Liquidity and Exchange Rates: An Empirical Investigation

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Supplementary Appendix B

1. The Model

We begin with a derivation of the home relative to foreign Phillips curve. We modify the standard Calvo-pricing equation in two ways. First, we assume that nominal prices must be set one period in advance. We make this assumption because, in practice, the response of nominal prices to current period shocks is so small relative to the response of nominal exchange rates, that a model with predetermined prices better represents reality in an open-economy framework. A fraction of firms, θ, are allowed to change their prices optimally each period, but the price they set at time $t-1$ is for the time t period. Let $p_t^{r,H}$ be the price for firms that reset their prices (which is identical for all such firms, because as in the standard New Keynesian framework, they face identical costs and demand functions.)

The remaining firms do not change their price optimally, but we assume that these firms build in an automatic price adjustment. We do not specify the trend term but impose a particular consistency restriction below. We let τ_t^H be the trend adjustment for home prices in the home country (set at time $t-1$.) The firms that adjust their price optimally consider any current disequilibrium in prices in planning their price increase, while the other firms simply adjust the price at the trend rate.

We have:

$$(1) \qquad p_t^H - p_{t-1}^H = \theta\left(p_t^{r,H} - p_{t-1}^H\right) + (1-\theta)\tau_t^H .$$[1]

The foreign currency price of home goods is set in a similar way:

$$p_t^{*H} - p_{t-1}^{*H} = \theta\left(p_t^{*r,H} - p_{t-1}^{*H}\right) + (1-\theta)\tau_t^{*H}$$

We now make two simplifying assumptions about the price setting process. The first is that firms, when they reset their price, set prices in such a way that there is no expected pricing to market: $p_t^{*r,H} = p_t^{r,H} - E_{t-1}s_t$. We can justify that assumption on the grounds that it is too costly for firms to calculate reset prices for each market they serve. As in the producer currency pricing model, we assume that firms calculate a single reset price, but then translate that price into the currency of each market they service. The local-currency price then remains unchanged until the next opportunity for price resetting. The second assumption is that, while we are agnostic about the process by which firms set the trend adjustment of their prices, we impose the following consistency requirement: $\tau_t^{*H} = \tau_t^H + E_{t-1}s_t - s_{t-1}$. That is, firms form a forecast of the exchange rate change, and then align their trend adjustments so that they are expected to be consistent, when expressed in a common currency, in the home and foreign market. These assumptions imply:

$$(2) \qquad p_t^{*H} - p_{t-1}^{*H} = \theta\left(p_t^{r,H} - E_{t-1}s_t - p_{t-1}^{*H}\right) + (1-\theta)\left(\tau_t^H - \left(E_{t-1}s_t - s_{t-1}\right)\right).$$

Subtracting (2) from (1), we find:

$$(3) \qquad E_{t-1}s_t - s_{t-1} + p_t^{*H} - p_{t-1}^{*H} - \left(p_t^H - p_{t-1}^H\right) = \theta\left(p_{t-1}^H - s_{t-1} - p_{t-1}^{*H}\right).$$

[1] See Engel (2019) for a study of the relationship of the price setting behavior in this model compared to the more standard Calvo pricing framework.

The expected change in the pricing to market arises from the adjustments of the fraction θ of firms that reset their prices each period.

An analogous equation can be derived for the prices set by the foreign firm: p_t^{*F} in foreign currency for sale in the foreign country, and p_t^F in home currency for sale in the home country:

(4) $$E_{t-1}s_t - s_{t-1} + p_t^{*F} - p_{t-1}^{*F} - \left(p_t^F - p_{t-1}^F\right) = \theta\left(p_{t-1}^F - s_{t-1} - p_{t-1}^{*F}\right).$$

We assume that consumption preferences over the two goods are identical so that the real exchange rate is driven entirely by the deviations from the law of one price that arise from pricing to market. The log of the consumer price basket in each country is a weighted average of the logs of the prices of foreign-produced and home-produced goods. Taking the weighted average of equations (3) and (4), we arrive at:

(5) $$\pi_t - \pi_t^* = \theta q_{t-1} + E_{t-1}s_t - s_{t-1}.$$

The equations of the model are then given by:

(6) $$i_t^{*m} + E_t s_{t+1} - s_t - i_t^m = r_t,$$

(7) $$\eta_t = \gamma_t - \gamma_t^* = \left(i_t^m - i_t\right) - \left(i_t^{m*} - i_t^*\right) = \left(i_t^m - i_t^{m*}\right) - \left(i_t - i_t^*\right).$$

(8) $$i_t^* + E_t s_{t+1} - s_t - i_t = \eta_t + r_t.$$

(9) $$\eta_t = \alpha\left(i_t - i_t^*\right) + v_t, \quad \alpha > 0.$$

(10) $\pi_t - \pi_t^* = \theta q_{t-1} + E_{t-1}s_t - s_{t-1}$.

(11) $i_t - i_t^* = \sigma\left(\pi_t - \pi_t^*\right) - \psi\eta_t + u_t - u_t^*$.

(12) $u_t - u_t^* = \delta\left(u_{t-1} - u_{t-1}^*\right) + \xi_t, \quad 0 \le \delta < 1$

(13) $v_t = \rho v_{t-1} + \varepsilon_t$,

(14) $r_t = \varsigma r_{t-1} + \omega_t, \qquad 0 \le \varsigma < 1$.

2. Derivation of solution for q_t

As a first step, using (11) and (9), we have

$$i_t - i_t^* = \sigma\left(\pi_t - \pi_t^*\right) - \psi\left(\alpha\left(i_t - i_t^*\right) + v_t\right) + u_t - u_t^*, \text{ so}$$

(15) $i_t - i_t^* = \dfrac{\sigma}{1+\alpha\psi}\left(\pi_t - \pi_t^*\right) - \dfrac{\psi}{1+\alpha\psi}v_t + \dfrac{1}{1+\alpha\psi}\left(u_t - u_t^*\right)$

We can solve the model by the method of undetermined coefficients. We guess a solution of the form:

(16) $q_t = a\left(\pi_t - \pi_t^*\right) + b\left(u_t - u_t^*\right) + cv_t + fr_t$,

where a, b, c and f are undetermined coefficients to be solved for.

We have:

(17)
$$\begin{aligned} E_t q_{t+1} &= aE_t\left(\pi_{t+1}-\pi^*_{t+1}\right)+bE_t\left(u_{t+1}-u^*_{t+1}\right)+cE_t v_{t+1}+fE_t r_{t+1} \\ &= aE_t\left(\pi_{t+1}-\pi^*_{t+1}\right)+b\delta\left(u_t-u^*_t\right)+c\rho v_t+f\varsigma r_t \end{aligned}$$

Now,

(18)
$$\begin{aligned} E_t\left(\pi_{t+1}-\pi^*_{t+1}\right) &= E_t s_{t+1}-s_t-\left(E_t q_{t+1}-q_t\right)=E_t s_{t+1}-s_t+\theta q_t \\ &= i_t-i^*_t+\eta_t+r_t+\theta q_t=\left(1+\alpha\right)\left(i_t-i^*_t\right)+v_t+r_t+\theta q_t \\ &= \theta q_t+\frac{\sigma\left(1+\alpha\right)}{1+\alpha\psi}\left(\pi_t-\pi^*_t\right)+\frac{1+\alpha}{1+\alpha\psi}\left(u_t-u^*_t\right)+\frac{1-\psi}{1+\alpha\psi}v_t+r_t \end{aligned}$$

where we have used, from (10) that $E_t q_{t+1}-q_t=-\theta q_t$.

