Econometrics 710
Midterm Exam
March 12, 2009

1. The data matrix is (Y, X) with $X = [X_1, X_2]$, and consider the transformed regressor matrix $Z = [X_1, X_2 - X_1]$. Suppose you do a LS regression of Y on X, and a LS regression of Y on Z. Let $\hat{\sigma}^2$ and $\tilde{\sigma}^2$ denote the residual variance estimates from the two regressions. Give a formula relating $\hat{\sigma}^2$ and $\tilde{\sigma}^2$? (Explain your reasoning.)

2. An equation is $Y = X_1\beta_1 + X_2\beta_2 + e$ where X_1 is $n \times 10$ and X_2 is $n \times 5$, and there are $n = 500$ observations. An economist estimates the equation by least-squares and tests the hypothesis $H_0 : \beta_2 = 0$ and obtains a Wald statistic $W_n = 0.34$.

   (a) What is the correct degrees of freedom for the χ^2 distribution to evaluate the significance of the Wald statistic?

   (b) Suppose the following are the quantiles of the appropriate χ^2 distribution

$P\left(\chi^2 \leq c\right)$	.01	.05	.10	.90	.95	.99
c	0.55	1.14	1.61	9.24	11.07	15.09

   Should you reject H_0 since W_n is less than the 0.01 quantile? Explain your reasoning.

3. Suppose for an economic model suggests

   $$g(x) = E\left(y_i \mid x_i = x\right) = \beta_0 + \beta_1 x + \beta_2 x^2$$

   where $x_i \in \mathbb{R}$. An economist has a random sample (y_i, x_i), $i = 1, ..., n$

   (a) Describe how to estimate $g(x)$ at a given value x.

   (b) Describe (be specific) an appropriate confidence interval for $g(x)$.

4. Take the model

   $$\begin{aligned} y_i &= x_i'\beta + e_i \\ E\left(x_i e_i\right) &= 0 \end{aligned}$$

   and suppose you have observations $i = 1, ..., 2n$. (The number of observations is $2n$.) You split the sample in half, (each has n observations), calculate $\hat{\beta}_1$ by LS on the first sample, and $\hat{\beta}_2$ by LS on the second sample. Assuming the observations are iid

   (a) What is the asymptotic distribution of $\sqrt{n}\left(\hat{\beta}_1 - \hat{\beta}_2\right)$?

   (b) Extra Credit: How could you use this to test the hypothesis of equal coefficients in the two samples?