Forecasting the U.S. Dollar in the 21st Century
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Online Appendix

Online Appendix 1: Data Appendix

Appendix Table 1: Data source

Variable	**Data source**	**Sample period**
Exchange rates	FRED	1999M1-2020M3 (end of month)
US treasury premium	Engel and Wu (2020) https://www.ssc.wisc.edu/~cengel/Data/LiquidityYield/	1999M1-2018M1
MAR global factor	Miranda-Agrippino and Rey (2020) (http://www.helenerey.eu/RP.aspx?pid=Published-Papers_en-GB&aid=291587444_67186463733)	1999M1-2019M4
GZ spread	Gilchrist and Zakrajsek (2012) https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/updating-the-recession-risk-and-the-excess-bond-premium-20161006.html	1999M1-2020M3
Dividend price ratio of SP500	https://data.nasdaq.com/data/MULTPL/SP500_DIV_YIELD_MONTH-sp-500-dividend-yield-by-month	1999M1-2020M3
Log VIX	FRED	1999M1-2020M3
US Term spread (5y-FF)	FRED	1999M1-2020M3
US Term spread (10y-2y)	FRED	1999M1-2020M3
TED	FRED	1999M1-2020M3
Intermediary leverage	He, Kelly, Manela (2017) http://apps.olin.wustl.edu/faculty/manela/data.html	1999M1-2018M11
Intermediary weighted return	He, Kelly, Manela (2017) http://apps.olin.wustl.edu/faculty/manela/data.html	1999M1-2018M11
Log Repo*	Board of Governors of the Federal Reserve System	1999M8-2020M3
Log Commercial Paper*	Board of Governors of the Federal Reserve System	2001M1-2020M3
CPI Index	IMF IFS	1999M1-2020M3

Notes: * We linearly detrended the data. For in sample forecasting, we detrended the variable using the whole sample. For out-of-sample forecasting, we detrended the variable using the 60 observations within the window.

Appendix Table 2: Persistence of global risk variables

AR(1) coefficient from the regression $X_t = \alpha + \beta X_{t-1} + e_t$

Variable	AR(1) coefficient	Standard errors*	Obs #
MAR global factor	0.97	(0.02)	243
GZ spread	0.96	(0.05)	254
Dividend price ratio of SP500	0.97	(0.03)	254
Log VIX	0.85	(0.03)	254
US Term spread (5y-FF)	0.90	(0.03)	254
US Term spread (10y-2y)	0.98	(0.01)	254
TED	0.84	(0.04)	254
Intermediary leverage	0.98	(0.01)	238
Intermediary weighted return	0.17	(0.10)	238
Log Repo	0.98	(0.01)	247
Log Commercial Paper	0.93	(0.03)	230

Notes: *Newey West standard errors with 5 lags are reported

AR(1) coefficient from the regression $X_t = \alpha + \beta X_{t-1} + e_t$ by currency

	Real exchange rate			US Treasury premium		
Currency	AR(1) coefficient	Standard errors*	Obs #	AR(1) coefficient	standard errors*	Obs #
AUD	0.99	(0.01)	254	0.78	(0.05)	228
CAD	0.98	(0.01)	254	0.90	(0.05)	228
CHF	0.97	(0.01)	254	0.82	(0.05)	228
EUR	0.98	(0.01)	254	0.82	(0.04)	228
GBP	0.97	(0.01)	254	0.72	(0.09)	228
JPY	0.99	(0.01)	254	0.81	(0.07)	228
NOK	0.98	(0.02)	254	0.84	(0.04)	228
NZD	0.98	(0.01)	254	0.78	(0.05)	228
SEK	0.99	(0.01)	254	0.85	(0.03)	228
Simple average	0.99	(0.01)	254	0.84	(0.04)	228

Notes: *Newey West standard errors with 5 lags are reported

Online Appendix 2: Empirical appendix

Due to space limitations, we do not report currency by currency regression for the global risk variables in the main text. We provide a summary of the currency-by-currency regression below.

