ECON 522 - SECTION 1 - MATH REVIEW AND EFFICIENCY DISCUSSION

I. Short Math Review: Derivatives

Recall the definition of a derivative for a function f defined on the real number line:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x) = \frac{df(x)}{dx}$$

This is just the slope of the function at the point x. We know that if a function is differentiable, concave, and has a maximum value on some convex open set, then at that maximum the slope must be zero. That is why we take "first order conditions" ($f'(x) = 0$) when we look for maximum values.

I.1 Derivative Rules for Univariate Calculus

Reminder: Your ability to use calculus to solve optimization problems is not the focus of this class, but may be useful.

- **Power Rule** $\frac{d}{dx}x^n = nx^{n-1}$
- **Product Rule** $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + g'(x)f(x)$
- **Chain Rule** $\frac{d}{dx}f(g(x)) = g'(x)f'(g(x))$
- **Logarithms and the Exponential Function** $\frac{d}{dx}e^x = e^x$; $\frac{d}{dx}\ln(x) = \frac{1}{x}$

I.2 Example: Contribution to a Public Good

Variables:

c	Your (monetary) contribution to public television
C	Total contributions to public television
$\underline{C}$	Total contributions to public television by other people
$u(C,c) = \ln(C+1) - c$	Your utility from watching \$$C$ worth of public television and contributing c
N	Number of contributors to public television

How much will you contribute? Remember, we can write $C = \underline{C} + c$. Then your decision problem is

$$\max_c\{\ln(\underline{C} + c + 1) - c\}$$

Taking a first-order condition yields

$$\frac{1}{1+\underline{C}+c} = \frac{1}{C+1} = 1 \Rightarrow C = 0 \Rightarrow u(0,0) = 0$$

since, if contributions must be nonnegative, everyone must contribute $c = 0$. Is this efficient? Suppose that a central government chooses a total contribution level C, and that this government is concerned with efficiency (and hence maximizing the size of "the pie"). Since each viewer's utility function is quasilinear in their contribution (and thus utility is transferable), all that matters for efficiency is the total contribution, and the government can add everyone's utility functions together and maximize the sum in order to find the efficient contribution level:

$$\max_C\{N\ln(C+1) - C\}$$

What is the difference here? Now the decision maker (here: the government) takes the positive externality of contributing into account: we say she *internalizes the externality*. The first-order condition is now

$$\frac{N}{C+1} = 1 \Rightarrow C = N - 1$$

the unique efficient total contribution level. Suppose contributions are divided evenly, as before. Then each viewer's contribution is $\frac{N-1}{N}$ and your utility (and everyone else's) is $u(C,c) = \ln(N) - \frac{N-1}{N}$, which is higher than when no collective action was possible, and thus a Pareto improvement. (If contributions were not divided evenly, this change in total contribution level might not be a Pareto improvement, but it would be a Kaldor-Hicks improvement since we can execute a set of monetary transfers to return to the case where contributions are evenly divided.)

II. Efficiency

- A **Pareto improvement** is a change that makes everyone at least as well off as before and at least one person better off.
- A **Kaldor-Hicks improvement** is a change that could be turned into a Pareto improvement by adding a set of monetary transfers. Hence, every Pareto improvement is a Kaldor-Hicks improvement as well, but the converse does not hold.
  Take the example from class/section: You own a car which you value at $4000, while I value your car at $5000. It's a K-H improvement for the government to sieze your car and give it to me, but it is not a Pareto improvement, since you're worse off. But if I purchase your car for any amount between $4000 and $5000 it's both a Pareto and K-H improvement, since we're both better off. However, once I have your car, no matter how I got it, the situation is both K-H and Pareto efficient, since there's no way to create more value in our little economy.
- An allocation is **Pareto efficient** if there exist no Pareto improvements upon it. Examples:
  - I own all of the wealth in the world and you have nothing
  - Everyone owns an equal amount of wealth, and nothing is being wasted.
  - Any allocation of wealth in which nothing is being wasted. No money is "left on the table."
- An allocation is **Kaldor-Hicks efficient** if and only if it is Pareto efficient. Why?
  - If an allocation is Kaldor-Hicks efficient, then there exist no Kaldor-Hicks improvements upon it. Since Pareto improvements are a subset of Kaldor-Hicks improvements, there cannot exist a Pareto improvement, and the allocation is Pareto efficient.
  - If an allocation is Pareto efficent, then there exist no Pareto improvements. Suppose there existed a Kaldor-Hicks improvement on a Pareto efficient allocation. Then there must exist some set of monetary transfers that when combined with the Kaldor-Hicks improvement forms a Pareto improvement. But such a Pareto improvement cannot exist. Then there cannot exist a Kaldor-Hicks improvement, and the allocation is Kaldor-Hicks efficient.

$$\text{Kaldor-Hicks Efficient} \quad \Leftrightarrow \quad \text{Pareto Efficient}$$

$$\text{Kaldor Hicks Improvement} \quad \begin{matrix} \not\Rightarrow \\ \Leftarrow \end{matrix} \quad \text{Pareto Improvement}$$

***IMPORTANT*.** If there are no externalities, an allocation of property is efficient if and only if every object is owned by the person who values it the most.

III. Next Week- Coase Theorem and Property Rights

Coase Theorem. If property rights are well defined and tradable, and transaction costs are low, then the initial allocation of property rights does not affect efficiency.