The Long-Run U.S./U.K. Real Exchange Rate

The paper estimates a model for the real U.S./U.K. exchange rate. The Kalman filter is used to identify a permanent and a transitory component. We find the variance of the transitory component shifts among three states according to a Markov-switching process. The model is estimated by Gibbs sampling. The transitory component appears to be driven by temporary monetary phenomena. The shifts of variance occur at times of historically significant monetary events. We find the permanent component is cointegrated with relative per capital income levels, as in the Balassa-Samuelson hypothesis. The data support a model that contains a transitory component driven by monetary phenomena and a permanent component driven by relative income levels.

Recent studies of purchasing power parity using long data series have found a tendency for real exchange rates among the major industrialized countries to converge in the long run. This literature, which typically examines time series with over one hundred years of data, rejects the null hypothesis of a unit root in real exchange rates using standard unit root tests, such as the Dickey-Fuller test.

Should we conclude, therefore, that there is no permanent component to these real exchange rates? There are at least two reasons to doubt this conclusion. First, it is likely there have been shifts in the data-generating process for the real exchange rate over this period. For example, as Mussa (1986) and Grilli and Kaminsky (1991) have noted, the stochastic process for real exchange rates appears to be different under regimes of fixed and floating nominal exchange rates. The literature has not established how robust unit root tests are to changes in regimes. Indeed, Kim, Nelson, and Startz (1998) have presented evidence in variance ratio tests for mean reversion in U.S. stock prices that the tests are sensitive to the pattern of heteroskedasticity in the

The authors thank Graciela Kaminsky and Nelson Mark for providing their data sets. Engel acknowledges assistance from a National Science Foundation grant to the National Bureau of Economic Research.


Charles Engel is professor of economics at the University of Washington. Chang-Jin Kim is associate professor of economics at Korea University.

Journal of Money, Credit, and Banking. Vol. 31, No. 3 (August 1999, Part 1)
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data. Frankel and Rose (1996) test for purchasing power parity using panels of real exchange rates beginning only in 1973, because "of concerns about the use of long time series, since they include potentially serious structural shifts." (p. 210).

The second reason why we should not rule out the presence of a permanent component based on the conclusion of the unit root tests is that the tests have a large size bias when a series contains both a permanent and a transitory component. In this case, the first difference of the real exchange rate has a moving average component in its ARMA representation. Schwert (1989) and others have shown that unit root tests tend to reject the null far too frequently in the presence of moving average components. Engel (2000) calibrates some unit root tests to real exchange rate data, and argues that tests with a nominal size of 5 percent may have a true size of about 80 percent. That is, the tests reject the null 80 percent of the time, even when a large fraction of the variance of real exchange rate movements in the sample are generated from the permanent component.

We investigate the behavior of the U.S./U.K. real exchange rate, using monthly data from January 1885 to November 1995. The data is plotted in Figure 1. Inspection of the graph suggests that the real exchange rate process has not been homoskedastic over the entire 111-year period. It is impossible to conclude visually whether there is a permanent or a transitory component to the real exchange rate. We note that, using an Augmented Dickey-Fuller test, we reject a unit root in this real exchange rate at the 5 percent level. But we do not accept this as conclusive evidence that there is no permanent component in this real exchange rate. Instead, we explore models in which there are unobservable permanent and transitory components. We allow both the transitory and the permanent component to switch among three states characterized by low, medium, and high variance. Our model, therefore, allows for the structural shifts that were the concern of, for example, Frankel and Rose (1996); and, it investigates the possibility that both transitory and permanent components exist, as Engel (1996) advocates.

We initially examine a generously parameterized model that allows three variance states for both the permanent and transitory components. However, we shall demonstrate in section 3 that the permanent component is overparameterized in this case. In fact, a homoskedastic permanent component, along with a three-state model for the transitory component, best describes the data. Hence, we can conclude that the pattern of shifting variances clearly visible in the data reflects heteroskedasticity in the transitory component.

Interestingly, as section 4 shows, the transitory component appears to shift among states at points where significant nominal events occur. Many of the regime switches occur when the nominal exchange rate regime changes—formalizing Mussa’s (1986) observation that the real exchange rate is not “nominal exchange rate regime neutral.”

2. The data is an updated version of the series that is used in Grilli and Kaminsky (1991).
3. Hegwood and Papell (1996) consider a different type of regime shift—a shift in the mean of the real exchange rate.
Other shifts occur during episodes of banking crises or temporary inflations in the United States and United Kingdom. We note the fact that it is the transitory component that switches at the points of nominal regime changes is consistent with models of real exchange rates in which nominal prices are sticky and adjust slowly. However, this paper develops no independent evidence in favor of sticky-price models, so we do not take a stand on why the transitory component switches regimes when there are significant monetary events.

The variance of innovations to the permanent component is much smaller than the variance of the transitory component in all but the case in which the transitory component is in the low-variance state. It is in such a circumstance that tests for a unit root tend to have large size biases, according to Engel (2000). We show in section 5, using a Monte Carlo exercise, that if the true data-generating process is our permanent-transitory model with the variances calibrated to our estimates, Dickey-Fuller type tests tend to reject the null of a unit root far too frequently.

In the next section, we briefly describe the model we estimate. The model is difficult to estimate using maximum likelihood techniques, but can be estimated relatively easily using Gibbs sampling. In section 2, we briefly review the concepts of Gibbs sampling, though we relegate the details of our estimation techniques to an appendix.\(^4\)

\(^4\) Available on request from the authors.
1. THE MODEL

The real exchange rate, $q_t$, is assumed to be comprised of a transitory component, $x_t$, and a permanent component, $y_t$:

$$q_t = x_t + y_t,$$  \hfill (1)

The permanent component, $y_t$, follows the process

$$y_t = y_{t-1} + u_t, \quad u_t \sim N(0, \sigma_{y_t}^2).$$  \hfill (2)

The stochastic shock, $u_t$, is assumed to be serially uncorrelated.

One of the identifying assumptions of our model is that the permanent component of real exchange rates has no drift. The real exchange rate is a relative price—the price level in one country relative to another. In the long run, its movements will be determined by taste changes and technological changes. We assume that there is no drift in those changes over periods as long as our 1111-year sample.

The transitory component is assumed to follow:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-1} + e_t, \quad e_t \sim N(0, \sigma_{e_t}^2),$$  \hfill (3)

where $e_t$ is serially uncorrelated.