In Appendix Table 3, we summarize the in-sample regression by reporting the number of currencies that have a significant coefficient at the 5% level. In Appendix Table 4, we summarize the out-of-sample regressions by reporting the number of currencies that have a significant coefficient at the 5% level. Overall, we can see that nominal exchange rate predictability is stronger than the global risk variables for dominant pairs of currency. For example, for in-sample regression, all of the currencies have a significant coefficient for nominal exchange rate except for one insignificant currency for the regression that includes GZ spreads at the 3-year horizon. For out-of-sample regression, at the 5-year forecast horizon, the nominal exchange rate outperforms five global risk variables for more than seven out of nine currencies in column (i).

Appendix Table 5 and 6 are supplementary information of the actual data analysis in Table 3 and Table 4.

Appendix Table 3:

Summary of regression statistics of in sample forecasting for each currency: For each horizon, we run regressions of $s_{t+h} - s_t = \alpha + \beta^X X_t + e_{t+h}$ and $s_{t+h} - s_t = \alpha + \beta^{XX} X_t + \beta^{SS} s_t + e_{t+h}$ for each currency. we report the count of significant coefficients of the first regression in the first column, and the count of significant coefficients of the second regression in the second column in the format of $\beta^{XX} - \beta^{SS}$.

		Univariate model	Multivariate model		Univariate model	Multivariate model
		β^X	$\beta^{XX} - \beta^{ss}$		β^X	$\beta^{XX} - \beta^{ss}$
Independent variables		(1)	(2)		(1)	(2)
US Treasury premium	1-month horizon forecast (h=1)	2	3 - 1	3-year horizon forecast (h=36)	4	1 – 9
MAR global factor		1	0 - 0		2	0 – 9
GZ spread		0	0 - 0		8	7 – 8
Log VIX		0	0 - 0		7	1 – 9
US Term spread (5y-FF)		1	1 - 0		0	1 – 9
US Term spread (10y-2y)		2	5 - 0		2	1 – 9
TED		2	2 - 0		2	2 – 9
Intermediary leverage		0	5 - 0		3	5 – 9
Interm. weighted return		0	0 - 0		6	6 – 9
Log Repo		0	6 - 3		0	8 – 9
Log Commercial Paper		2	1 - 0		3	8 - 9
US Treasury premium	1-year horizon forecast (h=12)	1	1 – 9	5-year horizon forecast (h=60)	5	0 - 9
MAR global factor		1	0 - 9		3	5 - 9
GZ spread		7	5 – 9		3	2 - 9
Log VIX		3	1 – 9		4	2 - 9
US Term spread (5y-FF)		2	2 – 9		2	1 - 9
US Term spread (10y-2y)		1	4 – 9		7	2 - 9
TED		1	1 – 9		3	7 - 9
Intermediary leverage		0	7 – 9		9	5 - 9
Interm. weighted return		0	0 – 9		0	0 - 9
Log Repo		0	7 – 9		2	8 - 9
Log Commercial Paper		4	2 - 9		2	7 - 9

Notes: Nine sample countries in total. We count the coefficient that is above 5% significance level (one-sided test based on Phillips Perron test statistics compared with Dickey Fuller distribution without drift (population value of $\alpha = 0$) for s_t and two-sidedd test based on t-distribution for macro variables). Significance are based on Newey West standard errors with h-1 lags. Regressions with s_t matches the sample period of each global variable. Log SP500, Log Repo and Log Commercial Paper are log linearly detrended. MAR global factor is Miranda-Agrippino and Rey (2020) global factor. GZ spread is U.S. corporate bond credit spread taken from Gilchrist and Zakrajšek (2012). Intermediary leverage ratio and Intermediary weighted return are taken from He et al. (2017). TED is the 3-month Treasury Eurodollar spread. Log Repo and Log Commercial Paper are linearly detrended.

Appendix Table 4:

Summary of individual country 5-year rolling window out-of-sample prediction error, comparing predictive accuracy between:

i) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{X}_{t-61,t-1} X_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{s}_{t-61,t-1} s_t$ (Univariate s_t vs univariate X_t) (Diebold Mariano West test)

ii) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{XX}_{t-61,t-1} X_t + \hat{\beta}^{ss}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{X}_{t-61,t-1} X_t$ (Multivariate v.s. univariate X_t) (Clark West test)

iii) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{XX}_{t-61,t-1} X_t + \hat{\beta}^{ss}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{s}_{t-61,t-1} s_t$ (Multivariate v.s. univariate s_t) (Clark West test)

iv) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{s}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = 0$ (Univariate s_t v.s. random walk model (r.w.) (Clark West test)

v) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{X}_{t-61,t-1} X_t$ and $\widehat{s_{t+h} - s_t} = 0$ (Univariate X_t v.s. random walk model (r.w.)) (Clark West test)