Substituting from (18) into (17), we get:

(19)
$$\begin{aligned} E_t q_{t+1} &= a\left[\theta q_t+\frac{\sigma\left(1+\alpha\right)}{1+\alpha\psi}\left(\pi_t-\pi^*_t\right)+\frac{1+\alpha}{1+\alpha\psi}\left(u_t-u^*_t\right)+\frac{1-\psi}{1+\alpha\psi}v_t+r_t\right]+b\delta\left(u_t-u^*_t\right)+c\rho v_t+f\varsigma r_t \\ &= a\theta q_t+\frac{a\sigma\left(1+\alpha\right)}{1+\alpha\psi}\left(\pi_t-\pi^*_t\right)+\left[\frac{a\left(1+\alpha\right)+b\delta\left(1+\alpha\psi\right)}{1+\alpha\psi}\right]\left(u_t-u^*_t\right)+\left[\frac{a\left(1-\psi\right)+c\rho\left(1+\alpha\psi\right)}{1+\alpha\psi}\right]v_t+\left[a+f\varsigma\right]r_t \\ &= a\theta\left[a\left(\pi_t-\pi^*_t\right)+b\left(u_t-u^*_t\right)+cv_t+fr_t\right]+ \\ &\quad \frac{a\sigma\left(1+\alpha\right)}{1+\alpha\psi}\left(\pi_t-\pi^*_t\right)+\left[\frac{a\left(1+\alpha\right)+b\delta\left(1+\alpha\psi\right)}{1+\alpha\psi}\right]\left(u_t-u^*_t\right)+\left[\frac{a\left(1-\psi\right)+c\rho\left(1+\alpha\psi\right)}{1+\alpha\psi}\right]v_t+\left[a+f\varsigma\right]r_t \\ &= \frac{a\left[\sigma\left(1+\alpha\right)+a\theta\left(1+\alpha\psi\right)\right]}{1+\alpha\psi}\left(\pi_t-\pi^*_t\right)+\frac{\left[a\left(1+\alpha\right)+b\left(\delta+a\theta\right)\left(1+\alpha\psi\right)\right]}{1+\alpha\psi}\left(u_t-u^*_t\right) \\ &\quad +\frac{\left[a\left(1-\psi\right)+c\left(\rho+a\theta\right)\left(1+\alpha\psi\right)\right]}{1+\alpha\psi}v_t+\left[a+f\varsigma+fa\theta\right]r_t \end{aligned}$$

But, also, using from (10) that $E_t q_{t+1}=\left(1-\theta\right)q_t$, we have from (16):

(20) $$E_t q_{t+1}=a\left(1-\theta\right)\left(\pi_t-\pi^*_t\right)+b\left(1-\theta\right)\left(u_t-u^*_t\right)+c\left(1-\theta\right)v_t+f\left(1-\theta\right)r_t.$$

Now, equating the right-hand sides of (19) and (20), we get:

$$\frac{a\left[\sigma(1+\alpha)+a\theta(1+\alpha\psi)\right]}{1+\alpha\psi}=a(1-\theta)$$

from which we derive:

$$a=\frac{-\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)}.$$

Then, from (19) and (20), we get:

$$a(1+\alpha)=b(1-\theta)(1+\alpha\psi)-b(\delta+a\theta)(1+\alpha\psi).$$

Substituting in the solution for *a*, we get:

$$b=\frac{-(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}.$$

From (19) and (20), we also get:

$\frac{\left[a(1-\psi)+c(\rho+a\theta)(1+\alpha\psi)\right]}{1+\alpha\psi}=c(1-\theta)$, from which we derive

$$c=\frac{-(1-\psi)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left[\sigma(1+\alpha)-\rho(1+\alpha\psi)\right]}$$

And, finally, from (19) and (20), we get:

$a+f\varsigma+fa\theta=f(1-\theta)$, which gives us:

$$f=\frac{-\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}$$

We can therefore write solution for the real exchange rate as:

(21)

$$q_t = -\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta(1+\alpha\psi)}\right)(\pi_t - \pi_t^*) - \left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)(u_t - u_t^*)$$
$$-\left(\frac{(1-\psi)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left[\sigma(1+\alpha)-\rho(1+\alpha\psi)\right]}\right)v_t - \left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}\right)r_t .$$

3. <u>Derivation of solution for</u> $s_t - s_{t-1}$

Use (15) to write $u_t - u_t^* = (1+\alpha\psi)(i_t - i_t^*) - \sigma(\pi_t - \pi_t^*) + \psi v_t$, and substitute this into (21)

:

$$q_t = -\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta(1+\alpha\psi)}\right)(\pi_t - \pi_t^*)$$
$$-\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)\left((1+\alpha\psi)(i_t - i_t^*) - \sigma(\pi_t - \pi_t^*) + \psi v_t\right)$$
$$-\left(\frac{(1-\psi)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left[\sigma(1+\alpha)-\rho(1+\alpha\psi)\right]}\right)v_t - \left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}\right)r_t$$

The coefficients on $\pi_t - \pi_t^*$ are

$$-\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta(1+\alpha\psi)}\right) + \sigma\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)$$
$$= \frac{\delta\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)} .$$

The coefficients on v_t are

$$
\begin{aligned}
&-\left(\frac{(1-\psi)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left[\sigma(1+\alpha)-\rho(1+\alpha\psi)\right]}\right)-\psi\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta(1+\alpha\psi)\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right) \\
&=\frac{-\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}
\end{aligned}
.
$$

We have:

$$
\begin{aligned}
q_t =& \frac{\delta\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\left(\pi_t-\pi_t^*\right)-\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)\left(i_t-i_t^*\right) \\
&-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right)v_t \\
&-\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}\right)r_t
\end{aligned}
.
$$

Now we can use the fact that $s_t - s_{t-1} = q_t - q_{t-1} + \pi_t - \pi_t^*$ and the equation above to write:

(22)

$$
\begin{aligned}
s_t - s_{t-1} =& \frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\left(\pi_t-\pi_t^*\right)-\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)\left(i_t-i_t^*\right) \\
&-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right)v_t \\
&-\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}\right)r_t - q_{t-1}
\end{aligned}
.
$$

But
$$
\begin{aligned}
\pi_t-\pi_t^* &= s_t - s_{t-1} - \left(q_t - q_{t-1}\right) = E_{t-1}s_t - s_{t-1} - \left(E_{t-1}q_t - q_{t-1}\right) \\
&= i_{t-1} - i_{t-1}^* + \eta_{t-1} + r_{t-1} + \theta q_{t-1}
\end{aligned}
.
$$

Also, we have $v_t = \eta_t - \alpha\left(i_t - i_t^*\right)$. Substituting these into (22), we get

(23)

$$
\begin{aligned}
s_t - s_{t-1} = {} & \frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\left(i_{t-1}-i_{t-1}^*+\eta_{t-1}+r_{t-1}+\theta q_{t-1}\right) \\
& -\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)\left(i_t-i_t^*\right) \\
& -\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right)\left(\eta_t-\alpha\left(i_t-i_t^*\right)\right). \\
& -\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}\right)r_t - q_{t-1}
\end{aligned}
$$

Rearrange terms. The coefficient on q_{t-1} is given by:

$$
\frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\sigma(1+\alpha)-\delta(1+\alpha\psi)}-1=-\left(\frac{\sigma(1+\alpha)(1-\theta-\delta)}{\sigma(1+\alpha)-\delta(1+\alpha\psi)}\right)
$$

The coefficient on $i_t - i_t^*$ is:

$$
\begin{aligned}
& -\left(\frac{(1+\alpha)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\right)+\alpha\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right) \\
& =-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[(1+\alpha)(\sigma-\rho)+\alpha\delta(1-\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right)
\end{aligned}
$$

So we can write:

$$s_t - s_{t-1} = -\left(\frac{\sigma(1+\alpha)(1-\theta-\delta)}{\sigma(1+\alpha)-\delta(1+\alpha\psi)} \right) q_{t-1}$$

$$-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[(1+\alpha)(\sigma-\rho)+\alpha\delta(1-\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)} \right)\left(i_t - i_t^*\right)$$

$$-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)} \right)\eta_t$$

$$+\frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}\left(i_{t-1}-i_{t-1}^*+\eta_{t-1}+r_{t-1}\right)-\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]} \right) r_t$$