The variances of the two independent shocks, $\sigma_{y_t}^2$ and $\sigma_{e_t}^2$, are assumed to be heteroskedastic. In particular, we assume that they depend on two independent discrete-valued first-order Markov switching variables, $S_{1,t}$ and $S_{2,t}$ ($S_{1,t} \in \{1, 2, 3\}$ and $S_{2,t} \in \{1, 2, 3\}$) which evolve independently of $u_t$ and $e_t$, according to the following transition probabilities:

$$\Pr[S_{1,t} = j | S_{1,t-1} = i] = p_{1,ij}; \quad i, j = 1, 2, 3; \quad \sum_{j=1}^{3} p_{1,ij} = 1;$$  \hfill (4)

$$\Pr[S_{2,t} = j | S_{2,t-1} = i] = p_{2,ij}; \quad i, j = 1, 2, 3; \quad \sum_{j=1}^{3} p_{2,ij} = 1.$$  \hfill (5)

Thus, we can write the variances of the shocks in the following ways:

$$\sigma_{y_t}^2 = \sigma_{v1}^2 S_{1,t} + \sigma_{v2}^2 S_{1,t} + \sigma_{v3}^2 S_{1,3}; \quad \sigma_{v1}^2 < \sigma_{v2}^2 < \sigma_{v3}^2,$$  \hfill (6)

$$\sigma_{e_t}^2 = \sigma_{e1}^2 S_{2,t} + \sigma_{e2}^2 S_{2,t} + \sigma_{e3}^2 S_{2,3}; \quad \sigma_{e1}^2 < \sigma_{e2}^2 < \sigma_{e3}^2,$$  \hfill (7)

where $S_{n,kt} = 1$ if $S_{nt} = k$ and $S_{n,kt} = 0$ otherwise ($k = 1, 2, 3; n = 1, 2$).

5. We have only two lags in the autoregression because higher-order terms are statistically insignificant.

The independence of \( u_t \) and \( e_t \) is an identifying assumption in our model. The permanent-transitory decomposition of a univariate series must make an assumption about the correlation of these variables. We argue in section 4 that our decomposition provides plausible series for the permanent and transitory components, but we do not explore, for example, the properties of permanent and transitory components that might be arrived at under alternative assumptions about the correlation of \( u_t \) and \( e_t \).

2. ESTIMATION

The model is estimated using the technique of Gibbs sampling. Because this technique is relatively new and has not been used much in applications in economics, we will briefly describe the technique here. More complete introductions to Gibbs sampling are available in Albert and Chib (1993), Casella and George (1992), and Gelfand and Smith (1990). Our use of the Gibbs sampler is closely related to that of Albert and Chib, who use it to estimate Hamilton’s (1989) autoregressive time-series model with Markov switching, and to that of Carter and Kohn (1994), who use it to make inferences on unobserved components in a state-space model. Our application extends the Gibbs sampling technique to general state-space models such as those estimated by Kim (1994) using maximum likelihood techniques.\(^7\)

Gibbs sampling is a technique for generating random variables from a joint distribution indirectly, without having to know the density. For example, suppose we would like to make inferences about two unobserved random variables, \( P \) and \( R \), given some data \( \bar{W}^T \), which is a vector of \( T \) observations on some random variable \( W_t \). In particular, we would like to obtain the marginal density \( f(p, r|\bar{W}^T) \), because we are interested in, say, the mean and variance of these variables.

The Gibbs sampling technique allows us to generate a sample from \( f(p, r|\bar{W}^T) \) by sampling from the conditional densities \( f(p|R, \bar{W}^T) \) and \( f(r|P, \bar{W}^T) \). As we shall see, frequently the latter densities are readily available, while \( f(p, r|\bar{W}^T) \) is not. Begin with an initial guess for a value of \( R \), call it \( r_0 \). Then, we can generate a draw, call it \( p_1 \), from the density \( f(p|r_0, \bar{W}^T) \). Take this value of \( p_1 \), and generate \( r_1 \) from \( f(r|p_1, \bar{W}^T) \). Continue this process iteratively to generate the sequence \( r_0, p_1, r_1, p_2, r_2, \ldots, p_k, r_k \). These values are realizations of the sequence of random variables \( R_0, P_1, R_1, P_2, R_2, \ldots, P_k, R_k \). Under fairly general conditions, the joint density of \( P_k \) and \( R_k \) converges to \( f(p, r|\bar{W}^T) \).

Thus, one might generate \( K \) values of \( p_i \) and \( r_i \). Then, if \( K \) is large enough, we can assume for values of \( i > K \) that \( p_i \) and \( r_i \) are drawn from their true joint distribution, \( f(p, r|\bar{W}^T) \). Then we can generate a sample of \( P \) and \( R \) drawn from this distribution by generating an additional \( N \) draws. These \( N \) draws are used to calculate moments of \( f(p, r|\bar{W}^T) \) that we are interested in.

For the model estimated here, we actually would like to find the density of \( 4T + 20 \)

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random variables, conditional on the vector of observed real exchange rates, $Q^T$. There are $T$ values of the permanent component, $y_i$ (call this vector $Y^T$); $T$ values of the transitory component, $x_i$ (call this vector $X^T$); $2T$ values of the state variables $S_{1i}$ and $S_{2i}$ (call the two $T \times 1$ matrices of states $S_{1i}^T$ and $S_{2i}^T$); and twenty parameters, $\theta = \{ \phi_1, \phi_2, \sigma_{v,1}^2, \sigma_{v,2}^2, \sigma_{v,3}^2, \sigma_{e,1}^2, \sigma_{e,2}^2, \sigma_{e,3}^2, p_{11}^2, p_{12}^2, p_{21}^2, p_{22}^2, p_{13}^2, p_{14}^2, p_{15}^2, p_{23}^2, p_{24}^2, p_{31}^2, p_{32}^2, p_{33}^2, p_{34}^2 \}$. Note that the parameters of the model are treated as random variables with prior distributions in the Bayesian context.

While the derivation of $4T + 20$ random samples based on appropriate conditional densities is more complicated than the above example with two random variables, the principles are the same. The data on real exchange rates, $Q^T$, play the role of the observable $W^T$ in the example above.