	Independent variables	Univariate s_t vs univariate X_t	Multivariate v.s. univariate X_t	Multivariate v.s. univariate s_t	Univariate s_t v.s. r.w.	Univariate X_t v.s. r.w.
		(i)	(ii)	(iii)	(iv)	(v)
1-month horizon forecast (*h*=1)	US Treasury premium	0	1	1	0	0
	MAR global factor	0	1	0	0	0
	GZ spread	0	0	0	0	0
	Log VIX	0	0	1	0	0
	US Term spread (5y-FF)	0	0	0	0	0
	US Term spread (10y-2y)	0	3	0	0	0
	TED	0	1	0	0	0
	Intermediary leverage	0	0	0	0	0
	Interm. weighted return	0	0	0	0	0
	Log Repo	0	2	0	0	0
	Log Commercial Paper	0	0	0	0	0
1-year horizon forecast (*h*=12)	US Treasury premium	0	8	5	8	4
	MAR global factor	0	7	5	8	4
	GZ spread	0	6	1	7	4
	Log VIX	0	8	2	8	3
	US Term spread (5y-FF)	0	9	4	8	2
	US Term spread (10y-2y)	1	8	3	8	0
	TED	0	6	7	8	4
	Intermediary leverage	0	9	3	8	4
	Interm. weighted return	0	9	0	8	0
	Log Repo	1	9	5	8	6
	Log Commercial Paper	0	9	4	7	6

Notes: Nine sample countries in total. We count the coefficient that is below 5% significance level. Significance are based on Newey West standard errors with *h*-1 lags. Regressions with s_t matches the sample period of each global variable. Regressions with s_t matches the sample period of each global variable. MAR global factor is Miranda-Agrippino and Rey (2020) global factor. GZ spread is U.S. corporate bond credit spread taken from Gilchrist and Zakrajšek (2012). Intermediary leverage ratio and Intermediary weighted return are taken from He et al. (2017). TED is the 3-month Treasury Eurodollar spread. Log Repo and Log Commercial Paper are linearly detrended.

Appendix Table 4: (continued)

Summary of individual country 5-year rolling window out-of-sample prediction error, comparing predictive accuracy between:

i) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{X}_{t-61,t-1} X_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{s}_{t-61,t-1} s_t$ (Univariate s_t vs univariate X_t) (Diebold Mariano West test)

ii) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{XX}_{t-61,t-1} X_t + \hat{\beta}^{ss}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{X}_{t-61,t-1} X_t$ (Multivariate v.s. univariate X_t) (Clark West test)

iii) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{XX}_{t-61,t-1} X_t + \hat{\beta}^{ss}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{s}_{t-61,t-1} s_t$ (Multivariate v.s. univariate s_t) (Clark West test)

iv) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{s}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = 0$ (Univariate s_t v.s. random walk model (r.w.) (Clark West test)

v) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{X}_{t-61,t-1} X_t$ and $\widehat{s_{t+h} - s_t} = 0$ (Univariate X_t v.s. random walk model (r.w.)) (Clark West test)

	Independent variables	Univariate s_t vs univariate X_t	Multivariate v.s. univariate X_t	Multivariate v.s. univariate s_t	Univariate s_t v.s. r.w.	Univariate X_t v.s. r.w.
		(i)	(ii)	(iii)	(iv)	(v)
3-year horizon forecast (*h*=36)	US Treasury premium	3	9	2	9	7
	MAR global factor	1	6	1	9	6
	GZ spread	2	9	3	8	8
	Log VIX	0	9	7	9	8
	US Term spread (5y-FF)	3	9	6	9	8
	US Term spread (10y-2y)	4	9	5	9	8
	TED	4	9	2	9	7
	Intermediary leverage	3	9	4	9	8
	Interm. weighted return	5	9	1	9	6
	Log Repo	7	9	2	9	5
	Log Commercial Paper	5	9	3	9	5
5-year horizon forecast (*h*=60)	US Treasury premium	3	9	6	9	8
	MAR global factor	7	8	3	9	6
	GZ spread	8	9	6	9	7
	Log VIX	6	9	4	9	7
	US Term spread (5y-FF)	5	9	6	9	9
	US Term spread (10y-2y)	4	9	4	9	8
	TED	6	9	7	9	8
	Intermediary leverage	6	9	7	9	7
	Interm. weighted return	7	9	0	9	7
	Log Repo	7	9	3	9	9
	Log Commercial Paper	7	8	2	9	8

