Treatment Effects

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Treatment Effects

The key theme in my part of the course will be causal estimation-we want to estimate the effect of some intervention or policy on an individual.

The standard way to think about these issues is in the "Treatment Effects" framework

We will start with the simplest binary case where the default is that an individual (or firm or state or whatever) either receives some treatment or they do not

The classic and easiest way to think about this is a drug

We have a patient who is sick and either we give them the drug or we do not

We let Y_{0i} be the outcome that individual i would receive if they did not get the drug

and Y_{1i} the outcome they would receive if they did receive the drug.

One of these must be counterfactual

We define the treatment effect for individual i as the change in their outcomes resulting from the treatment:

$$\alpha_i \equiv Y_{1i} - Y_{0i}$$

There are many many different treatments we can think of:

- a drug for Ebola
- a big fence between here and Mexico
- a year of school
- a month of training
- an increase in the federal minimum wage to $15
- a welfare program
- free tuition for public colleges
- inclusion of a public option in health exchanges
- exposure to polluted air
- access to clean water
- race or gender
- a cigarette a day while pregnant
- a glass of wine every night for dinner
- increased financial regulations on banks
- helmets on motorcyclists
- the death penalty

Our first issue: even if we knew the value of α_i for every person in the population that we study,

What do we present in the paper?

We must focus on a feature of the distribution

The most common:

- Average Treatment Effect (ATE)

$$E(\alpha_i)$$

- Treatment on the Treated (TT)

$$E(\alpha_i \mid T_i = 1)$$

- Treatment on the Untreated (TUT)

$$E(\alpha_i \mid T_i = 0)$$

(Heckman and Vytlacil discuss Policy Relevant Treatment effects, but I need more notation than I currently have to define those)

These each answer very different questions

I will ignore TUT for the rest of these lecture notes because it is symmetric with TT

In terms of identification they are symmetric.

What is identified from this type of data?

We can directly identify $Pr(T_i = 1)$

We can also observe

- the distribution of Y_{0i} conditional on $T_i = 0$
- the distribution of Y_{1i} conditional on $T_i = 1$

Identification of TT

By definition the TT parameter is

$$\begin{aligned} TT = E(\alpha_i \mid T_i = 1) &= E(Y_{1i} - Y_{0i} \mid T_i = 1) \\ &= E(Y_{1i} \mid T_i = 1) - E(Y_{0i} \mid T_i = 1) \end{aligned}$$

Is this identified?

Well one part we get immediately: $E(Y_{1i} \mid T_i = 1)$

The other part is counterfactual: $E(Y_{0i} \mid T_i = 1)$

It is the expected value the treatments would have received if the treatment option were not available to them.

This is not measured in the data

To a large extent what my part of this course and more generally causal estimation is about is trying to say something about $E(Y_{0i} \mid T_i = 1)$

All the different empirical approaches we examine do this in one way or another

Average Treatment Effect

This is slightly more complicated

$$
\begin{aligned}
ATE = E(\alpha_i) = & E(\alpha_i \mid T_i = 1)Pr(T_i = 1) + E(\alpha_i \mid T_i = 0)Pr(T_i = 0) \\
= & \left[E(Y_{1i} \mid T_i = 1) - E(Y_{0i} \mid T_i = 1)\right] Pr(T_i = 1) \\
& + \left[E(Y_{1i} \mid T_i = 0) - E(Y_{0i} \mid T_i = 0)\right] \left[1 - Pr(T_i = 1)\right]
\end{aligned}
$$

All we can directly identify from the data is :

$$E(Y_{1i} \mid T_i = 1), E(Y_{0i} \mid T_i = 0), Pr(T_i = 1)$$

In this case there are two missing pieces:

$$E(Y_{1i} \mid T_i = 0), E(Y_{0i} \mid T_i = 1)$$

If we could figure these out we could identify the ATE

So how do we deal with this general identification problem?