The analogs to $f(p|R, W^T)$ and $f(r|P, W^T)$ in the example are the densities of each row of $Z^T = [Y^T X^T]$, $S_{1i}^T$, $S_{2i}^T$, and $\theta$ conditional on $Q^T$ and on all of the other values of $Z^T$, $S_{1i}^T$, $S_{2i}^T$, and $\theta$. The above example was one of "single-move" Gibbs sampling. We actually make use of the "multimove" Gibbs sampling of Carter and Kohn (1994) and Kim and Nelson (1998). In multimove Gibbs sampling, a Gibbs bloc can be the whole $Z^T$, $S_{1i}^T$, or $S_{2i}^T$ matrices, as opposed to single-move Gibbs sampling, in which each row of $Z^T$, $S_{1i}^T$, or $S_{2i}^T$ is generated one at a time.

These conditional densities of each Gibbs bloc of $Z^T$, $S_{1i}^T$, and $S_{2i}^T$, and of each element of $\theta$, are easily derived. Conditional on $S_{1i}^T$, $S_{2i}^T$, and $\theta$, the model reduces to a standard linear unobserved components model. So, $f(Z^T|S_{1i}^T, S_{2i}^T, \theta, Q^T)$ can be generated as in Carter and Kohn (1994). Conditional on $Y^T$, the distribution of $S_{1i}^T$ is independent of the data, $Q^T$, and of the stationary component, $X^T$, so $f(S_{1i}^T|Z^T, S_{2i}^T, \theta, Q^T) = f(S_{1i}^T|Y^T, S_{2i}^T, \theta)$. Similarly, conditional on $X^T$, the distribution of $S_{2i}^T$ is independent of the data, $Q^T$, and of the permanent component, $Y^T$, so $f(S_{2i}^T|Z^T, S_{1i}^T, \theta, Q^T) = f(S_{2i}^T|X^T, S_{1i}^T, \theta)$. The problem of generating $S_{1i}^T$ and $S_{2i}^T$ as a bloc was solved by Kim and Nelson (1998). The derivation of $f(\theta|S_{1i}^T, S_{2i}^T, Z^T, Q^T)$ is a straightforward Bayesian exercise. The detailed derivation of each conditional distribution for our Gibbs sampler is given in an appendix which is available from the authors on request.

In our estimation, the Gibbs sampler appears to converge after about two hundred iterations. We run the Gibbs sampler for eleven thousand observations, and to be on the safe side, discard the first one thousand. Our distributions of $Z^T$, $S_{1i}^T$, and $S_{2i}^T$ are based on the last ten thousand observations. For our distribution of $\theta$, we take every fifth observation from the final ten thousand iterations (because of potential serial correlation across the iterations).

3. ESTIMATION RESULTS

Table 1a presents the results of the estimation of the model described in section 1. The real exchange rate is defined as the relative price of U.K. to U.S. producer goods (that is, the product of the dollar/pound nominal exchange rate and the U.K. producer price index, divided by the U.S. producer price index). The table reports are the mean and median of the Gibbs samples of each parameter, their standard deviations, and the
### TABLE 1A

**THREE-STATE MARKOV-SWITCHING VARIANCE FOR PERMANENT COMPONENT AND THREE-STATE MARKOV-SWITCHING VARIANCE FOR TRANSITORY COMPONENT**

<table>
<thead>
<tr>
<th></th>
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<td></td>
<td>Prior</td>
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<td>St. Dev.</td>
<td>Mean</td>
<td>St. Dev.</td>
<td>Median</td>
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<td></td>
<td></td>
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<tr>
<td>$p_{1,11}$</td>
<td>0.5</td>
<td>0.8837</td>
<td>0.1795</td>
<td>0.9655</td>
<td>0.4287</td>
<td>0.9948</td>
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<tr>
<td>$p_{1,12}$</td>
<td>0.25</td>
<td>0.0803</td>
<td>0.1478</td>
<td>0.0152</td>
<td>0.0009</td>
<td>0.4428</td>
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<tr>
<td>$p_{1,21}$</td>
<td>0.25</td>
<td>0.3768</td>
<td>0.2305</td>
<td>0.3543</td>
<td>0.0460</td>
<td>0.7988</td>
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<tr>
<td>$p_{1,12}$</td>
<td>0.5</td>
<td>0.3947</td>
<td>0.2273</td>
<td>0.3802</td>
<td>0.0494</td>
<td>0.7823</td>
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<tr>
<td>$p_{1,31}$</td>
<td>0.25</td>
<td>0.2507</td>
<td>0.1767</td>
<td>0.2154</td>
<td>0.0328</td>
<td>0.6128</td>
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<td>$p_{1,31}$</td>
<td>0.25</td>
<td>0.1827</td>
<td>0.1654</td>
<td>0.1342</td>
<td>0.0000</td>
<td>0.5255</td>
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<td>$\sigma_{v_1}^2$</td>
<td>—,b</td>
<td>1.0557</td>
<td>0.1782</td>
<td>1.0401</td>
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<td>$\sigma_{v_2}^2$</td>
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<td>1.9638</td>
<td>1.8739</td>
<td>1.3437</td>
<td>0.8638</td>
<td>5.1762</td>
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<tr>
<td>$\sigma_{v_3}^2$</td>
<td>—,b</td>
<td>5.1196</td>
<td>6.1515</td>
<td>2.9679</td>
<td>1.0301</td>
<td>9.5955</td>
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<table>
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<th>Transitory Component</th>
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<tr>
<td></td>
<td>Prior</td>
<td>Mean</td>
<td>St. Dev.</td>
<td>Mean</td>
<td>St. Dev.</td>
<td>Median</td>
<td>90% Posterior Band</td>
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<tr>
<td>$p_{2,11}$</td>
<td>0.5</td>
<td>0.9877</td>
<td>0.0043</td>
<td>0.9882</td>
<td>0.9801</td>
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<td>$p_{2,12}$</td>
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<td>0.0034</td>
<td>0.0028</td>
<td>0.0027</td>
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<td>$p_{2,21}$</td>
<td>0.25</td>
<td>0.0113</td>
<td>0.0169</td>
<td>0.0082</td>
<td>0.0008</td>
<td>0.0282</td>
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<tr>
<td>$p_{2,31}$</td>
<td>0.25</td>
<td>0.9688</td>
<td>0.0131</td>
<td>0.9741</td>
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<td>0.9921</td>
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<td>$p_{2,31}$</td>
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<td>0.1199</td>
<td>0.0570</td>
<td>0.1114</td>
<td>0.0467</td>
<td>0.2231</td>
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<td>1.1262</td>
<td>0.0400</td>
<td>1.1260</td>
<td>1.0606</td>
<td>1.1917</td>
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<td></td>
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<tr>
<td>$\phi_2$</td>
<td>0.0</td>
<td>-0.1400</td>
<td>0.0394</td>
<td>-0.1404</td>
<td>-0.2057</td>
<td>-0.0767</td>
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</tr>
<tr>
<td>$\sigma_{v_1}^2$</td>
<td>—,b</td>
<td>0.9592</td>
<td>0.1658</td>
<td>0.9371</td>
<td>0.7341</td>
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<tr>
<td>$\sigma_{v_2}^2$</td>
<td>—,b</td>
<td>6.1336</td>
<td>0.9290</td>
<td>6.0852</td>
<td>4.7800</td>
<td>7.7394</td>
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<tr>
<td>$\sigma_{v_3}^2$</td>
<td>—,b</td>
<td>23.3221</td>
<td>8.2048</td>
<td>22.2366</td>
<td>13.1953</td>
<td>36.5233</td>
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<tr>
<td>$\delta_1 + \delta_2$</td>
<td>—,b</td>
<td>0.9861</td>
<td>0.0052</td>
<td>0.9877</td>
<td>0.9758</td>
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<tr>
<td>$\delta_2$</td>
<td>—,b</td>
<td>0.9838</td>
<td>0.0062</td>
<td>0.9857</td>
<td>0.9712</td>
<td>0.9899</td>
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<td></td>
</tr>
</tbody>
</table>

