or people call out higher prices themselves
- someone starts the bidding at 50,
  someone else bids 55, someone bids 60,
  and it goes back and forth like this;
  at some point, someone bids 85, and nobody is willing to go any higher,
  so the auction ends at 85
- this type of auction is very common –
  art auctions work this way,
  Storage Wars works this way –
  but it's actually pretty hard to model
- so theorists often work with a couple of simpler auction formats that have somewhat similar properties

4.1 The Button Auction

- the key feature of an English auction is that if I win,

  the price I pay is determined by the price at which my toughest competitor stopped raising his bid,

  basically, the price at which all the other bidders had "dropped out"

- one way to formaize that intuition is with what's known as a "buttom auction"

- (button auctions are used in practice, but only a little bit)

- in a button auction, all the bidders press down a button at the beginning of the auction,

  which shows that they're "active,"

  and the price begins to rise continuously

- bidders hold down the button as long as they want to keep bidding at the current price;

  once they let go of the button, they're out of the auction

- (so "holding down your button" plays the same role as continuing to bid in an English auction)

- when the second-to-last bidder lets go of her button,

  the price stops rising and the auction is over,

  with the bidder who never dropped out winning at that price

- (button auctions are also called "clock auctions,"

  since the continuously-rising price looks like a clock gradually advancing the time)

- like with the English auction, you win when nobody else wants to keep bidding higher;

  the difference is that in order to stay in, you have to stay active at every price,

  you can't stop bidding for a while, watch what other people do,

  and then jump back in later

- (in a button auction, we can imagine varying the amount of information bidders have –

  we could run one where I know how many bidders are still active,

  and maybe even who they are;

  we could run one where all I know at each price is that the auction hasn't ended yet,

  but I don't know how many other bidders are still active)

4.2 The Second-Price Sealed Bid Auction

- Finally, if we think of the button auction where I don't get any new information about opponent bidding until the auction ends,

  then it's as if I'm just picking a single number at the start of the auction –

  what price I'll drop out at if the auction is still going

- In that case, we can think of a different format:

  the sealed-bid *second-price* auction

- here, each bidder writes down a bid, sends it in,

  and the highest bid wins;

  but the price the winner pays is not her own bid,

  but the second-highest

- (this is the price she'd pay in a button auction,

  since it's the price where all the other bidders would have dropped out)

- Theorists came up with the idea of a second-price sealed-bid auction,

  as a loose analogy for an English auction that was easier to model,

  and then found out these had occasionally been used in practice,

  for example, in the market for collectible stamps

- (the advantage of a sealed-bid auction is that the bidders don't all have to be available at the same time,

  these can even be done by mail)

- like I said, both the button auction and the second-price sealed-bid auction are at least somewhat loosely analogous to the ascending auction,

  and they're must easier to model, so theorists like to work with them

5 Other auction types

- Those are the most common types of auction, but there are tons of others

- candle auctions were popular in England in the mid-1600s –
  the auction would run until a candle burned out,
  so nobody would know the exact moment the auction would end,
  and therefore when to put in a final bid that nobody would be able to respond to

- in the formats we've considered so far, only the winner pays, but that need not be the case –
  another possible format is an all-pay auction,
  where everyone pays their bid whether or not they win
- (These aren't typically used to sell objects,
  but as a modeling analogy for situations where everyone expends effort –
  an election, where both candidates invest time and money, but only one wins the election)

- we could modify the formats we're considering so far by adding reserve prices –
  either minimum bids that must be met to bid at all,
  or a secret price level that must be met or the auction is cancelled

- Procurement auctions – government wants to buy something

  (examples: military procurement, e.g., air force needs some new fighter jets;

  local government wants to outsource garbage collection or road salting or paving)

- In procurement, the auctioneer is a buyer, and the bidders are sellers;

  if the service is standard enough, we can just think of the competition going the opposite way,

  with the lowest price bid being the winner instead of the highest,

  but aside from that change, the same auction formats are used

- (In more complex procurement settings,

  different sellers are submitting bids with different details for the project,

  and the procurer has to score these auctions on price and other characteristics to determine the winner)

- Some auctions could even use multiple stages,

  eliminating some buyers early on and having a smaller group continue to compete

- (I have a couple of papers on various multi-stage auction types)

- in fact, where we're going with the theory,

  is to consider the seller's problem of maximizing expected revenue

  over all possible auction formats they could conceivably try –

  subject only to the constraints that the rules are known at the beginning,

  participation is voluntary, and bidders play an equilibrium –

  and maybe surprisingly, we'll be able to solve that problem

- so, lots of possible auction formats, but we'll focus on the common ones

6 The Basic Model

- next, we're going to think about how to model this
- fundamentally, we want to game-theory this
- game theory assumes the game is common knowlege –
  I know everyone's payoffs, as a function of everyone's actions,
  and they know mine,
  and they know I know theirs, and so on
- but in an auction, I'm usually uncertain of exactly how my opponents are going to bid,
  in part because I don't actually know how much they want the thing
- how do we square this with a game being common knowledge?
- roughly, I think of my opponents' willingness to pay for the object being a random number
- and similarly, they think of my willingness to pay being a random number
- but since my strategy depends on how I think they'll bid,
  and that depends on how they think I'll bid,
  we also need to specify what I think about what they think about my valuations,
  and what they think about what I think about their valuations,
  and so on
- the easiest way to close the loop is to make each person's valuation a random variable,
  with the probability distribution it's drawn from being common knowledge –
  so we all agree on the distribution that my valuation is drawn from,
  but only I learn the realization before the auction –
  everyone else just knows the distribution
  (and I know they know the distribution, and they know I know that, and so on)