Notes: Nine sample countries in total. We count the coefficient that is below 5% significance level. Significance are based on Newey West standard errors with *h*-1 lags. Regressions with s_t matches the sample period of each global variable. Regressions with s_t matches the sample period of each global variable. MAR global factor is Miranda-Agrippino and Rey (2020) global factor. GZ spread is U.S. corporate bond credit spread taken from Gilchrist and Zakrajšek (2012). Intermediary leverage ratio and Intermediary weighted return are taken from He et al. (2017). TED is the 3-month Treasury Eurodollar spread. Log Repo and Log Commercial Paper are linearly detrended.

Appendix Table 5: Regression statistics of in sample forecasting: $s_{t+h} - s_t = \alpha + \beta s_t + e_{t+h}$

	1-month horizon forecast (h=1)		1-year horizon forecast (h=12)		3-year horizon forecast (h=36)		5-year horizon forecast (h=60)	
	Beta	Adjusted R^2	Beta	Adjusted R^2	Beta	Adjusted R^2	Beta	Adjusted R^2
Currency	(1)	(2)	(3)	(4)	(3)	(4)	(5)	(6)
AUD	-0.016	0.004	-0.224***	0.113	-0.672***	0.453	-1.030***	0.727
	(0.011)		(0.110)		(0.142)		(0.138)	
CAD	-0.017	0.006	-0.191***	0.111	-0.622***	0.401	-1.125***	0.732
	(0.010)		(0.091)		(0.204)		(0.178)	
CHF	-0.011	0.002	-0.123***	0.084	-0.404***	0.478	-0.606***	0.710
	(0.009)		(0.074)		(0.079)		(0.099)	
EUR	-0.019	0.005	-0.257***	0.131	-0.733***	0.517	-1.122***	0.776
	(0.013)		(0.118)		(0.143)		(0.105)	
GBP	-0.013	0.000	-0.216***	0.075	-0.618***	0.217	-1.250***	0.436
	(0.011)		(0.153)		(0.298)		(0.342)	
JPY	-0.023	0.008	-0.283***	0.143	-0.946***	0.490	-1.283***	0.682
	(0.013)		(0.140)		(0.198)		(0.263)	
NOK	-0.008	-0.002	-0.193***	0.073	-0.652***	0.328	-1.262***	0.600
	(0.012)		(0.112)		(0.240)		(0.217)	
NZD	-0.020	0.007	-0.252***	0.139	-0.712***	0.570	-0.964***	0.817
	(0.012)		(0.129)		(0.121)		(0.073)	
SEK	-0.018	0.003	-0.301***	0.127	-0.764***	0.417	-1.244***	0.689
	(0.012)		(0.141)		(0.206)		(0.174)	
SA	-0.014	0.002	-0.219***	0.111	-0.689***	0.475	-1.111***	0.759
	(0.011)		(0.114)		(0.169)		(0.152)	
Panel	-0.015	0.003	-0.217***	0.107	-0.657***	0.419	-1.030***	0.661
	(0.008)		(0.091)		(0.125)		(0.097)	
Observations								
Single	254		243		219		195	
Panel	2286		2187		1971		1755	

Notes: SA is the regression with simple average of all nine currencies. * p<0.1, ** p<0.05, *** p<0.01 for one-sided test based on Phillips Perron (1988) test statistics compared with Dickey Fuller distribution without drift (population value of $\alpha = 0$) and Choi (2001) test statistics for panel regressions. Panel regressions include country fixed effect. Newey-West standard errors and Driscoll Kraay (1998) standard errors (for panel regression) with h-1 lags in parentheses.

