Econometrics 710
Midterm Exam
March 12, 2013

This exam concerns the model

$$
\begin{aligned}
y_i &= m(x_i) + e_i && (1)\\
m(x) &= \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_p x^p && (2)\\
E(z_i e_i) &= 0 && (3)\\
z_i &= (1, x_i, ..., x_i^p)' && (4)\\
g(x) &= \frac{d}{dx} m(x) && (5)
\end{aligned}
$$

with iid observations (y_i, x_i), $i = 1, ..., n$. The order of the polynomial p is known.

1. How should we interpret the function $m(x)$ given the projection assumption (3)? How should we interpret $g(x)$? (Briefly)
2. Describe an estimator $\hat{g}(x)$ of $g(x)$.
3. Find the asymptotic distribution of $\sqrt{n}(\hat{g}(x) - g(x))$ as $n \to \infty$.
4. Show how to construct an asymptotic 95% confidence interval for $g(x)$.
5. Assume $p = 2$. Describe how to estimate $g(x)$ imposing the constraint that $m(x)$ is concave.
6. Assume $p = 2$. Describe how to estimate $g(x)$ imposing the constraint that $m(u)$ is increasing on the region $u \in [x_L, x_U]$.