- So, here's the "basic" model we'll be working with –

  what's known as the Independent Private Values model

- there is a fixed set of bidders, $i = 1, 2, \ldots, N$

- bidder i has valuation v_i for winning the auction, so i's payoff is 0 if she loses, and $v_i - p$ if she wins and pays p

- bidder i knows v_i, but from everyone else's point of view, it's a random variable drawn from a distribution F_i

  (these could be the same for all bidders, or they could be different across bidders)

- bidders are risk-neutral, so they maximize expected payoffs

- and different bidders' valuations are independent – v_1 and v_2 and v_3 are all uncorrelated, knowing one of them doesn't tell you anything about the others

- note the key assumptions:

  each bidder perfectly knows their own valuation,

  so I don't change my view of the object if I find out how other people are bidding

- (this is what "private values" means – not always true in reality,

  we'll come back to this)

- bidders are risk-neutral

- there's a fixed number of bidders

- the whole environment is common knowledge – we all know how many bidders there are,

  and the probability distribution of each bidder's valuation,

  but each bidder knows their own realized valuation and the others don't

- and the bidders' values are independent – there's no correlation

- (if I value the object highly relative to my possible range,

  that doesn't mean other bidders are more likely to have high valuations too)

- these are all substantive assumptions –

  they may sometimes be poor assumptions to make –

  but they give us a very tractable model

- next, what does equilibrium mean here?
- we start out – with the prisoner's dilemma – with the simplest definition of Nash equilibrium,

  where a strategy is just a choice of action,

  and an equilibrium is when each player's action maximizes their payoff, given the equilibrium actions of all the other players
- we already modified that once, to account for uncertainty –

  in the search model, we allowed players to maximize expected payoffs, rather than exact payoffs,

  since they didn't know who they'd randomly be matched with
- here, uncertainty is of a slightly different form, but the gist is the same
- a strategy is now a choice of action for each possible type I could be –

  if my valuation turns out to be v, I'll bid $b(v)$ –

  and an equilibrium is when each player's bid function

  maximizes their expected payoff,

  given the equilibrium bid functions of all the other players

- let's do an example
- we'll consider a first-price, sealed-bid auction,

  with, say, 5 bidders,

  and each bidder's valuation is uniformly drawn from the interval $[0, 100]$
- I claim it's an equilibrium for each bidder to bid four-fifths of their valuation

- why?
- well, suppose everyone else is bidding $\frac{4}{5}$ of their valuation,

  and my valuation turns out to be v
- I'm going to look for a bid b that solves

$$\max_b \{(v-b)\Pr(win|b)\}$$

- What's the probability?

  well, if each of my four opponents is bidding $\frac{4}{5}v_i$,

  it's the probability that $\frac{4}{5}v_i < b$ for each of my opponents,

  or $v_i < \frac{5}{4}b$ for each of my opponents,

  and since v_i are independent random variables,

  as long as $\frac{5}{4}b \leq 100$ (it will be – why?),

  this is $\left(\frac{\frac{5}{4}b}{100}\right)^4$
- So I solve

$$\max_b \left\{(v-b)\left(\frac{5}{400}b\right)^4\right\} \quad = \quad \left(\frac{5}{400}\right)^4 \max_b \left\{(v-b)b^4\right\}$$

- If we drop the constant and take a FOC, this is

$$4vb^3 - 5b^4 = 0 \qquad \longrightarrow \qquad \frac{4}{5}v = b$$

- So if everyone else is bidding four-fifths of their valuation,

  my best-response is to bid four-fifths of my valuation,

  so that's an equilibrium!

- how did I guess this equilibrium?
- the envelope theorem!
- let's suppose we expect there to be an efficient equilibrium –

  an equilibrium where bidders all play the same strategy, which is increasing in valuations, so the bidder with the highest valuation wins
- we know that

$$U(v) \quad = \quad \max f(v,b) \quad = \quad \max_b \{(v-b)\Pr(win|b)\}$$

  and therefore that

$$U'(v) \quad = \quad \frac{\partial f}{\partial v}\bigg|_{optimal\ b} \quad = \quad \Pr(win|b) \quad = \quad \Pr(v_i > v_{-i}) \quad = \quad \left(\frac{v}{100}\right)^4$$

- Since $U(0) = 0$ – a bidder with valuation 0 can't get positive payoff –

  we can write

$$U(v) \quad = \quad \int_0^v U'(s)ds \quad = \quad \int_0^v \frac{1}{100^4}s^4 ds \quad = \quad \frac{1}{5\cdot 100^4}v^5$$

- But we can also write $U(v)$ as

$$U(v) \quad = \quad (v - b(v))\left(\frac{v}{100}\right)^4$$

- setting these equal,

$$\frac{1}{5\cdot 100^4}v^5 \quad = \quad (v - b(v))\left(\frac{v}{100}\right)^4$$

  simplifies to

$$\frac{v}{5} \quad = \quad v - b(v)$$

  or

$$b(v) \quad = \quad \frac{4}{5}v$$

7 what's next?

- that's where we got to so far
- next time: thinking about optimal play in the second-price auction or button auction
- and after that: stating, and beginning to solve, the seller's problem!