ECON 522 - SECTION 2 - NASH EQUILIBRIUM, COASE THEOREM, TRANSACTION COSTS AND BARGAINING

I Nash Equilibrium

A strategy profile is a **Nash equilibrium** if each player's strategy is optimal, given his opponents' strategies. To solve for pure strategy Nash equilibria, look for each player's best response to each of his opponents' actions, and then determine where the best responses coincide.

Example: A Coordination Game Suppose two friends are planning to meet up at a bar. One prefers the bars on the Capitol square, the other prefers the bars near campus. However, neither one wants to go to a bar alone. We can model this situation via the following game:

	Campus	Capitol
Campus	$(2^*, 3^*)$	(0,0)
Capitol	(0,0)	$(3^*, 2^*)$

Example: Cournot Duopoly Suppose that two firms in a duopoly are making output decisions. Each firm has identical costs $C(q_i) = cq_i$ for some $c > 0$. The firms face the inverse demand curve $P(Q) = a - q_1 - q_2$ for some $a > c$. Suppose that the firms choose their outputs q_i simultaneously. What is the Nash equilibrium of the duopoly game?

Solution. Each firm's objective is (letting q_{-i} denote the other firm's output choice)

$$\pi_i(q_i, q_{-i}) = (a - q_i - q_{-i})q_i - cq_i$$

This is a concave function of output, hence a first-order condition is necessary and sufficient:

$$a - 2q_i - q_{-i} - c = 0$$

Symmetrically, we have

$$a - 2q_{-i} - q_i - c = 0 \Rightarrow q_{-i} = \frac{a - q_i - c}{2}$$

Solving for equilibrium q_i yields

$$a - 2q_i - c - \frac{a - q_i - c}{2} = 0 \Rightarrow \frac{a - 3q_i - c}{2} = 0$$

and so each firm chooses $q_i = \frac{a-c}{3}$.

II Coase Theorem

The point of the Coase Theorem is that *if there are no transaction costs*, then as long as property rights are well defined and tradable, then for *efficiency* it doesn't matter who owns property rights initially: people will trade with each other until the individuals who value certain property the highest own that property (this is the intuition behind Adam Smith's "invisible hand" of the market). However, even with zero transaction costs, who has initial property rights definitely matters for distribution; if I own a car that you value more, I will sell you the car and earn a profit, but it is just as efficient for you to own the car in the beginning and reap the entire surplus, leaving me with less.

III Transaction Costs

Think of transaction costs as anything that makes it difficult to trade, or as any cost you have to pay that you're not exchanging for something you value. For example, if you want to buy a car you have to find someone to buy it from. Searching for a seller is a real cost, but you don't really gain any value from paying that cost; you would have had higher net utility making the same purchase if you didn't have to search for a seller. There are three cost categories:

- **Search:** you have to find someone to buy from, educate yourself about a product, etc.
- **Enforcement:** there must be ways to enforce the laws governing property rights and exchange.
- **Bargaining:** this seems to be most relevant for the property rights segment of our class.
  - **Private information** is when individuals have secret but symmetric information, such as their own valuations.
  - **Asymmetric information** is when one individual has more/less information than the other.
  - **Uncertainty** about the law and property rights can be a problem (one way to reduce this transaction cost is by relying on *precedent* in rulings).
  - **Large numbers of people** can also lead to problems. Example- Smoking bans in bars: As far as the Coase Theorem is concerned, who has smoking rights is irrelevant for efficiency, but since there would be many people involved in any negotiation for smoking rights in a public area, it may be more efficient for the government to try to give the rights to the people who value them most.

IV Threat Points

Often when doing a bargaining problem you'll be asked to calculate threat points. An individual's **threat point** is the payoff they can guarantee themselves by not participating in a bargain (their payoff in autarky); this is their opportunity cost for the bargain. When defining threat points (or initial payoffs), in some sense the starting value you pick is arbitrary, since what we really care about is the *relative* payoffs between different options. For example, suppose I have a cupcake which I value at \$3 and I trade it to you for a donut which I value at \$4. What was my payoff in the allocation when I owned the cupcake? Was it \$3? What if I also had a banana which I valued at \$2, then was my payoff \$5? The answer is: it doesn't matter what my payoff was, only that my payoff after I trade you for the donut is \$1 more. The following example should help illustrate the point.

IV.1 Example: Buying a Car

Bob owns a car which he values at \$4,000. Hal has \$15,000 and needs a car to get to work, an he values Bob's car at \$7,000. (1) What are Hal and Bob's threat points? (2) What are the potential gains form trade? (3) What price would they agree to and what would their payoffs be if they decided to split the gains from trade?