Appendix Table 6: 5 year rolling window out-of-sample prediction error: $s_{t+h} - s_t = \alpha + \beta_{t-61,t} s_t$ vs random walk model ($s_{t+h} - s_t = 0$)

Clark West Statistics	1-month horizon forecast (*h*=1)	1-year horizon forecast (*h*=12)	3-year horizon forecast (*h*=36)	5-year horizon forecast (*h*=60)
Currency	(1)	(2)	(3)	(3)
AUD	-1.067	1.292*	2.927***	4.476***
CAD	-0.127	2.896***	3.861***	3.066***
CHF	-0.024	2.813***	2.257**	2.756***
EUR	0.043	3.988***	3.833***	7.981***
GBP	-1.501	2.275**	2.192**	4.101***
JPY	-0.903	1.839**	2.355***	3.918***
NOK	-1.360	2.495***	2.769***	2.087**
NZD	-0.438	1.693**	2.289**	3.830***
SEK	-0.679	2.102**	2.311**	1.988**
SA	-0.975	1.848**	3.444***	2.957***
Panel	-0.493	2.001**	4.300***	5.018***

Notes: SA is the regression with simple average of all nine currencies. * p<0.1, ** p<0.05, *** p<0.01 for one-sided test. Panel regressions include country fixed effect. Newey-West standard errors and Driscoll Kraay (1998) standard errors (for panel regression) with *h*-1 lags in parentheses.

Online Appendix 3: Simulation Methods

First we describe the simulation methods for the in-sample forecasts (Table 3), then for the out-of-sample exercises in Tables 4.

<u>Data</u>

For any given currency, we have data running from January 1999 to March 2020, for 255 data points. Let January 1999 be date $t=1$. T=255.

<u>In-sample</u>

For h=1,12,36,60, we estimate

(1) $s_{t+h} - s_t = a + b\, s_t + u_{t+h}$

We then calculate the $\hat{b}$ and the t-statistics of $\hat{b}$ using Newey-West standard error of $h-1$ lags.

For the panel specification, we estimate

$s_{i,t+h} - s_{i,t} = a_i + b s_{i,t} + u_{i,t+h}$

We then calculate the $\hat{b}_i$ and the t-statistics of $\hat{b}_i$ using Driscoll-Kraay standard error of $h-1$ lags.

Also, we record the adjusted R^2 for $h = 1, 12, 36, 60$.

<u>Out-of-sample</u>

For h=1,12,36,60, we use 60 data points to estimate

(2) $s_{t+h} - s_t = a_{h,t} + b_{h,t} s_t + u_{t+h}$

for $t = 1, \ldots, T-h$. That is, we run rolling regressions with 60 observations each.

We forecast h periods ahead using the formula

$\tilde{s}_{t+1+h} - s_{t+1} = \hat{a}_{h,t} + \hat{b}_{h,t} s_{t+1} + \hat{u}_{t+h}$

for $t = 1, \ldots, T-h$, where the ^ over variables refers to the estimated values, and $\tilde{s}_{t+1+i}$ is the forecasted value. That is, for $h = 1, 12, 36, 60$, we make h-period ahead forecasts.

We then calculate the $T-60-h$ forecast errors, $\tilde{s}_{t+1+h} - s_{t+1+h}$, and calculate their mean-squared error. We use the Clark-West statistic to compare that m.s.e. to the m.s.e. of the forecast of no change in the exchange rate, for which the forecast error is $s_{t+1+h} - s_{t+1}$. We use Newey-West standard error of $h-1$ lags to correct for the Clark-West statistics.

For the panel specification, we forecast *h* periods ahead using the following formula and apply the same procedure above

$$\tilde{s}_{i,t+1+h} - s_{i,t+1} = \hat{a}_{i,h,t} + \hat{b}_{h,t} s_{i,t+1} + \hat{u}_{i,t+h}$$

We use Driscoll-Kraay standard error of $h-1$ lags to correct for the panel Clark-West statistics.

Also, we record the $T-60-h$ values of adjusted R^2 and record the min, max, 50th, 90th, 95th, and 99th percentile of those $T-60-h$ regressions for $h = 1, 12, 36, 60$.