Now, we get the equation in the form that is used for estimating, which involves $i_t - i_t^* - \left(i_{t-1}-i_{t-1}^*\right)$ and $\eta_t - \eta_{t-1}$. Also, use the fact that $r_t = \varsigma r_{t-1} + \omega_t$ from equation (14).

$$s_t - s_{t-1} = -\left(\frac{\sigma(1+\alpha)(1-\theta-\delta)}{\sigma(1+\alpha)-\delta(1+\alpha\psi)} \right) q_{t-1}$$

$$-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[(1+\alpha)(\sigma-\rho)+\alpha\delta(1-\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)} \right)\left(i_t - i_t^* - \left(i_{t-1}-i_{t-1}^*\right)\right)$$

$$-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)} \right)\left(\eta_t - \eta_{t-1}\right)+$$

$$\left[\begin{array}{l} \dfrac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)} - \\ \left(\dfrac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[(1+\alpha)(\sigma-\rho)+\alpha\delta(1-\rho)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)} \right) \end{array} \right]\left(i_{t-1}-i_{t-1}^*\right)$$

$$+\left[\begin{array}{l} \dfrac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)} - \\ \left(\dfrac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)} \right) \end{array} \right]\eta_{t-1}$$

$$-\left(\frac{\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]} \right)\omega_t + \left[\frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)} - \frac{\zeta\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]} \right] r_{t-1}$$

We can rewrite the term on $i_{t-1}-i^*_{t-1}$ as:

$$\frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[(1+\alpha)(\sigma-\rho)+\alpha\delta(1-\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right)$$

$$=\frac{(\theta+\delta-1)\left[\sigma(1+\alpha)-(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\rho(1+\alpha\psi)\right]+(\rho-\delta)\alpha(1-\psi)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}$$

The term on η_{t-1} can be written as:

$$\frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}-\left(\frac{\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\left(\delta(1-\psi)+\rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}\right)=$$

$$\frac{(\theta+\delta-1)\left[\sigma(1+\alpha)-(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\rho(1+\alpha\psi)\right]-(\rho-\delta)(1-\psi)\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha)-\rho(1+\alpha\psi)\right)}$$

To simplify the term on r_{t-1}, we can write:

$$\frac{(\theta+\delta)\sigma(1+\alpha)-\delta(1+\alpha\psi)}{\theta\left(\sigma(1+\alpha)-\delta(1+\alpha\psi)\right)}-\frac{\zeta\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}=$$

$$\frac{\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]\left[(\theta+\delta-\zeta)\sigma(1+\alpha)-(1+\alpha\psi)(\delta-\zeta(1-\theta))\right]+(\delta-\zeta)(1+\alpha\psi)\zeta\left[\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)\right]}{\theta\left[\sigma(1+\alpha)-\delta(1+\alpha\psi)\right]\left[\sigma(1+\alpha)-\zeta(1+\alpha\psi)\right]}$$

With these simplifications, we can arrive at the equation in the text:

(24) $$s_t-s_{t-1}=\beta_1 q_{t-1}+\beta_2(\eta_t-\eta_{t-1})+\beta_3\left(i_t-i^*_t-\left(i_{t-1}-i^*_{t-1}\right)\right)+\beta_4\eta_{t-1}+\beta_5\left(i_{t-1}-i^*_{t-1}\right)+z_{j,t}$$

where

(25) $$\beta_1=-\left(\frac{\sigma(1+\alpha)(1-\theta-\delta)}{\sigma(1+\alpha)-\delta(1+\alpha\psi)}\right),$$

(26) $$\beta_2 = -\left(\frac{\left[\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi) \right] \left[\sigma(1+\alpha) - \left(\delta(1-\psi) + \rho\psi(1+\alpha) \right) \right]}{\theta\left(\sigma(1+\alpha) - \delta(1+\alpha\psi) \right)\left(\sigma(1+\alpha) - \rho(1+\alpha\psi) \right)} \right),$$

(27) $$\beta_3 = -\left(\frac{\left[\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi) \right] \left[(1+\alpha)(\sigma-\rho) + \alpha\delta(1-\psi) \right]}{\theta\left(\sigma(1+\alpha) - \delta(1+\alpha\psi) \right)\left(\sigma(1+\alpha) - \rho(1+\alpha\psi) \right)} \right)$$

(28)
$$\beta_4 = \frac{(\theta+\delta-1)\left[\sigma(1+\alpha) - (1+\alpha\psi) \right]\left[\sigma(1+\alpha) - \rho(1+\alpha\psi) \right] - (\rho-\delta)(1-\psi)\left[\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi) \right]}{\theta\left(\sigma(1+\alpha) - \delta(1+\alpha\psi) \right)\left(\sigma(1+\alpha) - \rho(1+\alpha\psi) \right)}$$

(29)
$$\beta_5 = \frac{(\theta+\delta-1)\left[\sigma(1+\alpha) - (1+\alpha\psi) \right]\left[\sigma(1+\alpha) - \rho(1+\alpha\psi) \right] + (\rho-\delta)\alpha(1-\psi)\left[\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi) \right]}{\theta\left(\sigma(1+\alpha) - \delta(1+\alpha\psi) \right)\left(\sigma(1+\alpha) - \rho(1+\alpha\psi) \right)}$$

and,

$$z_t = -\left(\frac{\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi)}{\theta\left[\sigma(1+\alpha) - \zeta(1+\alpha\psi) \right]} \right)\omega_t + z_1 r_{t-1}, \text{ where}$$

$$z_1 = \frac{\left[\sigma(1+\alpha) - \zeta(1+\alpha\psi) \right]\left[(\theta+\delta-\zeta)\sigma(1+\alpha) - (1+\alpha\psi)\left(\delta - \zeta(1-\theta) \right) \right] + (\delta-\zeta)(1+\alpha\psi)\zeta\left[\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi) \right]}{\theta\left[\sigma(1+\alpha) - \delta(1+\alpha\psi) \right]\left[\sigma(1+\alpha) - \zeta(1+\alpha\psi) \right]}$$

Recall the restrictions we have assumed: $0<\theta<1$, $0\le\rho<1$, $0\le\delta<1$, $0\le\varsigma<1$, $\sigma>1$, $\alpha>0$, $\delta<1-\theta$ and $\rho<1-\theta$..

Under these conditions, first one can see that $\beta_1<0$. The denominator of β_1 must be positive since $\sigma>1$. The numerator is positive since $\delta<1-\theta$.

We must have $\beta_2<0$. From $\sigma>1$, we can see that the terms $\sigma(1+\alpha)-(1-\theta)(1+\alpha\psi)$, $\sigma(1+\alpha)-\delta(1+\alpha\psi)$, and $\sigma(1+\alpha)-\rho(1+\alpha\psi)$ are all positive. Also from $\sigma>1$,

$$\sigma(1+\alpha) - \left(\delta(1-\psi) + \rho\psi(1+\alpha) \right) = \sigma - \left(\delta(1-\psi) + \rho\psi \right) + \alpha(\sigma - \rho\psi) > 0.$$

It is also immediate that $\beta_3<0$.

The coefficients on $i_{t-1}-i^*_{t-1}$ and η_{t-1} are, under these assumptions, ambiguous in sign. Note also that the coefficients on $i_t - i_t^* - \left(i_{t-1} - i^*_{t-1}\right)$ and $\eta_t - \eta_{t-1}$ would be much larger than the coefficients on $i_{t-1}-i^*_{t-1}$, and η_{t-1} when θ is small. In our baseline regression, as we have emphasized, the coefficients on q_{t-1}, $i_t - i_t^* - \left(i_{t-1} - i_{t-1}^*\right)$ and $\eta_t - \eta_{t-1}$ are all negative, and almost uniformly highly statistically significant. We note here that also almost all of the coefficients on $i_{t-1} - i_{t-1}^*$, and η_{t-1} are estimated to be negative, and they are smaller in magnitude then the coefficients on $i_t - i_t^* - \left(i_{t-1} - i_{t-1}^*\right)$ and $\eta_t - \eta_{t-1}$.