*p* Prior constrained as discussed in appendix.

*Standard errors of the prior distribution of variance terms are improper.

*eigen* represents the largest eigenvalue of the transitory component.

90 percent posterior probability bands. This information should help give a pretty clear picture of the distribution of each parameter.

In the standard maximum likelihood approach to estimation, there are difficulties testing for the null of two states versus the alternative of three states for each of the permanent and transitory components. The problem is that some of the transition probabilities are not identified under the null—for example, of the “Davies (1977) problem.” For example, suppose the null is that the variance of the permanent component in the second state is equal to the variance of that component in the third state: $\sigma_{v_2}^2 = \sigma_{v_3}^2$. We can interpret the null as saying that there is no state 3 for the permanent component—only states 1 and 2. In that case, there are only two distinct transition probabilities under the null, $p_{1,11}$ and $p_{1,22}$. However, in the model of section 1, there are six transition probabilities. Four of these are not identified under the null hypothesis. In that case, the likelihood ratio test of $\sigma_{v_2}^2 = \sigma_{v_3}^2$ does not have the standard chi-square distribution because of the parameters that are estimated under the alternative but not identified under the null hypothesis. Andrews and Ploberger (1994) and
Hansen (1996b) have recently proposed a solution to this problem when the parameters are estimated in the classical context.\footnote{Their solutions cannot be applied to this problem, however, because the score is identically zero under the null. Hansen (1992, 1996a) provides a method for calculating conservative \(p\)-values for this model which are, however, very computationally burdensome.}

The estimates in Table 1a indicate that the model for the permanent component may be overparameterized. Note that the 90 percent posterior probability bands for the variance estimates overlap. That is, the band for \(\sigma_{v1}^2\) overlaps the band for \(\sigma_{v2}^2\); the band for \(\sigma_{v2}^2\) overlaps the band for \(\sigma_{v3}^2\); and, in fact, the band for \(\sigma_{v1}^2\) even overlaps the band for \(\sigma_{v3}^2\). This strongly suggests that there are not three separate states of variance for the permanent component. In contrast, there is no overlap in the 90 percent posterior bands for the variance of the states of the transitory component.

Another indication that the model of the permanent component is overparameterized is the wide confidence intervals on the transition probabilities. If the permanent component should be modeled with fewer states, then some of the transition probabilities are redundant. They will not be estimated with much precision. In fact, the standard deviations of the transition probabilities for the permanent component \((p_{1,11},\ldots)\) are much larger than the estimates for the transitory component \((p_{2,11})\). For the permanent component, the standard deviations range from 0.1478 to 0.2305, while they take on values of 0.0028 to 0.0624 for the transitory component.

From this evidence, we conclude that the permanent component can be modeled with fewer than three states, but there are three distinct states for the transitory component. Table 1b reports estimates of a model with two states for the permanent component and three for the transitory component. We reach the same conclusion here: there appear to be three distinct states for the temporary component, but not as many as two for the permanent part. Again, we see that the 90 percent posterior probability bands for \(\sigma_{v1}^2\) and \(\sigma_{v2}^2\) overlap, and the standard error for the transition probabilities \(p_{11,11}\) and \(p_{11,22}\) are large (0.1748 and 0.2516, respectively). In contrast, the posterior probability bands for \(\sigma_{v1}^2\), \(\sigma_{v2}^2\), and \(\sigma_{v3}^2\) do not overlap, and the standard errors for the transition probabilities of the transitory component \((p_{2,11})\) have small standard errors (0.0029 to 0.0613).

We settle on the model presented in Table 1c: three-state Markov switching for the transitory component, and a constant variance for the permanent component. Again, we find for this model that the posterior probability bands for \(\sigma_{v1}^2\), \(\sigma_{v2}^2\), and \(\sigma_{v3}^2\) do not overlap, and the standard errors for the transition probabilities of the transitory component \((p_{2,11})\) have small standard errors (0.0028 to 0.0554). When we fit a model with only two states for the transitory component (and one for the permanent component), we find that the posterior distribution of the estimates of the variances of the transitory component is bimodal. This indicates that that model has not allowed for enough states for the transitory component. In contrast, we find that the posterior distributions of \(\sigma_{v1}^2\), \(\sigma_{v2}^2\), and \(\sigma_{v3}^2\) for the model of Table 1c are single-peaked, suggesting that three states are adequate for the transitory component.

So, we can conclude that the transitory component switches among states with different variances, while the permanent component does not exhibit state switching.
### TABLE 1B

**TWO-STATE MARKOV-SWITCHING VARIANCE FOR PERMANENT COMPONENT AND THREE-STATE MARKOV-SWITCHING VARIANCE FOR TRANSIENT COMPONENT**

<table>
<thead>
<tr>
<th></th>
<th>Permanent Component</th>
<th>Transitory Component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prior</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{1,11}$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$p_{1,22}$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma^2_{1}$</td>
<td>~b</td>
<td>0.9966</td>
</tr>
<tr>
<td>$\sigma^2_{2}$</td>
<td>~b</td>
<td>2.9003</td>
</tr>
</tbody>
</table>

*Priors constructed as discussed in appendix.  
Standard errors of the prior distribution of variance terms are improper.  
eigen represents the largest eigenvalue of the transitory component.

