Econ 522 In-Class Examples

Fall 2013

1 Broad vs Narrow Taxes

Two goods: beer (x), pizza (y)
One consumer with budget of \$60 and utility $u(x, y) = \sqrt{xy}$

- Given prices p for beer and q, calculate demand.
  Our consumer's problem is

  $$\max_{x,y} \sqrt{xy} \text{ such that } px + qy = 60$$

  We can substitute the budget constraint into the objective function to convert this multivariate problem into a univariate one:

  $$\max_{x} \sqrt{x\frac{60 - px}{q}}$$

  Remember, (x, y) *maximizes the agent's objective function if and only if it maximizes a monotone transformation of the agent's objective function. Knowing this, we square the objective, yielding*

  $$\max_{x} x\frac{60 - px}{q}$$

  Our first-order condition is

  $$\frac{60}{q} - \frac{2px}{q} = 0$$

  Hence we have $x = \frac{30}{p}$, and thus $y = \frac{30}{q}$. *Alternatively, note that our consumer has Cobb-Douglas utility, so we know he will spend a constant fraction of income (here,* $\frac{1}{2}$*) on each good.*

Now suppose beer and pizza are produced at a constant cost of \$1 per unit and sold in a perfectly competitive market.

- Calculate the quantity demanded and utility with no tax.
  $(x, y) = (30, 30)$, $u(x, y) = \sqrt{30^2} = 30$

- Calculate demand and utility with \$0.50 per unit tax on beer.
  $(x, y) = (\frac{30}{1.5}, 30) = (20, 30)$, $u(x, y) = \sqrt{20 \times 30} \approx 24.49$

- How much revenue does the tax raise?
  $20 \times 0.5 =$\$10

- Calculate demand and utility with a \$0.20 per unit tax on both goods.
  $(x, y) = (\frac{30}{1.2}, \frac{30}{1.2}) = (25, 25)$, $u(x, y) = \sqrt{25^2} = 25$
- How much revenue does the tax raise?
  $50 \times 0.2 =$\$10

2 Adverse Selection and Unraveling

2.1 Valuation as a Discrete Random Variable

Suppose that we have a seller with a poker chip that is worth 2x the number shown on a die to him, and knows that number. Suppose that there is also a buyer who values the poker chip at 3x the number shown on the die, but does not know that number. Let's look at how this leads to unraveling.

- The most the buyer would ever be willing to pay for the chip is $(1{+}2{+}3{+}4{+}5{+}6)/6{\cdot}3 = 10.5$
- But then the seller would only take the offer if the roll was less than 6. So no trade will occur if the die roll is 6.
- Then the most the buyer would be willing to pay is $(1 + 2 + 3 + 4 + 5)/5 \cdot 3 = 9$
- But then the seller would only take the offer if the roll was less than 5
- Then the most the buyer would be willing to pay is $(1 + 2 + 3 + 4)/4 \cdot 3 = 7.5$
- But then the seller would only take the offer if the roll was less than 4
- Then the most the buyer would be willing to pay is $(1 + 2 + 3)/3 \cdot 3 = 6$
- The seller would be willing to take this offer for a die roll of 3 or less. So at best, we can only get trade half the time.

2.2 Valuation as a Continuous Random Variable (not done in class)

Suppose we have a buyer and a seller of a car, both risk neutral. The value that each places on the car depends on a random variable θ. In particular, the seller's valuation is θ and the buyer's valuation is $\frac{3\theta}{2}$, so there are gains from trade for every $\theta > 0$. The fact that θ is distributed uniformly on $[0, 1]$ is common knowledge, but only the seller observes the realization of θ. Is there any price p at which trade will occur?
First note that the seller is only willing to sell his car for the market-clearing price p if $\theta \leq p$. Now note that the buyer knows this, and so is only willing to buy a car for p if his expected valuation conditional on that fact is greater than p, that is, if

$$\begin{aligned} E\left(\frac{3\theta}{2}|\theta \leq p\right) &\geq p \\ \frac{3}{2}E(\theta|\theta \leq p) &\geq p \\ \frac{3}{2}\left(\frac{p}{2}\right) &\geq p \\ p &\leq 0 \end{aligned}$$

So it is clear that trade can only occur with $\theta = 0$, which is a probability zero event. Thus, asymmetric information causes the market to completely unravel, despite the fact that gains from trade are present with probability one. Note that this problem would not be present if the seller did not know θ: his expected valuation would be $\frac{1}{2}$ and the buyer's would be $\frac{3}{4}$, so clearly trade could occur at any price between $\frac{1}{2}$ and $\frac{3}{4}$.