4. Structural interpretation

We describe in more detail the structural moment matching exercise in section 3.2. The purpose of the exercise is to illustrate that the coefficients estimated from the empirical regressions are reasonable numbers that can be reproduced from our model with fairly plausible structural parameter values. For example, the model would predict coefficient of β_2 and β_3 are numerically similar.

There are six structural parameters in the model, θ, σ, ψ, α, ρ and δ. We can make use of the five coefficients estimated from the empirical regression ($\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$) and price adjustment equation to discipline the parameters. Throughout the exercise, we use the average of the G10 coefficient estimate to infer the parameter values.

According to the pricing adjustment equation, θ governs the persistence of real exchange rate with a persistent parameter of $1-\theta$. We first estimate the persistence of real exchange rate using a panel regression of $q_{i,t} = \rho q_{i,t} + u_{i,t}$ and the take the average of G10 currencies. The average of estimated ρ is 0.965. We conduct a simple correction to account for the small sample issue in estimating a persistent process. The Rudebusch (1993) bias adjustment is $\rho_{adj} = \frac{T\times\rho+1}{T-3}$ and the biased adjusted ρ is 0.983. Therefore, the unbiased estimate of θ is 0.017.

We use the five coefficients from the empirical regression to infer σ, ψ, α, ρ and δ. The G10 average of the coefficient estimates are -0.02315, -4.5835, -5.07993, 0.37851 and -0.19015. We look for the parameter value of σ, ψ, α, ρ and δ that minimizes the following objective function of the weighted average of the absolute value of % deviation from the empirical estimate.

$$Objective = W_1 \left|\frac{-0.02315-\text{RHS eq(25)}}{-0.02315}\right| + W_2 \left|\frac{-4.5835\times 12-\text{RHS eq(26)}}{-4.5835\times 12}\right| +$$

$$W_3 \left|\frac{-5.07993\times 12-\text{RHS eq(27)}}{-5.07993\times 12}\right| + W_4 \left|\frac{0.37851\times 12-\text{RHS eq(28)}}{0.37851\times 12}\right| + W_5 \left|\frac{-0.19015\times 12-\text{RHS eq(29)}}{-0.19015\times 12}\right|$$

Each absolute value deviation is weighted by the inverse of the standard deviation of the coefficient estimate to the inverse of standard deviation all five coefficient estimates. i.e. $W_i = \left[SD(\widehat{\beta_i})\right]^{-1} / \sum_{i=1}^{5}\left[SD(\widehat{\beta_i})\right]^{-1}$. Note that coefficient estimate of β_2 to β_5 is scaled by 12 to convert the annualized rates to monthly rates.

The minimized value of the objective function is 0.0135. The implied parameters are {σ, ψ, α, ρ, δ}={4.69, 0.241, 0.211, 0.276, 0.964}. The model implied coefficient estimates of the regression are -0.02315, -4.3269, -4.8956, 0.37851 and -0.19015. As discussed in the main text, these implied parameters are within the plausible range that is commonly used or estimated in the literature. These reasonable parameter values generate model implied regression coefficient estimates that are all very close to the empirical estimates.

5. The serial correlation of the residual

From equation (24), the regression residual implied by our model is given by

$$z_t = z_0\omega_t + z_1 r_{t-1},$$

where, dividing numerator and denominator by $1+\alpha\psi$, and letting $x = \frac{1+\alpha}{1+\alpha\psi}$, we have

$$z_0 = -\left(\frac{\sigma x - (1-\theta)}{\theta[\sigma x - \zeta]}\right)$$

$$z_1 = \frac{[\sigma x - \zeta]\left[(\theta+\delta-\zeta)\sigma x - \left(\delta - \zeta(1-\theta)\right)\right] + (\delta-\zeta)\zeta\left[\sigma x - (1-\theta)\right]}{\theta[\sigma x - \delta][\sigma x - \zeta]}$$

The serial correlation of this error term is given by $\frac{\operatorname{cov}\left(z_0\omega_t + z_1 r_{t-1}, z_0\omega_{t+1} + z_1 r_t\right)}{\operatorname{var}\left(z_0\omega_t + b r_{t-1}\right)}$.

For the numerator of this expression, we have:

$$
\begin{aligned}
\operatorname{cov}\left(z_0\omega_t + z_1 r_{t-1}, z_0\omega_{t+1} + z_1 r_t\right) &= \operatorname{cov}\left(z_0\omega_t + z_1 r_{t-1}, z_1 r_t\right) \\
&= \operatorname{cov}\left(z_0\omega_t + z_1 r_{t-1}, z_1\left(\varsigma r_{t-1} + \omega_t\right)\right) \\
&= z_0 z_1 \operatorname{var}\left(\omega_t\right) + \left(z_1\right)^2 \varsigma \operatorname{var}\left(r_{t-1}\right) \\
&= \left(z_0 z_1 + \frac{\left(z_1\right)^2 \varsigma}{1-\varsigma^2}\right) \operatorname{var}\left(\omega_t\right)
\end{aligned}
$$

Then, $\operatorname{var}\left(z_0\omega_t + z_1 r_{t-1}\right) = \left(\left(z_0\right)^2 + \frac{\left(z_1\right)^2}{1-\varsigma^2}\right)\operatorname{var}\left(\omega_t\right)$. So,

$$
\frac{\operatorname{cov}\left(z_0\omega_t + z_1 r_{t-1}, z_0\omega_{t+1} + z_1 r_t\right)}{\operatorname{var}\left(z_0\omega_t + z_1 r_{t-1}\right)} = \frac{z_0 z_1\left(1-\varsigma^2\right) + \left(z_1\right)^2 \varsigma}{\left(z_0\right)^2\left(1-\varsigma^2\right) + \left(z_1\right)^2}.
$$

We can calibrate this expression for the serial correlation of the error term as follows: Engel (2019) measures the monthly serial correlation of the real exchange rate for the six non-U.S. countries in the G7 relative to the U.S., equal on average to 0.98. In our model, the serial correlation of the real exchange rate is given by $1-\theta$, so we set $\theta = 0.98$. Engel (2019) also estimates simple Taylor rules for these countries and finds the monthly serial correlation of the error term to be 0.96, so we set $\delta = 0.96$. The serial correlation of the ex ante risk premium is more problematic to calibrate because we need a measure of the serial correlation of the expected excess return on the market (LIBOR) investments. A natural measure is to equilibrate this serial correlation to the serial correlation of the nominal interest rate differential, which is also equal to 0.96 for the same set of countries, because the literature has generally found that the excess return is correlated with the interest rate differential and little else.

Note that the serial correlation of the residual does not depend on ρ, and it only depends on α as it influences σx. Our stability condition is $\sigma x > 1$. With that restriction in place, the serial correlation of the residual is practically independent of the value of σx as this table shows.

Appendix Table					
σx	θ	ς	δ		Serial Correlation of Residual
1	0.02	0.96	0.96		-0.020
2	0.02	0.96	0.96		-0.015
4	0.02	0.96	0.96		-0.015
100	0.02	0.96	0.96		-0.015
2	0.02	0.90	0.96		-0.044
2	0.02	0.95	0.96		-0.024
2	0.02	0.98	0.96		0.027
4	0.02	0.90	0.96		-0.050
4	0.02	0.95	0.96		-0.022
4	0.02	0.98	0.96		0.007
2	0.01	0.96	0.96		-0.009
2	0.03	0.96	0.96		-0.019
2	0.04	0.96	0.96		-0.020
2	0.05	0.96	0.96		-0.019
2	0.06	0.96	0.96		-0.016

6. Less Restrictive Formulations and "Internally Consistent" Regressions

We estimate a model for the exchange rate of a given currency, such as the Australian dollar or the Swiss franc. Our results in Tables 1A – 1G present strong support for this model for each of the G10 currencies. But, as in all the empirical exchange rate literature of which we are aware, our estimates allow the parameters to be different for different base currencies. If our estimating equation, (24), were the "true" model of exchange rates, the coefficients should be invariant across exchange rates. In fact, if the parameters were invariant, there would be no need to estimate a separate panel for each currency. We could, for example, take the panel estimates with the Swiss franc as the base currency, and from the equations for the Swiss franc/Australian dollar and Swiss franc/Norwegian krone, infer the model for the Australian dollar/Norwegian krone.