1. Since Bob starts with a car which he values at \$4,000 it seems reasonable to say that his threat point is \$4,000. Likewise, since Hal starts with \$15,000 it's reasonable to say that that is Hal's threat point. **Bob's TP: \$4,000; Hal's TP: \$15,000**

2. The potential gains from trade are the increase in total social welfare if a trade took place, which is the difference in how much Hal values the car versus Bob, which is \$7,000-\$4,000=\$3,000.
   Gains from Trade: \$3,000

3. If they split the gains from trade then they should each get half of the gains from trade *more than their threat points* as payoffs. Thus each gets $\frac{1}{2}(\$3000) = \$1,500$ above their threat points, which is \$4,000+\$1,500=\$5,500 for Bob, and \$15,000+\$1,500=\$16,500 for Hal. The price they agree to leaves Bob with a \$5,500 payoff, so we know that:

   $$\text{Bob's Threat Point } - \$4,000 + p = \$5,500$$

   So $p = \$5,500$ (the -\$4000 is because Bob is giving up his car which he values at \$4,000). Let's check that that makes sense for Hal's payoff being \$1,500:

   $$\begin{aligned} \text{Hal's payoff } &= \text{ Hal's threat point } + \$7,000 - p \\ &= \$15,000 + \$7,000 - \$5,500 \\ &= \$16,500 \end{aligned}$$

   $\mathbf{p = \$5,500}$; Bob's payoff: \$5,500; Hal's payoff: \$16,500

Alternative Solution

1. Since Bob and Hal don't gain or loose anything from not bargaining, let's say both of their threat points are \$0.
   Bob's TP: \$0; Hal's TP: \$0

2. The potential gains from trade are the increase in total social welfare if a trade took place, which is the difference in how much Hal values the car versus Bob, which is \$7,000-\$4,000=\$3,000.
   Gains from Trade: \$3,000

3. If they split the gains from trade then they should each get half of the gains from trade *more than their threat points* as payoffs. Thus each gets $\frac{1}{2}(\$3000) = \$1,500$ above their threat points, which is \$0+\$1,500=\$1,500 for Bob, and \$0+\$1,500=\$1,500 for Hal. The price they agree to leaves Bob with a \$1,500 payoff, so we know that:

$$\begin{aligned} \text{Bob's Threat Point } - \$4,000 + p &= \$1,500 \\ \$0 - \$4,000 + p &= \$1,500 \end{aligned}$$

   So $p = \$5,500$. Let's check that that makes sense for Hal's payoff being \$16,500:

$$\begin{aligned} \text{Hal's payoff } &= \text{ Hal's threat point } + \$7,000 - p \\ &= \$0 + \$7,000 - \$5,500 \\ &= \$1,500 \end{aligned}$$

   p $= \$5,500$; Bob's payoff: \$1,500; Hal's payoff: \$1,500

In both solutions we get that $p = \$5,500$, and the gains from trade are \$3000, even though we used different threat points in the two problems. This will always be the case, no matter what numbers you pick for threat points. When dealing with payoffs (roughly utility), the payoff form one outcome is only meaningful when comparing it to other outcomes. No matter what initial payoffs you pick for Hal and Bob (i.e. what threat points you assign), you will always find that they are each \$1,500 better off after trading and splitting the gains evenly. This is because the values that they each assign to the car, \$4,000 and \$7,000, are absolute and cannot be arbitrarily assigned. Of course, some threat points are more natural picks than others. Generally is easiest to assume the threat point of the person originally holding the object in question is whatever he values that object at (e.g. Bob's TP=\$4,000).

IV.2 Step-by-step guide to solving bargaining problems

To solve a bargaining problem we can see that we need two (possibly three) things: we need to know the individuals' threat points, the gains from trade, and sometimes we may need to know what happens after bargaining is concluded (such as in the previous example, where we needed to know where the car ended up). Here is a step-by-step guide to finding what we need.

1. Define payoffs.
2. Figure out what happens in autarky.
3. Find each individual's payoff when (2) happens. These are your threat points.
4. Figure out the efficient allocation. Remember, the Coase Theorem implies that when two people can bargain, they will arrive at the efficient allocation, at least if there is no one else in society who is impacted by their decision. Hence, this is what will happen after bargaining.
5. Figure out the sum of the players' payoffs in the efficient allocation.
6. Subtract the sum of the players' threat points from (5). This is the gains from trade.