Econometrics 710
Midterm Exam
March 6, 2003

1. The model is

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0. \end{aligned}$$

   The *ridge regression* estimator for β is

$$\hat{\beta} = \left(X'X + \lambda I_k\right)^{-1}\left(X'Y\right).$$

   Suppose that $\lambda = cn$ with c fixed as $n \to \infty$. Find the probability limit of $\hat{\beta}$ as $n \to \infty$.

2. The model is

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0 \\ \Omega &= E\left(x_i x_i' e_i^2\right). \end{aligned}$$

   (a) Find the method of moments estimators $(\hat{\beta}, \hat{\Omega})$ for (β, Ω).
   (b) In this model, are $(\hat{\beta}, \hat{\Omega})$ efficient estimators of (β, Ω)?
   (c) If so, in what sense are they efficient?

3. Suppose we have an estimate $\hat{\beta}$ of $\beta \in R$ such that $\sqrt{n}\left(\hat{\beta} - \beta\right) \to^d N(0, V)$ as $n \to \infty$, we have a consistent estimator $\hat{V}$ of V, and the parameter of interest is $\theta = \beta^2$.

   (a) Find the asymptotic distribution of $\hat{\theta} = \hat{\beta}^2$.
   (b) Use result (a) to form a confidence interval for θ.
   (c) What are the consequences if $\beta = 0$?

4. The model is

$$\begin{aligned} y_i &= x_i\beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

   where $x_i \in R$. Consider the two estimators

$$\begin{aligned} \hat{\beta} &= \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2} \\ \tilde{\beta} &= \frac{1}{n}\sum_{i=1}^n \frac{y_i}{x_i}. \end{aligned}$$

   (a) Under the stated assumptions, are both estimators consistent for β?
   (b) Are there conditions under which either estimator is efficient?