Even if equation (24) were the true model, the parameter estimates for the Australian dollar/Norwegian krone derived from the panel with the Swiss franc as the base currency almost certainly would be different than the model for that exchange rate when the Australian dollar is the base currency in finite samples because of estimation error. But even if the parameter estimates were the same, it is still interesting to ask whether the model can explain non-dollar exchange rates. That is, even if the model has explanatory power for one currency, it may not for others.

We can consider each of the panel estimates we present as restricted versions of a general unrestricted model in which each nominal exchange rate change depends on all lagged nine real exchange rates, all nine relative convenience yields (and lags), and all ten interest rates (and lags). That unrestricted panel requires estimation of 432 parameters, but such an unrestricted model is the only way to have an equation for each exchange rate that is the same irrespective of which currency is used as the base currency. Each of our panels, which estimate 14 parameters (nine intercepts and five slope coefficients) is a restricted version of the general panel. We show here that we can strongly reject our restricted panels in favor of the general panel.

However, we have followed the practice of all the empirical exchange rate literature, which might be described as regularization – the process of selecting the level of complexity of the model to avoid overfitting and to advance the interpretation of the economic forces at work. The model of a bilateral exchange rate that depends on 47 variables (and an intercept) has obviously been overfit, and theory does not motivate such a model. Yet, the "true" model is still not the unrestricted model – if it were, the fit of the unrestricted model would be perfect.

The key lesson from our estimates is that the relative convenience yield matters for exchange rates, and this set of results provides evidence that government bond liquidity at the individual country level plays an important role in exchange rate determination. The relative liquidity yield matters for non-U.S. dollar exchange rates, as well as for the dollar.

Consider our panel regression for currency *i*. A typical equation for the exchange rate relative to currency *j*, $j = 1, 2, \ldots, 10$, $j \neq i$, is

(1) $$s_{ij,t} - s_{ij,t-1} = \beta_{ij,0} + \beta_{i1} q_{ij,t-1} + \beta_{i2}\left(\eta_{ij,t} - \eta_{ij,t-1}\right) + \beta_{i3}\left(i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)\right) \\ + \beta_{i4}\eta_{ij,t-1} + \beta_{i5}\left(i_{i,t-1} - i_{j,t-1}\right) + z_{ij,t}$$

In some specifications we replace $\eta_{ij,t}$ with $\eta_{ij,t} = \gamma_{i,t} - \gamma_{j,t}$.

This formulation places three different types of restrictions on a more general panel specification:

1. There are cross-equation restrictions. That is, more generally, the coefficients in the panel could be different for each bilateral exchange rate of currency *i*. We could have, for example, a different coefficient on each real exchange rate, so we have $\beta_{ij,1}$, $j = 1, 2, \ldots, 10$, $j \neq i$, but we have imposed $\beta_{ij,1} = \beta_{ik,1}$ for all $j = 1, 2, \ldots, 10$, $j \neq i \neq k$
2. There are exclusion restrictions. The explanatory variables for $s_{ij,t} - s_{ij,t-1}$ only include country *i* and *j* variables.
3. The explanatory variables in the baseline regression are all expressed in terms of country *i* relative to country *j*. In some cases, that is the only way it is possible to express the variables (such as with $q_{ij,t}$, or when we use the $\tilde{\eta}_t$ measure for η_t). In other cases, we can separate out country *i* and country *j* variables, and not impose that the coefficient on one be equal and of opposite sign to the coefficient on the other (such as with $\hat{\eta}_{ij,t} = IRS_{it} - IRS_{j,t} + i_{j,t} - i_{i,t}$, which can be separated into $IRS_{i,t} - i_{i,t}$ and $IRS_{j,t} - i_{j,t}$, or $i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)$ can be separated into $i_{i,t} - i_{i,t-1}$ and $i_{j,t} - i_{j,t-1}$.)

The object of the note is twofold. First, it is to display the form of the more general panel regressions, and to test the restricted panels against the more general panels. Second, is to find whether the model for $s_{ij,t} - s_{ij,t-1}$ is "internally consistent" – that is the model for $s_{ij,t} - s_{ij,t-1}$ is consistent with the model we would get by taking the equation of $s_{ik,t} - s_{ik,t-1}$ and subtracting off the equation for $s_{kj,t} - s_{kj,t-1}$, for $i \neq j \neq k$. In checking for internal consistency, we will ask if the model is "internally consistent if the restrictions are true". The most unrestricted model is internally consistent, as we will show.

All models we consider have fixed effects – that is, exchange-rate specific intercepts.

Relative Variables only

First take the case where all variables are expressed in terms of country *i* relative to country *j*. In this section, assume that this is not a restriction – that the data do not allow for country-specific terms, only relative terms, so that the "unrestricted" Model C can be understood to be unrestricted when the data are only in relative form. It is clearer to consider this case first, before allowing country-specific variables.

Model A

The most restricted case is the baseline model in equation (1). Our findings for this model are reported in Table 2A for the $\tilde{\eta}_t$ measure, and in Table 2F for the $\hat{\eta}_t$ measure. If the restrictions are true, the model is internally consistent. To see this, we can write the model for $s_{ik,t} - s_{ik,t-1}$

(2)
$$\begin{aligned} s_{ik,t} - s_{ik,t-1} = \beta_{ik,0} + \beta_{i1} q_{ik,t-1} + \beta_{i2}\left(\eta_{ik,t} - \eta_{ik,t-1}\right) + \beta_{i3}\left(i_{i,t} - i_{k,t} - \left(i_{i,t-1} - i_{k,t-1}\right)\right) \\ + \beta_{i4}\eta_{ik,t-1} + \beta_{i5}\left(i_{i,t-1} - i_{k,t-1}\right) + z_{ik,t} \end{aligned}$$

Then, if both (1) and (2) are true, subtracting the former from the latter, we get a model for $s_{jk,t} - s_{jk,t-1}$, where we use the fact that $q_{jk,t} = q_{ik,t-1} - q_{ij,t-1}$, $\eta_{jk,t} = \eta_{ik,t} - \eta_{ij,t}$, and $i_{j,t} - i_{k,t} = i_{i,t} - i_{k,t} - \left(i_{i,t} - i_{j,t}\right)$:

(3)
$$\begin{aligned} s_{jk,t} - s_{jk,t-1} = \beta_{jk,0} + \beta_{i1} q_{jk,t-1} + \beta_{i2}\left(\eta_{jk,t} - \eta_{jk,t-1}\right) + \beta_{i3}\left(i_{j,t} - i_{k,t} - \left(i_{j,t-1} - i_{k,t-1}\right)\right) \\ + \beta_{i4}\eta_{jk,t-1} + \beta_{i5}\left(i_{j,t-1} - i_{k,t-1}\right) + z_{jk,t} \end{aligned}$$

where $\beta_{jk,0} = \beta_{ik,0} - \beta_{ij,0}$, and $z_{jk,t} = z_{ik,t} - z_{ij,t}$. Clearly this equation is of the same form as (1) and (2). The exchange rate for *jk* depends only on relative *jk* variables.