### TABLE 1C

**CONSTANT VARIANCE FOR PERMANENT COMPONENT AND THREE-STATE MARKOV-SWITCHING VARIANCE FOR TRANSIENT COMPONENT**

<table>
<thead>
<tr>
<th></th>
<th>Permanent Component</th>
<th>Transitory Component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prior</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>~b</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Prior</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Median</th>
<th>90% Posterior Band</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{2,11}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2887</td>
<td>0.9864</td>
<td>0.0042</td>
<td>0.9869</td>
<td>0.9786</td>
</tr>
<tr>
<td>$p_{2,12}$</td>
<td>0.25</td>
<td>~a</td>
<td>0.0037</td>
<td>0.0028</td>
<td>0.0031</td>
<td>0.0004</td>
<td>0.0004</td>
</tr>
<tr>
<td>$p_{2,21}$</td>
<td>0.25</td>
<td>~a</td>
<td>0.0109</td>
<td>0.0085</td>
<td>0.0090</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
<tr>
<td>$p_{2,22}$</td>
<td>0.5</td>
<td>0.2887</td>
<td>0.9701</td>
<td>0.0160</td>
<td>0.9724</td>
<td>0.9410</td>
<td>0.9910</td>
</tr>
<tr>
<td>$p_{2,31}$</td>
<td>0.25</td>
<td>~a</td>
<td>0.1267</td>
<td>0.0524</td>
<td>0.1175</td>
<td>0.0582</td>
<td>0.2216</td>
</tr>
<tr>
<td>$p_{2,32}$</td>
<td>0.25</td>
<td>~a</td>
<td>0.0745</td>
<td>0.0554</td>
<td>0.0618</td>
<td>0.0084</td>
<td>0.1789</td>
</tr>
<tr>
<td>$f_{1}$</td>
<td>0.0</td>
<td>2.0</td>
<td>1.1275</td>
<td>0.0402</td>
<td>1.1286</td>
<td>1.0618</td>
<td>1.1944</td>
</tr>
<tr>
<td>$f_{2}$</td>
<td>0.0</td>
<td>2.0</td>
<td>0.1398</td>
<td>0.0399</td>
<td>0.1401</td>
<td>0.2057</td>
<td>0.0750</td>
</tr>
<tr>
<td>$\sigma^2_{1}$</td>
<td>~b</td>
<td>0.8169</td>
<td>0.0962</td>
<td>0.8132</td>
<td>0.6644</td>
<td>0.9763</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_{2}$</td>
<td>~b</td>
<td>6.0803</td>
<td>0.8582</td>
<td>6.0143</td>
<td>4.8252</td>
<td>7.5603</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_{\phi}$</td>
<td>~b</td>
<td>26.2161</td>
<td>6.9803</td>
<td>25.0158</td>
<td>17.2506</td>
<td>39.8036</td>
<td></td>
</tr>
<tr>
<td>$\phi_{1} + \phi_{2}$</td>
<td>0.9877</td>
<td>0.0037</td>
<td>0.9889</td>
<td>0.9799</td>
<td>0.9914</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eigen$^c$</td>
<td></td>
<td>0.9856</td>
<td>0.0044</td>
<td>0.9871</td>
<td>0.9763</td>
<td>0.9899</td>
<td></td>
</tr>
</tbody>
</table>

*Priors constructed as discussed in appendix.  
Standard errors of the prior distribution of variance terms are improper.  
eigen represents the largest eigenvalue of the transitory component.

343
Visual inspection of Figure 1 clearly indicates there are shifts in the variance. Here we can conclude that it is switches in the variance of the transitory component that are revealed in the graph.

There are several interesting things to note about the parameter estimates from Table 1c. First, the variance of the permanent component is relatively small compared to the variances of the transitory component. $\sigma_\pi^2$ is about 1.0, which is slightly larger than $\sigma_{\pi 1}^2 (=0.817)$, but much smaller than $\sigma_{\pi 2}^2 (=6.080)$ and $\sigma_{\pi 3}^2 (=26.216)$. When the transitory component is in states 2 or 3, a very large proportion of the variance of the real exchange rate is attributable to the transitory component.

States 1 and 2 for the transitory component are quite persistent, but state 3 is not. The expected duration of the low-variance state, given by $1/1 - p_{2,11}$ is equal to 73.53 months, or over six years. The expected duration of the medium-variance state, $1/1 - p_{2,22}$, is 33.44 months. The high-variance state has an expected duration of 4.97 ($=1/P_{2,31} + P_{2,32}$) months. We shall see shortly that the medium-variance regime generally corresponds to the post-1973 era of floating nominal exchange rates; the low-variance regime is associated with periods of fixed nominal exchange rates prior to 1973; and, the high-variance regime matches either times of exchange rate revaluation or periods of rapid inflation in one of the two countries prior to 1973.

The transitory component, $x_t$, is itself highly persistent. The median estimate of the largest eigenvalue for the AR(2) model of $x_t$ is 0.987. This implies a half-life of transitory shocks of fifty-five months. This estimate of the persistence of temporary shocks is consistent with other estimates in the literature (see Rogoff 1996). However, the persistence of the transitory shock makes it difficult to extract a permanent component. We shall discuss this problem further in section 5.

Figures 2.1 and 2.2 plot the permanent and transitory components of the log of the real exchange rate. In Figure 2.1, the permanent component is plotted along with its 95 percent posterior probability band. The diagram shows that the permanent component evolves gradually over time (low innovation variance) and has had a tendency to rise since World War II. The permanent component is also graphed in Figure 1 along with the actual value of the real exchange rate. It is clear that very little of the volatility of the real exchange rate comes from the permanent component.

Figure 2.2 shows the transitory component, $x_t$, and its 95 percent posterior probability band. The transitory component has periods of high volatility, which match the periods of high volatility in the real exchange rate itself. Examination of Figure 1 and Figure 2.2 reveals that the transitory component accounts for the period of sustained high volatility in the real exchange rate after 1973.

4. INTERPRETATION OF THE TRANSITORY AND PERMANENT COMPONENTS

In this section we examine the $x_t$ and $y_t$ series generated from the Gibbs sampler. The series have been generated from a pure time-series analysis, but here we ask what are the economic properties of these series.