3 Continuous Reliance

I have agreed to purchase a plane from you for $350,000. Your price to build it is $250,000 with probability $1-p$ and $1,000,000 with probability p; when the high cost realizes, you will choose to breach. My valuation for the plane is $500,000. I can also choose to invest x dollars in building a hangar, which provides me with benefit of $600\sqrt{x}$.

1. What is the efficient level of reliance?
We maximize the expected value of the hangar minus cost, which is $600(1-p)\sqrt{x}-x$. *First order condition is*
$$\frac{300(1-p)}{\sqrt{x}} - 1 = 0 \Rightarrow x = 90,000(1-p)^2$$
2. What will I do if expectation damages include anticipated benefit from reliance? *I will get benefit from reliance no matter what, so I will maximize* $600\sqrt{x} - x$. *First order condition is*
$$\frac{300}{\sqrt{x}} - 1 = 0 \Rightarrow x = 90,000$$
3. What will I do if expectation damages exclude anticipated benefit from reliance? *Now I only benefit from reliance when breach does not occur, so I maximize the expected value of the hangar minus cost, which is* $600(1-p)\sqrt{x} - x$. *First order condition is*
$$\frac{300(1-p)}{\sqrt{x}} - 1 = 0 \Rightarrow x = 90,000(1-p)^2$$
the efficient level of reliance.

4 Continuous Investment in Performance

You hire me to build you a plane, and I have the ability to reduce the probability of breach by investing in performance. My ability to reduce the probability of breach is described by the function:
$$p(z) = \frac{1}{2}e^{\frac{-z}{40000}}$$
Where $p(z)$ is the probability of breach, and z is the amount in dollars that I invest in performance. Your expected payoff from the plane is $150,000, but on top of that you've built a hangar (this is an example of reliance) that will give you a return of $180,000 if you get the plane. My payoff from the contract is $100,000 minus however much I decide to invest in performance, but if I breach I have to pay some damages D.

Point: We haven't decided what D should be. We're going to solve this problem to figure out what D has to be so that I have the incentive to invest the efficient amount in performance.

Efficiency requires that social welfare is maximized. With probability $p(z)$ the contract is broken, so I must pay you damages D, but that is a transfer and has no effect on efficiency. With probability $(1-p(z))$ we get a combined payoff of $150000+180000+100000 = 430000$. And for sure I have to pay z (I choose what z to pay, but I spend it before the realization of breach/no breach). Thus, social utility is:

$$U = (1 - p(z))(430000) - z$$

So, we figure out the optimal z by taking first order conditions. Note that:

$$p'(z) = -\frac{1}{2}\frac{1}{40000}e^{\frac{-z}{40000}} = -\frac{1}{40000}p(z)$$

So the first order condition is:

$$\begin{aligned} U'(z) = 0 &= -430000p'(z) - 1 \\ &= \frac{430000}{40000}p(z) - 1 \end{aligned}$$

$$\text{Therefore the optimal } z \text{ is such that } p(z) = \frac{40000}{430000}$$

When I decide how much to invest in performance, I don't care at all that you've relied. All I care about is that if I breach I'll have to pay D. So my utility is

$$u(z) = (1 - p(z))(100000) + p(z)(-D) - z$$

since if I breach I pay D, and if I don't I get \$100000, and in either case I pay z. So I take first order conditions to get:

$$u'(z) = -100000p'(z) - Dp'(z) - 1 = -p'(z)(100000 + D) - 1 = \frac{100000 + D}{40000}p(z) - 1 = 0$$

$$\Rightarrow p(z) = \frac{40000}{100000 + D}$$

Take a look at the efficient probability of breach $p(z) = \frac{40000}{430000}$ versus the probability that I choose $p(z) = \frac{40000}{100000+D}$. In order to get me to invest the optimal z, we must have $D = \$330,000$, which is *exactly* what expectation damages are if we include reliance! Thus we need to include reliance in expectation damages if we want efficient investment in performance, but we've already seen that this will result in over-reliance.