In fact, we can see that the slope coefficients, β_{i1}, β_{i2}, β_{i3}, β_{i4}, and β_{i5}, are the same as in equations (1) and (2). Since equation (3) is for currency *j*, not currency *i*, but the slope coefficients are the same, we can drop the *i* subscript on them. We can call them β_1, β_2, β_3, β_4, and β_5.

That is, if the cross-equation restrictions for the $s_{ij,t}-s_{ij,t-1}$ and $s_{ik,t}-s_{ik,t-1}$ equations are true, then the model is internally consistent. And, if we estimated the model as a panel for currency *j* instead of for currency *i*, we would get the same slope coefficients.

Concretely, if currency *i* is the AUD, currency *j* is CAD, and currency *k* is EUR, then we can estimate β_1, β_2, β_3, β_4, and β_5 for the panel in which AUD is the base currency. But if CAD or EUR were the base currency, we would find the same parameters β_1, β_2, β_3, β_4, and β_5.

We clearly do not find that to be true in the data, because we report different values for these slope parameters for the panels with each currency as the base. However, the fact that the estimated β_1, β_2, β_3, β_4, and β_5 are not the same across the different panel estimates does not invalidate the hypothesis that the cross-equation restrictions are valid, because there is estimation error in finite samples. Below we test the restriction (Model A vs. Model B.)

Model A is internally consistent if the restrictions are true.

There are 14 parameters to estimate: five slope parameters (β_1, β_2, β_3, β_4, and β_5), and nine intercept parameters in each panel. Even if the restrictions are true, we will find numerically different estimates for the slope parameters in each panel (and $\beta_{jk,0}=\beta_{ik,0}-\beta_{ij,0}$ will not be true also), because of finite samples.

<u>Model B</u>

Now we modify equations (1) and (2) so we do not impose the cross-equation restrictions (but still impose the exclusion restrictions.)

(4)
$$s_{ij,t}-s_{ij,t-1}=\beta_{ij,0}+\beta_{ij,1}q_{ij,t-1}+\beta_{ij,2}\left(\eta_{ij,t}-\eta_{ij,t-1}\right)+\beta_{ij,3}\left(i_{i,t}-i_{j,t}-\left(i_{i,t-1}-i_{j,t-1}\right)\right)$$
$$+\beta_{ij,4}\eta_{ij,t-1}+\beta_{ij,5}\left(i_{i,t-1}-i_{j,t-1}\right)+z_{ij,t}$$

(5)
$$s_{ik,t}-s_{ik,t-1}=\beta_{ik,0}+\beta_{ik,1}q_{ik,t-1}+\beta_{ik,2}\left(\eta_{ik,t}-\eta_{ik,t-1}\right)+\beta_{ik,3}\left(i_{i,t}-i_{k,t}-\left(i_{i,t-1}-i_{k,t-1}\right)\right)$$
$$+\beta_{ik,4}\eta_{ik,t-1}+\beta_{ik,5}\left(i_{i,t-1}-i_{k,t-1}\right)+z_{ik,t}$$

This model is not internally consistent. If we take the model for $s_{jk,t}-s_{jk,t-1}$ by subtracting (4) from (5) – so that the model for $s_{jk,t}-s_{jk,t-1}$ is equal to the model for $s_{ik,t}-s_{ik,t-1}$ minus that for $s_{ik,t}-s_{ik,t-1}$ – we do not get a model that depends only on *jk* variables. The exclusion restrictions would not hold for $s_{jk,t}-s_{jk,t-1}$.

If, for example, we estimate the model with AUD as a base, and s_{ij} is the AUD/CAD exchange rate and s_{ik} is the AUD/EUR exchange rate, we can infer a model for s_{jk}, the CAD/EUR exchange rate by subtracting (4) from (5). But that model for the CAD/EUR exchange rate would depend separately on the AUD/CAD real exchange rate and the AUD/EUR real exchange rate, not just the CAD/EUR exchange rate. But if we estimated a panel with CAD as a base, and only used *jk* variables in the *jk* regression, the CAD/EUR exchange rate would depend only on the CAD/EUR real exchange rate (and other CAD/EUR variables.)

The panel regression for currency *i* as represented in equations (4) from (5) has 54 parameters to estimate – an intercept and five slope coefficients for each *ij* exchange rate, and there are 9 exchange rates, so $9\times 6=54$. Each panel should give us 54 different parameter estimates.

Since there are no cross-equation restrictions, this model is essentially identical to the equation-by-equation estimates reported in Table 5B.

Model C

In this model, we have no exclusion restrictions. The *ij* exchange rate depends on all variables, including all *ik* variables:

(6)
$$s_{ij,t}-s_{ij,t-1}=\beta_{ij,0}+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{ij,k,1}q_{ik,t-1}+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{ij,k,2}\left(\eta_{ik,t}-\eta_{ik,t-1}\right)+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{ij,k,3}\left(i_{i,t}-i_{k,t}-\left(i_{i,t-1}-i_{k,t-1}\right)\right)$$
$$\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{ij,k,4}\eta_{ik,t-1}+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{ij,k,5}\left(\left(i_{i,t-1}-i_{k,t-1}\right)\right)+z_{ij,t}$$

In this equation, there are 45 explanatory variables, plus an intercept variable, for the exchange rate $s_{ij,t}-s_{ij,t-1}$.

For exchange rate $s_{im,t}-s_{im,t-1}$, we have:

(7)
$$s_{im,t}-s_{im,t-1}=\beta_{im,0}+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{im,k,1}q_{ik,t-1}+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{im,k,2}\left(\eta_{ik,t}-\eta_{ik,t-1}\right)+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{im,k,3}\left(i_{i,t}-i_{k,t}-\left(i_{i,t-1}-i_{k,t-1}\right)\right)$$
$$\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{im,k,4}\eta_{ik,t-1}+\sum_{\substack{k=1\\k\neq i}}^{N}\beta_{im,k,5}\left(\left(i_{i,t-1}-i_{k,t-1}\right)\right)+z_{im,t}$$

This equation has another 46 parameters. The entire panel for currency *i* has $9\times 46=414$ parameters.

This formulation is internally consistent. We can take the equation for $s_{jm,t} - s_{jm,t-1}$ from this panel by subtracting (6) from (7), or we could estimate an equation for $s_{jm,t} - s_{jm,t-1}$ from a panel for currency *j*, and they would both involve the same 45 explanatory variables with no cross-equation restrictions.

Country-Specific Variables

We can allow country-specific, rather than relative variables, to replace $\hat{\eta}_{ij,t} = IRS_{it} - IRS_{j,t} + i_{j,t} - i_{i,t}$ with $IRS_{i,t} - i_{i,t}$ and $IRS_{j,t} - i_{j,t}$, or $i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)$ with $i_{i,t} - i_{i,t-1}$ and $i_{j,t} - i_{j,t-1}$. We are most interested in whether the individual convenience yields matter, rather than their relative values, as we have already allowed in some specifications. To keep the analysis a bit simpler, we will only consider $i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)$ in relative form, instead of splitting it into $i_{i,t} - i_{i,t-1}$ and $i_{j,t} - i_{j,t-1}$. Define $\gamma_{i,t} \equiv IRS_{i,t} - i_{i,t}$

Model D

This is the most restricted form of this class of models. We report estimates from this model in Table 5A. We only include variables for country *i* and country *j* in the regression for $s_{ij,t} - s_{ij,t-1}$, and for the panel for currency *i*, we impose cross-equation restrictions on the parameters. We have:

$$
\begin{aligned}
(8) \qquad s_{ij,t} - s_{ij,t-1} = {} & \beta_{ij,0} + \beta_{i1} q_{ij,t-1} + \beta_{i2}\left(\gamma_{i,t} - \gamma_{i,t-1}\right) + \beta_{i3}\left(\gamma_{j,t} - \gamma_{j,t-1}\right) + \beta_{i4}\left(i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)\right) \\
& + \beta_{i5}\gamma_{i,t-1} + \beta_{i6}\gamma_{j,t-1} + \beta_{i7}\left(i_{i,t-1} - i_{j,t-1}\right) + z_{ij,t}
\end{aligned}
$$