9. We do not construct these bands by taking the central 95 percent of draws from the Gibbs sampler for each point in time because this requires saving a large amount of data in memory on the computer. Instead, we calculate the 95 percent posterior probability band as the 1.96 standard deviation band (which requires saving only $x_t$ and $y_t^2$). Under the normality assumption, the two methods are asymptotically equivalent.
The Transitory Component

First, we plot the smoothed probabilities that the transitory component is in each of states 1, 2, and 3 in Figures 3.1, 3.2, and 3.3, respectively. First, consider Figure 3.2, which graphs the probability of being in the medium-variance state. For almost the entire period after 1973, the probability that $S_{2,t} = 2$ is near unity. It appears we can associate the medium-variance state with the period of floating nominal exchange rates.
In fact, the other periods in which there is a high probability that \( S_{2,t} = 2 \) also correspond to times in which the dollar/pound nominal exchange rate was floating. There was a period of controlled floating from December 1914 to March 1919, and a regime with a more volatile float from April 1919 to April 1925.\(^{10}\) During this period, our estimates show that the probability that \( S_{2,t} = 2 \) hovers between 0.25 and 0.80. According to our model, the beginning of the period of free floating might have been a period in which the real exchange rate was in its most volatile state (\( S_{2,t} = 3 \)).

There is one other period in which the probability that \( S_{2,t} = 2 \) rises above 0.5: in the early 1930s, when the dollar/pound exchange rate returned to floating after the gold exchange standard for the pound which prevailed from May 1925 to August 1931. This episode of floating rates lasted until September of 1939. Our model suggests during most of this period \( S_{2,t} = 2 \), except in September 1939 when Britain abandoned the gold standard and mid-1933 when the United States ceased stabilizing the price of gold (during which \( S_{2,t} = 3 \)).

Other than the three periods mentioned in which it is likely that \( S_{2,t} = 3 \) (1919–1920, September 1931, and mid-1933), every period of floating exchange rates (1914–1925, early 1930s, and post-1973) correspond to the periods in which the probability \( S_{2,t} = 2 \) is greater than 0.5.

Figure 3.1 shows that for most of the time prior to 1973, the probability that \( S_{2,t} = 1 \) is very nearly unity. Those times in which the real exchange rate is in its low-variance state correspond to times in which the nominal exchange rate was fixed. There are a few exceptions to this rule, which we turn to next.

Figure 3.3 shows several short periods in which the probability that \( S_{2,t} = 3 \) is high, while most of the time that probability is near zero. Specifically, there are twelve short periods in which this probability is greater than 0.5. Each of these episodes is associated with a monetary event:

1. **Mid-1898.** This period is concurrent with gold discoveries in Alaska and South Africa (see Friedman and Schwartz, pp. 135–38). During this period, nominal price levels in both the United States and the United Kingdom fluctuated wildly from month to month (while the nominal exchange rate remained fixed) as the money supplies of the two countries were jolted by increases in their stocks of gold.

2. **Late 1902.** There was a rapid inflation in the United States through September of 1902. This in part was driven by an expansionary monetary policy followed by the U.S. Treasury, which increased its deposits at major banks and persuaded the larger national banks to increase currency in circulation. The business cycle peaked in September 1902, according to the NBER chronology, and prices fell from then until the end of the year. (See Friedman and Schwartz, pp. 149–52.)

3. **Mid- to late-1914.** As World War I began, the gold standard was abandoned. (See Grilli and Kaminsky, pp. 193–94.)

4. **Mid-1919 to late-1920.** The British ceased intervention in the foreign ex-

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\(^{10}\) Grilli and Kaminsky (1991) summarize the exchange rate history for most of our sample period.
change market that had begun in December 1914, and the dollar began to float freely against the pound. This period of floating lasted until May 1925, as was noted above, but initially there was a large realignment of the exchange rate as the pound depreciated from $4.76 to less than $3.40 by February 1920. (See Grilli and Kaminsky, p. 194.)

5. September 1931. Britain abandons the gold standard (see Friedman and Schwartz, pp. 380–84).


7. September 1939, Britain devalues the pound (see Grilli and Kaminsky, p. 195).


10. Late-1967. The pound is devalued.


12. Late-1992. The United Kingdom leaves the Exchange Rate Mechanism of the EMU. Between August and November 1992, the price of the pound falls from $1.95 to $1.52.

There is one other small spike in the graph of the probability that \( S_{2,t} = 3 \), which occurs in 1973, as the modern floating exchange rate era began.

All of the episodes in which the real exchange rate appears to shift to \( S_{2,t} = 3 \) are associated with monetary events.11 The periods in which it is likely \( S_{2,t} = 1 \) are periods in which the nominal exchange rate is fixed, and the periods in which it is probable that \( S_{2,t} = 2 \) are periods when the nominal exchange rate is floating. The switches between states all appear to be precipitated by monetary events. This pattern is reminiscent of Mussa’s (1986) finding that real exchange rate volatility increased directly with nominal exchange rate volatility for a large number of time periods and currencies. However, we find that a few of the periods of extreme real exchange rate volatility are associated with large nominal price movements, which are nonetheless associated with dramatic monetary events.

The Permanent Component

We next turn to the permanent component, \( y_t \). We investigate whether the permanent series generated by the Gibbs sampler corresponds to other measures of the permanent real exchange rate that have been proposed in the literature. Specifically, we examine measures that are based on economic models and that use data other than the real exchange rate in their construction.

11. With the exception of the “speculative bubble” in the 1980s.
Five of the alternative permanent series are from Mark and Choi (1997) and the sixth comes from Engel (1999).

The series from Mark and Choi are constructed by first positing models of the long-run real exchange rate as functions of economic variables. Let $w_i$ represent the vector of these variables for model $i$. Mark and Choi estimate the cointegrating vector between $q_s$ and $w_i$, and construct the permanent component as the fitted value from the cointegrating regression.\(^{12}\)

Call these measures of the long-run real exchange rate $v_s$.

1. $v_{pu}$ is a function of productivity in U.K. manufacturing relative to productivity in U.S. manufacturing. If relative productivity in the United Kingdom increases faster than in the United States, relative U.K./U.S. prices should decline because the U.K.'s relative costs decline.

2. $v_{pu}$ is a function of the relative productivities and of the difference in the ratio of government consumption to income in the United Kingdom and the United States. Following Rogoff (1992), government spending has an effect on the long-run real exchange rate under intersectoral factor immobility and poor international capital market integration.

3. $v_{pu}$ is a function of per capita output in the United Kingdom relative to the United States. Following Balassa (1964) and Samuelson (1964), higher-income countries should have higher relative price levels.