Monte Carlo

We perform $K = 5000$ iterations of the following procedure:

In iteration *k*, $k = 1, \ldots, K$, we create an artificial time series that has $2000+T$ elements as follows:

Under the null of a zero-drift random walk, we calculate the variance of $s_{t+1} - s_t$ for the sample of $T-1$ observations of $s_{t+1} - s_t$. We then construct an artificial time series of $2000+T-1$ random variables drawing from a normal distribution with mean zero and variance equal to the sample variance of $s_{t+1} - s_t$. Call each of these ε_j, $j = 1, \ldots, 2000+T-1$. Then we construct a series of length $2000+T$ with the following properties: The first element, call it x_1 is equal to zero. Then for $j = 1, \ldots, 2000+T-1$, we have $x_{j+1} = x_j + \varepsilon_j$. Now take the last *T* values of x_j. Call these $\dot{s}_t$ for $t = 1, \ldots, T$, which is the simulated exchange rate series for iteration *k*.

For the panel specification, under the null of a zero-drift random walk, we calculate the variance covariance matrix of $s_{i,t+1} - s_{i,t}$ for the sample of $T-1$ observations of $s_{i,t+1} - s_{i,t}$ where *i* is the index of a currency (*i={1,2…I}*). We then construct an artificial time series of $(2000+T-1)\times I$ random variables drawing from a multivariate normal distribution with mean zero and variance and covariance equal to the sample variance covariance of $s_{i,t+1} - s_{i,t}$. Call each of these $\varepsilon_{i,j}$, $j = 1, \ldots, 2000+T-1$. Then we construct a series of length $2000+T$ with the following properties: The first element, call it $x_{i,1}$ is equal to zero. Then for $j = 1, \ldots, 2000+T-1$

, we have $x_{i,j+1} = x_{i,j} + \varepsilon_{i,j}$. Now take the last *T* values of $x_{i,j}$. Call these $\dot{s}_{i,t}$ for $t = 1,\ldots,T$, which is the simulated panel exchange rate series for iteration *k*.

Next proceed exactly as in the Data section, but use $\dot{s}_t$ as the "data" rather than s_t.

So in each iteration *k*, for in-sample, we record a coefficient estimate $\hat{b}$, t-statistic of $\hat{b}$ and adjusted R^2. For out-of-sample, we record a Clark-West statistic and adjusted R^2 for the min, max, 50th, 90th, 95th, and 99th percentile of those $T-60-h$ rolling regressions.

Repeat this *K* times so we have the Monte Carlo distribution of these statistics.

<u>Bootstrap (with replacement)</u>

We essentially follow the same steps as above for the Monte Carlo, but the creation of the artificial data is different.

We perform $K = 5000$ iterations of the following procedure:

In iteration *k*, $k = 1,\ldots,K$, we create an artificial time series that has $2000+T$ elements as follows:

Under the null of a zero-drift random walk, we collect the $T-1$ observations of $s_{t+1} - s_t$. We then use a random number generator that chooses a value from 1 to $T-1$ with equal probability. We construct an artificial time series of $2000+T-1$ random variables, calling each element of this series ε_j, $j = 1,\ldots,2000+T-1$. ε_j is created as follows: For each *j*, we use the random number generator to choose a numeral *n* with equal probability, and then we set ε_j to be the *n*th element of $s_{t+1} - s_t$ Then we construct a series of length $2000+T$ with the following properties: The first element, call it x_1 is equal to zero. Then for $j = 1,\ldots,2000+T-1$, we have $x_{j+1} = x_j + \varepsilon_j$. Now take the last *T* values of x_j. Call these $\ddot{s}_t$ for $t = 1,\ldots,T$, which is the simulated exchange rate series for iteration *k*.

For the panel specification, under the null of a zero-drift random walk, we collect $T-1$ observations of $s_{i,t+1} - s_{i,t}$, where *i* is the index of a currency (*i={1,2…I}*) and for each *t* there are *I*-tuple of exchange rates. We use a random number generator that chooses an *I*-tuple from 1 to $T-1$ with equal probability. We construct an artificial time series of $(2000+T-1)\times I$ random variables, calling each element of this series $\varepsilon_{i,j}$, $j=1,\ldots,2000+T-1$. $\varepsilon_{i,j}$ is created as follows: For each *j*, we use the random number generator to choose a numeral *n* with equal probability, and then we set each element in ε_j to be the *n*th tuple of $s_{i,t+1}-s_{i,t}$. Then we construct a series of length $2000+T$ with the following properties: The first *I*-tuple, call it $x_{i,1}$ is equal to zero. Then for $j=1,\ldots,2000+T-1$, we have $x_{i,j+1}=x_{i,j}+\varepsilon_{i,j}$. Now take the last *T* values of $x_{i,j}$. Call these $\dot{s}_{i,t}$ for $t=1,\ldots,T$, which is the simulated panel exchange rate series for iteration *k*.

