ECON 522 - SECTION 1 - MATH REVIEW AND EFFICIENCY DISCUSSION

I. Very Short Math Review: Derivatives
Recall the definition of a derivative for a function f defined on the real number line:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \frac{df(x)}{dx}$$

Which is just the slope of the function at the point x. Since we know that if a function is differentiable and has a maximum value, then at that maximum the slope must be zero. That is why we take "first order conditions" ($f'(x) = 0$) when we look for maximum values.

I.1 Example: Contribution to a Public Good

Variables:

c	Your contribution to public television
C	Total contributions to public television
$\underline{C}$	Total contributions to public television by other people
$u(C, c) = \ln(C+1) - c$	Your utility from watching \$$C$ worth of public television and contributing c
N	Number of contributors to public television

How much will you contribute? Remember, we can write $C = \underline{C} + c$. Then your decision problem is

$$\max_c \{\ln(\underline{C} + c + 1) - c\}$$

Taking a first-order condition yields

$$\frac{1}{C+1} = 1 \Rightarrow C = 0 \Rightarrow u(C, c) = 0$$

if everyone contributes the same amount. Is this efficient? Suppose that a central government chooses a total contribution level C, and that this government is concerned with Kaldor-Hicks efficiency (and hence maximizing the size of "the pie"). Since each viewer's utility function is denominated in dollars (and thus utility is transferable), the government can add them all together and maximize the sum:[1]

$$\max_C \{N \ln(C+1) - C\}$$

What is the difference here? Now the decision maker (here: the government) takes the positive externality of contributing into account: we say she *internalizes the externality*. The first-order condition is now

$$\frac{N}{C+1} = 1 \Rightarrow C = N - 1$$

the unique K-H efficient total contribution level.[2] Suppose contributions are divided evenly, as before. Then each viewer's contribution is $\frac{N-1}{N}$ and your utility (and everyone else's) is $u(C, c) = \ln(N) - \frac{N-1}{N}$, which is higher than when no collective action was possible.

[1] Even if they weren't all denominated in dollars, we could add the utility functions together and get the K-H efficient total contribution level, but this is only true because everyone has the same preferences in this model.

[2] As it turns out, it is the unique Pareto efficient total contribution level as well, but this is only because everyone has the same preferences.

II. Efficiency

Pareto Efficiency is one of the most important concepts in economics. Recall the definition: An allocation is **pareto efficient** if there does not exist any other allocation such that *everyone is at least as well off* as before, and *at least one person is strictly better off*.

Examples:

- I own all of the wealth in the world and you have nothing
- Everyone owns an equal amount of wealth, and nothing is being wasted.
- Any allocation of wealth in which nothing is being wasted. No money is "left on the table."

Kaldor-Hicks (K-H) Efficiency is very similar to Pareto efficiency, and as long as utility is transferable (for example via monetary transfers) then they are equivalent. The main difference between the two concepts is that a Pareto *imporvement* is not the same as a K-H *improvement*. Recall that a K-H improvement is any change in an allocation which can be turned into a Pareto improvement via monetary transfers. Thus, any time a situation is K-H efficient it is also Pareto efficient, and there are no Pareto or K-H improvements available. Also, any Pareto improvement is also a K-H improvement, but <u>not every K-H improvement is a Pareto improvement</u>. Take the example from class/section: You own a car which you value at \$4000, while I value your car at \$5000. It's a K-H improvement for the government to sieze your car and give it to me, but it is not a Pareto improvement, since you're worse off. But if I purchase your car for any amount between \$4000 and \$5000 it's both a Pareto and K-H improvement, since we're both better off. However, once I have your car, no matter how I got it, the situation is both K-H and Pareto efficient, since there's no way to create more value in our little economy.

$$\text{Kaldor-Hicks Efficient} \quad \underset{\Leftarrow \text{ if utility is transferable}}{\Rightarrow} \quad \text{Pareto Efficient}$$

$$\text{Kaldor Hicks Improvement} \quad \underset{\Leftarrow}{\not\Rightarrow} \quad \text{Pareto Improvement}$$

***IMPORTANT*.** If there are no externalities, an allocation of property is Kaldor-Hicks efficient if and only if every object is owned by the person who values it the most.

III. Next Week- Coase Theorem and Property Rights

Coase Theorem. If property rights are well defined and tradable, and transaction costs are low, then the initial allocation of property rights does not affect efficiency.