The equation for $s_{ik,t} - s_{ik,t-1}$ is

$$
\begin{aligned}
(9) \qquad s_{ik,t} - s_{ik,t-1} = {} & \beta_{ik,0} + \beta_{i1} q_{ik,t-1} + \beta_{i2}\left(\gamma_{i,t} - \gamma_{i,t-1}\right) + \beta_{i3}\left(\gamma_{k,t} - \gamma_{k,t-1}\right) + \beta_{i4}\left(i_{i,t} - i_{k,t} - \left(i_{i,t-1} - i_{k,t-1}\right)\right) \\
& + \beta_{i5}\gamma_{i,t-1} + \beta_{i6}\gamma_{k,t-1} + \beta_{i7}\left(i_{i,t-1} - i_{k,t-1}\right) + z_{ik,t}
\end{aligned}
$$

Unlike the simplest case (Model A) when only relative variables are present, this model is not internally consistent. If we subtract equation (8) from equation (9) to get a model for $s_{jk,t} - s_{jk,t-1}$, the equation would become:

$$s_{ik,t}-s_{ik,t-1}=\beta_{jk,0}+\beta_{i1}q_{jk,t-1}+\beta_{i3}\left[\left(\gamma_{k,t}-\gamma_{k,t-1}\right)-\left(\gamma_{j,t}-\gamma_{j,t-1}\right)\right]+\beta_{i4}\left(i_{ji,t}-i_{k,t}-\left(i_{j,t-1}-i_{k,t-1}\right)\right)$$
$$+\beta_{i6}\left(\gamma_{k,t-1}-\gamma_{i,t-1}\right)+\beta_{i7}\left(i_{i,t-1}-i_{k,t-1}\right)+z_{ik,t},$$

but the internally consistent equation, which we would get from a panel in which *j* is the base currency, is:

$$s_{jk,t}-s_{jk,t-1}=\beta_{jk,0}+\beta_{j1}q_{jk,t-1}+\beta_{j2}\left(\gamma_{j,t}-\gamma_{j,t-1}\right)+\beta_{j3}\left(\gamma_{k,t}-\gamma_{k,t-1}\right)+\beta_{j4}\left(i_{j,t}-i_{k,t}-\left(i_{j,t-1}-i_{k,t-1}\right)\right)$$
$$+\beta_{j5}\gamma_{j,t-1}+\beta_{j6}\gamma_{k,t-1}+\beta_{j7}\left(i_{j,t-1}-i_{k,t-1}\right)+z_{ik,t}$$

The structures of these last two equations are different, so Model D is not internally consistent. For each currency used as the base currency, the panel involves estimating 7 slope parameters, and 9 intercepts, for a total of 16 parameters.

<u>Model E</u>

This model imposes cross-equation restrictions on the parameters of the relative variables, but not on the country-specific variables for country *j* in the regression for the *ij* exchange rate. It continues to assume the zero restrictions that only variables for country *i* and country *j* in the regression for $s_{ij,t}-s_{ij,t-1}$. It is internally consistent, if the restrictions are true.

(10)
$$s_{ij,t}-s_{ij,t-1}=\beta_{ij,0}+\beta_{i1}q_{ij,t-1}+\beta_{i2}\left(\gamma_{i,t}-\gamma_{i,t-1}\right)+\beta_{j3}\left(\gamma_{j,t}-\gamma_{j,t-1}\right)+\beta_{i4}\left(i_{i,t}-i_{j,t}-\left(i_{i,t-1}-i_{j,t-1}\right)\right)$$
$$+\beta_{i5}\gamma_{i,t-1}+\beta_{j6}\gamma_{j,t-1}+\beta_{i7}\left(i_{i,t-1}-i_{j,t-1}\right)+z_{ij,t}$$

Then for $s_{ik,t}-s_{ik,t-1}$, we have:

(11)
$$s_{ik,t}-s_{ik,t-1}=\beta_{ik,0}+\beta_{i1}q_{ik,t-1}+\beta_{i2}\left(\gamma_{i,t}-\gamma_{i,t-1}\right)+\beta_{k3}\left(\gamma_{k,t}-\gamma_{k,t-1}\right)+\beta_{i4}\left(i_{i,t}-i_{k,t}-\left(i_{i,t-1}-i_{k,t-1}\right)\right)$$
$$+\beta_{i5}\gamma_{i,t-1}+\beta_{k6}\gamma_{k,t-1}+\beta_{i7}\left(i_{i,t-1}-i_{k,t-1}\right)+z_{ik,t}$$

If we subtract equation (10) from equation (11) to get a model for $s_{jk,t}-s_{jk,t-1}$, the equation would become:

(12)
$$s_{jk,t}-s_{jk,t-1}=\tilde{\beta}_{jk,0}+\tilde{\beta}_{j1}q_{jk,t-1}+\tilde{\beta}_{j2}\left(\gamma_{j,t}-\gamma_{j,t-1}\right)+\tilde{\beta}_{k3}\left(\gamma_{k,t}-\gamma_{k,t-1}\right)+\tilde{\beta}_{j4}\left(i_{j,t}-i_{k,t}-\left(i_{j,t-1}-i_{k,t-1}\right)\right)$$
$$+\tilde{\beta}_{j5}\gamma_{j,t-1}+\tilde{\beta}_{k6}\gamma_{k,t-1}+\tilde{\beta}_{j7}\left(i_{j,t-1}-i_{k,t-1}\right)+z_{jk,t}$$

where $\tilde{\beta}_{jk,0} = \beta_{ik,0} - \beta_{ij,0}$, $\tilde{\beta}_{j1} = \beta_{i1}$. $\tilde{\beta}_{j2} = -\beta_{j3}$, $\tilde{\beta}_{k3} = \beta_{k3}$, $\tilde{\beta}_{j4} = \beta_{i4}$, $\tilde{\beta}_{j5} = -\beta_{j6}$, $\tilde{\beta}_{k6} = \beta_{k6}$, and $\tilde{\beta}_{j7} = \beta_{i7}$.

If the restricted Model E was true, then if equation (12) were estimated from a panel with currency *j* as the base currency, the estimated parameters would satisfy the parameter relationships from the panel with *i* as a base currency in an infinitely large sample (but not in a finite sample.)

That is, Model E is internally consistent if the restrictions are true. For each panel, there are 32 parameters to estimate. There are 9 intercept terms, 3 coefficients on the relative variables ($q_{ij,t-1}$, $i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)$, and $i_{i,t-1} - i_{j,t-1}$, 10 coefficients on the $\gamma_{j,t} - \gamma_{j,t-1}$, and 10 coefficients on the $\gamma_{j,t-1}$ variables.

<u>Model F</u>

This model puts on no cross-equation restrictions, but it imposes the zero restrictions (that only variables for country *i* and country *j* in the regression for $s_{ij,t} - s_{ij,t-1}$). It is not internally consistent, for the same reasons that Model B is not:

$$\begin{aligned} s_{ij,t} - s_{ij,t-1} = {} & \beta_{ij,0} + \beta_{ij,1} q_{ij,t-1} + \beta_{ij,2}\left(\gamma_{i,t} - \gamma_{i,t-1}\right) + \beta_{ij,3}\left(\gamma_{j,t} - \gamma_{j,t-1}\right) + \beta_{ij,4}\left(i_{i,t} - i_{j,t} - \left(i_{i,t-1} - i_{j,t-1}\right)\right) \\ & + \beta_{ij,5}\gamma_{i,t-1} + \beta_{ij,6}\gamma_{j,t-1} + \beta_{ij,7}\left(i_{i,t-1} - i_{j,t-1}\right) + z_{ij,t} \end{aligned}$$

This equation has 8 parameters to estimate (1 intercept and 7 slope coefficients.) The panel has 9 exchange rates, so the panel entails estimation of $9 \times 8 = 72$ parameters.