4. $v_{pu}$ is a function of the real interest differential between the United Kingdom and the United States. Mark and Choi mention that in Dornbusch (1976), the real exchange rate is related to the real interest differential, though they acknowledge that the relationship is posited to be a short-run one in Dornbusch.

5. $v_{pu}$ is a function of the relative money supplies and relative levels of output in the United Kingdom and the United States. Mark and Choi speculate that monetary effects on the real exchange rate might be permanent (or so long-lasting as to be indistinguishable from permanent).

6. $v_{pu}$ comes from Engel (1999), which examines various measures of the relative price of nontraded goods. Many models of real exchange rates relate their movement to the price of nontraded goods to traded goods in one country relative to the other country. This measure takes the producer price index relative to the consumer price index as the measure of the price of nontraded to traded goods in each country. It is unlikely that this measure of the real exchange rate will track the measure of $y_s$ from the model. In fact, our measure of the real exchange rate, $q_s$, in this paper uses $\ln(S/E) + \ln(ukppi) - \ln(usppi)$, while $v_{pu}$ is measured as $\ln(ukcpi) - \ln(ukppi) - \ln(uscpi) - \ln(usppi)$, so that $\ln(ukppi) - \ln(usppi)$ enters with opposite signs in $q_s$ and $v_{pu}$.

The measures of the long-run real exchange rate from Mark and Choi are monthly from 1960:1 to 1993:11. The relative price from Engel is monthly from 1963:1 to 1995:11.

\(^{12}\) This "permanent" component, of course, need not be a pure random walk.
We expect our measure of $y_t$ to be cointegrated with these other measures of the permanent component, with a cointegrating vector of $(1, -1)$. As a first step in assessing the relationship between $y_t$ and the $v_{it}$ measures, we perform Johansen cointegration tests (under the assumption of no time trend).\footnote{We include twelve lags in this test for all variables.} We reject the null of no cointegration for only a single case—$v_{3t}$, relative per capita output. The value of those statistics is reported in the first column of Table 2.

We next estimate the cointegrating vector of $y_t$ and each of the $v_{it}$, using the method proposed by Stock and Watson.\footnote{We use $y_t$ as the dependent variable and $v_{it}$ as the independent variable. We include twelve leads and lags of the change in $v_{it}$ for all variables.} The first column of Table 2 reports the point estimates from those regressions. We see that the estimated slope coefficient is close to one when the measure of the long-run real exchange rate is $v_{3t}$, the relative per capita income levels. The estimated slope coefficient is 0.746. However, we would reject the null that the cointegrating vector is $(1, -1)$ at a 5 percent level. The estimated "cointegrating vectors" for $y_t$ and each of the other $v_{it}$ is also reported in Table 2. But, given that we failed to reject the null of no cointegration for these variables, the parameter estimates are meaningless.

The test based on the standard errors of the cointegrating vector estimated by the Stock-Watson procedure assumes that it is known that the variables are cointegrated. A formal test of the null that $v_{it}$ (for each $i = 1, 2, \ldots, 6$) is not cointegrated with $y_t$ with cointegrating vector $(1, -1)$ is proposed by Horvath and Watson (1995).\footnote{We include eight lags of each variable in all tests.} We report the test statistics from these tests in the third column of Table 2. Here, we reject the null only for $v_{3t}$.

Thus, the permanent component from our univariate time series analysis, $y_t$, ap-
pears to be cointegrated with $v_{3t}$, with cointegrating vector $(1, -1)$, although the evidence is not entirely unambiguous. As in the Balassa-Samuelson hypothesis, the real exchange rate changes in the direction of relative output levels in the long run.

5. ARE THERE REALLY SEPARATE PERMANENT AND TRANSITORY COMPONENTS?

The estimates of the model presented in Table 1a indicate a very persistent transitory component, with an innovation variance that, on average, is much larger than the innovation variance for the permanent component. Is it possible that the data are not really generated by a process with a permanent and a transitory component? Might the process really be stationary? Or might it be a random walk?

Unfortunately, it is not possible to get a definitive answer to those questions. Cochrane (1991) and Blough (1992) have noted that for any process with a unit root, there is a stationary process that is nearly observationally equivalent. Engel (2000) has discussed this issue in the context of real exchange rates and tests of purchasing power parity. As we shall see in this section, it is difficult to make a definitive conclusion on the “correct” model of the real exchange rate. However, we shall present some evidence that our approach is, at least, a plausible representation of the data-generating process.

First, there is a fairly good case that can be made against the claim that the real exchange rate is a pure random walk. In terms of our model, if the $x_t$ component were a random walk, then $q_t = x_t + y_t$ would be the sum of two random walks and so would itself be a random walk. How strong is the evidence against the hypothesis that $x_t$ follows a random walk? If that hypothesis were true, $\phi_1$ and $\phi_2$ would sum to unity. The mean of the posterior distribution of $\phi_1 + \phi_2$ is 0.9877, so clearly the process is not too far from a random walk. Note that the 90 percent posterior probability band (0.9799 to 0.9914) does not include unity, but that is not a definitive statement of the odds against the hypothesis that $x_t$ is a random walk. Nelson (1988) presents evidence that if one performs a state-space decomposition of artificial random walk time series into permanent and transitory components, the state-space model tends to indicate incorrectly that much of the short-term variance of the series comes from a transitory component.

However, we have noted that many researchers have rejected a random walk in the real exchange rate using the Dickey-Fuller test. In fact, as mentioned above, we reject a unit root using the Augmented Dickey-Fuller (ADF) test. Our Dickey-Fuller statistic is 3.007, which just exceeds the 5 percent critical value of 2.86.

We noted in the introduction that the heteroskedasticity of the data-generating process may influence the size of the Dickey-Fuller test. Although asymptotically there should be no size bias introduced by heteroskedasticity, Kim, Nelson, and Startz (1998) have shown that the particular pattern of heteroskedasticity in a given sample could influence the rejection rate for tests for unit roots. We examine that issue by performing a Monte Carlo exercise to calculate the size of the ADF test when the data are

16. We settle on a lag length of two by using a data-based method.
generated by a random walk with a pattern of heteroskedasticity similar to the one in our data on the real exchange rate.