In my view there are 5 primary “design based " ways people solve this problem

1. Randomized Control Trials
2. Selection only on Observables
3. Instrumental Variables
4. Fixed Effects/Difference in Differences
5. Regression Discontinuity

Lets focus on the constant treatment effect model ($\Delta_i = \Delta$)

$$\begin{aligned} Y_i = & T_i Y_{1i} + (1 - T_i) Y_{0i} \\ = & \alpha_i T_i + Y_{0i} \\ = & \alpha T_i + u_i \end{aligned}$$

If u_i is uncorrelated with T_i we can run a regression

Randomized Control Trials

Probably the best way to solve the identification problem in estimating the treatment effect is to use a randomized control trial

The key is that if T_i is randomly assigned then it is uncorrelated with u_i by design and we can just run a regression (or look at Treatment minus controls which is the same thing)

This is often the ideal thing to do, but is also generally too costly or not feasible so I will focus on the other methods in this class

Selection Only on Observables

Consider the case in which we only have selection only on observables by which I mean:

For all x in the support of X_i and $t \in \{0, 1\}$,

$$E(u_i \mid X_i = x, T_i = t) = E(u_i \mid X_i = x)$$

This means that if I condition on people with the same $x's$, I can think of T_i as randomly assigned

- Generally a really strong assumption
- We can use OLS in this case
- We will also discuss matching

Instrumental Variables

Assume that

$$cov(Z_i, T_i) \neq 0$$
$$cov(Z_i, u_i) = 0$$

Then we can identify α.

To see how this works note that direction we can identify $cov(Z_i, T_i)$ and

$$\begin{aligned} cov(Z_i, Y_i) =& cov(Z_i, \alpha T_i + u_i) \\ =& cov(Z_i, \alpha T_i) + cov(Z_i, u_i) \\ =& \alpha cov(Z_i, T_i) \end{aligned}$$

Thus we can identifiy

$$\alpha = \frac{cov(Z_i, Y_i)}{cov(Z_i, T_i)}$$

We can implement this using GMM or 2SLS

Difference Model

Lets think about a simple evaluation of a policy.

If we have data on a bunch of people right before the policy is enacted and on the same group of people after it is enacted we can try to identify the effect.

Suppose we have two years of data 0 and 1 and that the policy is enacted in between

We could try to identify the effect by simply looking at before and after the policy

That is we can identify the effect as

$$\bar{Y}_1 - \bar{Y}_0$$

The problem is that this attributes any changes in time to the policy

That is suppose something else happened between times 0 and 1 other than just the program.

We will attribute whatever that is to the program.

If we added time dummy variables into our model we could not separate a time effect from T_{it}

Difference in Differences

To solve this problem, suppose we have two groups:

- People who are affected by the policy changes ($\blacklozenge$)
- People who are not affected by the policy change ($\clubsuit$)

and only two time periods before ($t = 0$) and after ($t = 1$)

We can think of using the controls to pick up the time changes:

$$\bar{Y}_{\clubsuit 1} - \bar{Y}_{\clubsuit 0}$$

Then we can estimate our policy effect as a difference in difference:

$$\widehat{\alpha} = (\bar{Y}_{\blacklozenge 1} - \bar{Y}_{\blacklozenge 0}) - (\bar{Y}_{\clubsuit 1} - \bar{Y}_{\clubsuit 0})$$

Regression Discontinuity

The idea of regression discontinuity goes way back, but it has gained in popularity in recent years

The basic idea is to recognize that in many circumstances policy rules vary at some cutoff point

To think of the simplest case suppose the treatment assignment rule is:

$$T_i = \begin{cases} 0 & X_i < x^* \\ 1 & X_i \geq x^* \end{cases}$$

$$E(Y_i \mid x) = \alpha E(T_i \mid x) + E(u_i \mid x)$$

Using Selection on Observables to think about Selection on Unobservables

Time permitting I want to talk about an approach I have worked on

More of a robustness check than something you want to take literally.

Suppose

$$Y_i = \Delta T_i + X_i'\beta + u_i$$

The problem is

$$E(u_i \mid T_i = 1) \neq E(u_i \mid T_i = 0)$$

But random assignment would imply

$$E(X_i'\beta \mid T_i = 1) = E(X_i'\beta \mid T_i = 0)$$

The idea here is to suppose what would happen if “selection on unobservables was like selection on observables"

Then we can use the relationship between $E(X_i'\beta \mid T_i = 1)$ and $E(X_i'\beta \mid T_i = 0)$ to tell us something about the relationship between $E(u_i \mid T_i = 1)$ and $E(u_i \mid T_i = 0)$