<u>Model G</u>

Model G has no cross-equation constraints, and no zero constraints, and is therefore internally consistent:

$$\begin{aligned} s_{ij,t} - s_{ij,t-1} = {} & \beta_{ij,0} + \sum_{\substack{k=1 \\ k \neq i}}^{N} \beta_{ij,k,1} q_{ik,t-1} + \sum_{k=1}^{N} \beta_{ij,k,2}\left(\gamma_{k,t} - \gamma_{k,t-1}\right) + \sum_{\substack{k=1 \\ k \neq i}}^{N} \beta_{ij,k,3}\left(i_{i,t} - i_{k,t} - \left(i_{i,t-1} - i_{k,t-1}\right)\right) \\ & \sum_{k=1}^{N} \beta_{ij,k,4}\gamma_{k,t-1} + \sum_{\substack{k=1 \\ k \neq i}}^{N} \beta_{ij,k,5}\left(\left(i_{i,t-1} - i_{k,t-1}\right)\right) + z_{ij,t} \end{aligned}$$

This equation has 1 intercept, 9 coefficients on each of the three relative variables ($q_{ik,t-1}$, $i_{i,t} - i_{k,t} - \left(i_{i,t-1} - i_{k,t-1}\right)$, and $i_{i,t-1} - i_{k,t-1}$), and 10 coefficients each on $\gamma_{k,t} - \gamma_{k,t-1}$ and $\gamma_{k,t-1}$, for a total of 48. Since there are 9 exchange rates in each panel, there are $9 \times 48 = 432$ parameters.

The possible nested tests (with number of parameters in each model in parentheses) are:

Model A (14) is nested in Models B (54), C (414), D (16), E (32), F (72), G (432)
Model B (54) is nested in Models C (414), F (72) and G (432).
Model C (414) is nested in Model G (432).
Model D (16) is nested in Models E (32), F (72). G (432)
Model E (32) is nested in Models F (72), G (432)
Model F (72) is nested in Model G (432).

So there are 16 possible F-tests, with each of the ten currencies serving as a base currency.

These are the p-values for those tests:

P value of the F tests

Test	constraints	AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	USD
A vs B	40	0.000	0.004	0.000	0.000	0.071	0.138	0.000	0.018	0.139	0.059
A vs C	400	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A vs D	2	0.014	0.156	0.006	0.045	0.000	0.000	0.169	0.122	0.100	0.000
A vs E	18	0.003	0.006	0.005	0.011	0.000	0.001	0.022	0.011	0.017	0.000
A vs F	58	0.000	0.014	0.000	0.000	0.000	0.005	0.000	0.008	0.108	0.004
A vs G	418	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B vs C	360	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B vs F	18	0.027	0.526	0.069	0.562	0.000	0.003	0.173	0.102	0.238	0.007
B vs G	378	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C vs G	18	0.039	0.001	0.003	0.720	0.075	0.067	0.029	0.000	0.834	0.103
D vs E	16	0.121	0.008	0.023	0.017	0.260	0.058	0.032	0.039	0.046	0.011
D vs F	56	0.000	0.017	0.000	0.000	0.136	0.066	0.000	0.020	0.186	0.229
D vs G	416	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
E vs F	40	0.000	0.177	0.000	0.001	0.167	0.206	0.001	0.088	0.526	0.809
E vs G	400	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F vs G	360	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

We generally reject the more restrictive models in favor of the less restrictive models. In the text, we have reported estimation results only for models A (Tables 2A and 2F), B (Table 5B), and D (Table 5A). There is a tradeoff. Our baseline model is more parsimonious; however, it is not internally consistent. There is always a tradeoff between the econometric gains in power by having a more parsimonious model, versus the distortions that arise when imposing restrictions that are not true. When we consider that we know we do not really have the “true” model (for example, the *ij* exchange rate may not depend just on *i* and *j* variables, or they may be other left-out variables), than the consistency criterion might not be a good one in practice.

As the following table shows, the relative liquidity yields remain highly significant in the more highly parameterized models, B, E and F, even though there are many more parameters estimated in these models. (We did not test for individual significance of coefficients for Models C and G, which have 414 and 432 parameters in each panel.)

Liquidity measure ($\Delta\hat{\eta}_{j,t}$) is significant at (cumulative):	Model B	Model E		Model F
	Total pair: 45	Home: 9	Foreign: 90	Total pair: 90
10%	35	8	62	56
5%	30	8	59	50
1%	25	7	49	43

7. Accounting for default risk in exchange rate determination

This section focuses on the role of default risk with a slight extension of the baseline model. We can interpret the government interest rate in the baseline model as the risk-free government interest rate that investors can obtain by buying a government bond but subtracting the cost of buying default protection.

From equation (3) in the main text, assuming risk premium is zero for simplicity, we have:

(3) $$i_t^* + E_t s_{t+1} - s_t - i_t = \eta_t$$

As in the main text, we can define the decomposed liquidity measure by using CDS rate to measure the default risk of a government as in Du et al (2018a):

(19) $$\eta_t \equiv \lambda_t - l_t^R \quad \text{where } l_t^R = CDS_t - CDS_t^*$$

This is the same as our definition of equation (19) in the main text but setting the deviation of CIP to zero to simplify the exposition.

With these two equations, we can rewrite equation (3) as:

$$i_t^* + E_t s_{t+1} - s_t - i_t = \lambda_t - l_t^R$$

Define $i_t^{*G} \equiv i_t^* - CDS_t^*$ as the risk-free interest rate. It is the government promised interest rate minus the cost of the CDS rate.

$$\lambda_t = E_t s_{t+1} - s_t + i_t^* - i_t + CDS_t - CDS_t^* = E_t s_{t+1} - s_t + (\underbrace{i_t^* - CDS_t^*}_{\equiv i_t^{*G}}) - (\underbrace{i_t - CDS_t}_{\equiv i_t^G})$$

Simply iterate forward this equation gives:

$$s_t = -E_t \sum_{j=0}^{\infty}\left(i_{t+j} - i_{t+j}^* - \left(\overline{i - i^*}\right)\right) - E_t \sum_{j=0}^{\infty}\left(\lambda_{t+j} - \bar{\lambda}\right) + E_t \sum_{j=0}^{\infty}\left(CDS_{t+j} - CDS_{t+j}^* - \overline{CDS - CDS^*}\right)$$
$$+ \lim_{k\to\infty}\left(E_t s_{t+k} - k\left(\overline{s_{+1} - s}\right)\right)$$

An increase in default risk, which is captured by an increase in CDS rate, has the opposite effect to an increase in the liquidity and interest rate.

To solve the model, we know $\eta_t \equiv \lambda_t - l_t^R$ so we know the coefficients on λ_t is β_2 and l_t^R is $-\beta_2$ where β_2 is solved earlier as:

$$\beta_2 = -\left(\frac{\left[\sigma(1+\alpha) - (1-\theta)(1+\alpha\psi)\right]\left[\sigma(1+\alpha) - \left(\delta(1-\psi) + \rho\psi(1+\alpha)\right)\right]}{\theta\left(\sigma(1+\alpha) - \delta(1+\alpha\psi)\right)\left(\sigma(1+\alpha) - \rho(1+\alpha\psi)\right)}\right)$$

We can allow for differences in persistence parameter ρ for λ_t and l_t^R (ρ_λ and ρ_l). We can also set α to zero for CDS differential l_t^R because the CDS does not respond to the interest rate differential as the Treasury liquidity yield does. With these assumptions, the response of change of nominal exchange rate to change of CDS rate becomes:

$$\beta_{CDS} = \left(\frac{\left[\sigma - (1-\theta)\right]\left[\sigma - \left(\delta(1-\psi) + \rho_l\psi\right)\right]}{\theta(\sigma - \delta)(\sigma - \rho_l)}\right)$$

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