Next proceed exactly as in the Data section, but use $\dot{s}_t$ as the "data" rather than s_t .

So in each iteration *k*, for in-sample, we record a coefficient estimate $\hat{b}$, t-statistic of $\hat{b}$ and adjusted R^2 . For out-of-sample, we record a Clark-West statistic and adjusted R^2 for the min, max, 50th, 90th, 95th, and 99th percentile of those $T-60-h$ rolling regressions.

Repeat this *K* times so we have the bootstrap distribution of these statistics.

Online Appendix 4: Additional notes to the tables

Table 1:

Table 1 does three in-sample regressions:

$s_{t+h} - s_t = \alpha + \beta^X X_t + e_t$, $s_{t+h} - s_t = \alpha + \beta^s s_t + e_t$ and $s_{t+h} - s_t = \alpha + \beta^{XX} X_t + \beta^{ss} s_t + e_t$

Parameters estimated by OLS with Newey-West standard errors of h-1 lags.

Inference of significance for s_t is based on Phillips Perron test statistics and compare to the Dickey Fuller distribution

Significance inference for macro variable is based on usual t statistics.

Table 2:

The regression is out-of-sample rolling regression, rolling window is 5 year, compared across 5 models. The first comparison involves two non-nested model, for which we compare MSPE of: model1 based on X_t minus model 2 based on s_t. Therefore, a positive statistic means the MSPE is larger for X_t than s_t. We use the Diebold Mariano statistics. The significance stars are based on a two-sided test.

i) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^X_{t-61,t-1} X_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^s_{t-61,t-1} s_t$ (Univariate s_t vs univariate X_t) (Diebold Mariano West test)

The rest of the tests involve nested model comparisons, we applied the Clark West statistics. The significance stars are based on one-sided tests. A positive statistic indicates the larger model is better in Clark West sense.

ii) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{XX}_{t-61,t-1} X_t + \hat{\beta}^{ss}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^X_{t-61,t-1} X_t$ (Multivariate v.s. univariate X_t) (Clark West test)

iii) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^{XX}_{t-61,t-1} X_t + \hat{\beta}^{ss}_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^s_{t-61,t-1} s_t$ (Multivariate v.s. univariate s_t) (Clark West test)

iv) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^s_{t-61,t-1} s_t$ and $\widehat{s_{t+h} - s_t} = 0$ (Univariate s_t v.s. random walk model (r.w.) (Clark West test)

v) $\widehat{s_{t+h} - s_t} = \hat{\alpha} + \hat{\beta}^X_{t-61,t-1} X_t$ and $\widehat{s_{t+h} - s_t} = 0$ (Univariate X_t v.s. random walk model (r.w.)) (Clark West test)

The Clark West statistics is calculated as follows:

A positive statistic indicates the larger model is the better one in the Clark West sense.

The squared regression error is obtained for each prediction,

$f_{1,t} = (y_t - \hat{y}_{1,t})^2$, $f_{2,t} = (y_t - \hat{y}_{2,t})^2$ and $adj = (\hat{y}_{1,t} - \hat{y}_{2,t})^2$

For each period, we compute: $\hat{f}_t = f_{1,t} - (f_{2,t} - adj)$

We regress $\hat{f}_t$ on a constant and test if the constant is significantly bigger than zero (one-sided test). (Clark West 2007) The inference is based on usual t-statistics of the constant term. We use Newey West standard errors for accounting serial correlation in the Clark West test.

<u>Table 3:</u>

The regression is in sample: $s_{t+h} - s_t = \alpha + \beta s_t + e_t$

Actual data

Single country

Parameters estimated by OLS, Newey-West standard errors of *h*-1 lags in parentheses. Significance inference is based on Phillips Perron test statistics and compared to the Dickey Fuller distribution.

Panel

The panel allows for country fixed effect.