We create ten thousand artificial series of random walk random variables: \( q_t = q_{t-1} + \sigma_{2,t} \epsilon_t \). The shock, \( \epsilon_t \), is drawn from a \( N(0, 1) \) distribution. We derive \( \sigma_{2,t} \) from our model estimate in section 3. That is, we use the variance estimate of the transitory component from section 3 to create an artificial random walk series with heteroskedasticity. Since the variance of the permanent component we estimated was constant and small, if we were to estimate a random walk model with Markov-switching variances on our data, the variances would essentially equal a constant plus the variance of our transitory component. For each run of the Gibbs sampler on the model of section 3, we get estimates of \( \sigma^2_{u1}, \sigma^2_{v2}, \) and \( \sigma^2_{u3} \) for the transitory component. In addition, each run of the Gibbs sampler simulates values of \( S_{2,1}, S_{2,2}, \) and \( S_{2,3} \), where \( S_{2,j} = 1 \) if \( S_{2,j} = j \), \( S_{2,j} = 0 \) otherwise, \( j = 1, 2, 3 \). Then \( \sigma_{2,t} \) is the square root of \( \sigma^2_{2,t} \), where \( \sigma^2_{2,t} \) is defined in equation (7).

We run the ADF test as described above, and calculate the frequency with which the Dickey-Fuller statistic exceeds 2.86. We find the actual size of the test is 0.0386—so there actually appears to be a slight downward bias in the size of the Dickey-Fuller statistic with this pattern of heteroskedasticity. We conclude that \( q_t \) probably does not follow a random walk.

While these results confirm the presence of a stationary component to the real exchange rate, why do we also allow for a permanent component? Engel (2000) shows that the ADF test has a very large size bias when the real exchange rate is generated from the sum of a random walk and a very volatile transitory component. Engel presents an example in which the real exchange rate is posited to be the sum of a random walk and an AR(1). Both components are assumed to be homoskedastic, but the innovation variance of the stationary component is assumed to be very large compared to that of the random walk. The stationary representation for the real exchange rate has the first differences following an ARMA(1, 1) process. The root of the MA component is near unity in absolute value. The ADF test, which is designed to test against the null of stationary AR processes (that is, with no MA component), has a very large size bias even when the random walk component accounts for a large fraction of the variance of the real exchange rate within the sample.

Engel’s (2000) examples are all for homoskedastic permanent and stationary components. In this section, we repeat some of Engel’s Monte Carlo exercises to assess the size bias of the ADF test under the null hypothesis that the real exchange rate is generated by the model of section 1.

The first Monte Carlo experiment generates ten thousand draws from the model of section 1, using the parameter estimates from Table 1c. We run the ADF test as described above. We find the true size of a 5 percent test is 29.51 percent. This result corresponds closely to the example in Engel (2000) which calibrates a homoskedastic model to one hundred years of U.S./U.K. real exchange rates.\(^\text{17}\) Depending on the pa-

\(^{17}\) Engel’s real exchange rates were based on annual personal consumption deflators, rather than the PPI.
rameter values chosen for the model, the ADF test with a nominal size of 5 percent had actual sizes ranging from 29.86 percent to 35.66 percent.

Since the “5 percent” ADF test actually rejects nearly 30 percent of the time when the model of section 1 is correct, we should not take the marginal rejection of a unit root at the “5 percent” level as very strong evidence against the model.

The first Monte Carlo exercise was undertaken assuming that the parameter estimates reported in Table 1c are the true values of the parameters. We also assess the size of the ADF test taking into account our uncertainty about the parameters. To do this, we generate a draw of the parameters from the Gibbs sampler. We then use the parameter draw to generate an artificial real exchange rate series. We run the ADF test on the artificial series. We repeat this procedure ten thousand times. So our ten thousand ADF tests are on series generated from different parameter values each time, with the distribution of parameter values corresponding to the distribution generated by the Gibbs sampler. There is still a substantial size bias in the ADF test, although less than when we assumed parameter certainty. A nominal 5 percent test has a true size of 19.08 percent taking into account parameter uncertainty, but 29.51 percent when we assume the parameters are known.

6. CONCLUSIONS

We have estimated a univariate time series model of the U.S./U.K. real exchange rate. Our model has a transitory component and a permanent component. The transitory component exhibits heteroskedasticity, switching among three states with different variances. We find the following:

1. The transitory component is highly persistent.
2. The regimes of variance of the transitory component of the real exchange rate are closely related to monetary phenomena, as Mussa (1986) has claimed.
3. Innovations in the permanent component have relatively low variance.
4. Relative per capital output levels may be important in understanding the behavior of the long-run real exchange rate.

Although we do not attempt a formal test of any specific model, we note that the model we estimate shares many of the characteristics of Mussa’s (1982) generalization of the Dornbusch (1976) model. In Mussa, the movement of the real exchange rate can be decomposed into an equilibrium component and a transitory component. The behavior of the transitory component depends on the nominal exchange rate regime—it is more volatile when the nominal exchange rate is more volatile. This component is stationary (and, thus transitory) because it arises from the slow adjustment of nominal prices. In the long run, the exchange rate is expected to converge to the equilibrium component. The equilibrium component is determined by “real factors” that would arise in equilibrium models, such as the Balassa-Samuelson formulation.

Our model was estimated as a univariate time series model, so we did not make use
of other information such as relative income levels, the timing of monetary shocks, etc., in honing our estimates of the model. We arrive at our model by considering the generalizations to the univariate model suggested recently in the literature on testing for purchasing power parity. Specifically, we allow for regime switching to allow for changes in the variance of the real exchange rate [as suggested by Frankel and Rose (1996) and others], and we allow both a transitory and permanent component [as suggested by Engel (2000)] because the unit root tests do not very conclusively rule out the presence of a permanent component when the transitory component is very volatile. However, we find that the model we have estimated seems to share these salient characteristics of Mussa’s (1982) model: the transitory component of the real exchange rate is associated with monetary phenomena [as in Dornbusch (1976)], and the permanent component appears to be associated with trends in relative income levels, as in the Balassa-Samuelson hypothesis.

So, we take the middle ground. We do not conclude, as many have done with long time series data on real exchange rates, that purchasing power parity deviations are only temporary. Nor do we conclude that real exchange rate movements are entirely permanent, as in many equilibrium models. Although we cannot make a definitive statement, the data appear to support a model that contains both a transitory component (whose regime switches are driven by monetary phenomena) and a permanent component (driven by relative per capita income levels.)

LITERATURE CITED


Engel, Charles, and James D. Hamilton. "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?" American Economic Review 80 (September 1990), 689–713.


Hansen, Bruce. "Inference When a Nuisance Parameter Is Not Identified under the Null Hypothesis." Econometrica 64 (March 1996b), 413–30.


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