Parameter are estimated by OLS, the standard errors are Driscoll Kraay (1998) standard errors with h-1 lags in parentheses.

Significance inference is based on Choi (2001), which is a panel version of Phillips Perron test statistics and compared to inverse chi square distribution.

Simulation

The regression is in sample: $s_{t+h} - s_t = \alpha + \beta s_t + e_t$

We report the beta, NW *t* statistics and adjusted R2.

Single country

The data are simulated with no drift

Parameters are estimated by OLS, the standard errors and *t* statistics are based on Newey-West standard errors with *h*-1 lags in parentheses.

Panel

The panel are simulated with no drift, but simulated with cross country covariance matrix

We regress restricting the slope coefficients to be the same. Parameters are estimated by OLS, the standard errors are Driscoll Kraay (1998) standard errors with *h*-1 lags in parentheses.

Table 4:

The regression is out of sample rolling regression, rolling window is 5 year: $s_{t+h} - s_t = \alpha + \beta_{t-61,t-1} s_t + e_t$ vs a random walk model

Panel

The panel allows for country fixed effect.

We use Driscoll Kraay (1998) for accounting serial correlation in the Clark West test.

For the Clark West statistics, we still regress on one single constant, (i.e. no country specific constant) Significance is based on one-sided test.

Simulation

The regression is out of sample: $s_{t+h} - s_t = \alpha + \beta_{t-61,t-1} s_t + e_t$ vs random walk

Single country

Parameters are estimated by OLS, the standard errors are Newey West standard errors with *h*-1 lags in parentheses.

Panel

We regress restricting the beta coefficient to be the same. Parameters are estimated by OLS, the standard errors are Driscoll Kraay (1998) standard errors with *h*-1 lags in parentheses.

Then we calculate the Clark West statistics based on the prediction of this regression.

The test statistics are calculated as in Table 2.

Table 5 and 6

We first estimate two equations from the actual data:

$$s_t - s_{t-1} = u_{1,t}$$

$$X_t = \alpha + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \delta_1 s_{t-1} + \delta_2 s_{t-2} + u_{2,t}$$

We correct for the bias using Kilian (1998) method. We compute the empirically estimated coefficients ($\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\delta}_1$ and $\hat{\delta}_2$) and variance covariance matrix of the error terms. We use these parameter estimates to simulate the series $\dot{s}_t$ and $\dot{X}_t$ with empirical sample size (*T*) 5000 times using a Monte Carlo method. Within each simulation *k*, we re-estimate β_1, β_2, δ_1 and δ_2. Call it $\tilde{\beta}_{1,k}$, $\tilde{\beta}_{2,k}$, $\tilde{\delta}_{1,k}$ and $\tilde{\delta}_{2,k}$. We then take average of these estimates across simulation, i.e.

$\tilde{\beta}_1 = \frac{\sum_{k=1}^{5000} \tilde{\beta}_{1,k}}{5000}$. Our empirical estimates of the bias are $\tilde{\beta}_1 - \hat{\beta}_1$, $\tilde{\beta}_2 - \hat{\beta}_2$, $\tilde{\delta}_1 - \hat{\delta}_1$, $\tilde{\delta}_2 - \hat{\delta}_2$. Finally, the bias adjusted estimates are $\widehat{\beta}_1 \equiv \hat{\beta}_1 - [\tilde{\beta}_1 - \hat{\beta}_1]$, $\widehat{\beta}_2 \equiv \hat{\beta}_2 - [\tilde{\beta}_2 - \hat{\beta}_2]$, $\widehat{\delta}_1 = \tilde{\delta}_1 - [\tilde{\delta}_1 - \hat{\delta}_1]$, $\widehat{\delta}_2 = \tilde{\delta}_2 - [\tilde{\delta}_2 - \hat{\delta}_2]$.

We use $s_t - s_{t-1} = e_{1,t}$ and $X_t = \alpha + \widehat{\beta}_1 X_{t-1} + \widehat{\beta}_2 X_{t-2} + \widehat{\delta}_1 s_{t-1} + \widehat{\delta}_2 s_{t-2} + e_{2,t}$ to generate the artificial data for the regression of interest: $s_{t+h} - s_t = \alpha + \beta_1 X_t + e_t$

Table 5 reports the in-sample estimates and Table 6 reports the out-of-sample estimates.