Econometrics 710
Midterm Exam
March 6, 2001

1. The model is

$$y_i = x_i'\beta + e_i, \qquad E(e_i \mid x_i) = 0, \qquad E\left(e_i^2 \mid x_i\right) = \sigma_i^2.$$

Assume the σ_i^2 are known. Let $D = diag\{\sigma_1^2, ..., \sigma_n^2\}$. Let $\hat{\beta}$ be the OLS estimator of β, and let $\tilde{\beta}$ be the (infeasible) GLS estimator of β.

(a) Show that $Cov\left(\hat{\beta} - \tilde{\beta}, \tilde{\beta} \mid X\right) = 0$.

(b) Deduce that $Cov\left(\hat{\beta}, \tilde{\beta} \mid X\right) = Var\left(\tilde{\beta} \mid X\right)$.

(c) Deduce that $Var\left(\hat{\beta} - \tilde{\beta} \mid X\right) = Var\left(\hat{\beta} \mid X\right) - Var\left(\tilde{\beta} \mid X\right)$.

(d) Write $V_n = Var\left(\hat{\beta} - \tilde{\beta} \mid X\right)$ as a function of X and D.

2. The model is

$$y_i = x_i'\beta + e_i \qquad E(e_i \mid x_i) = 0.$$

An econometrician is worried about the impact of some unusually large values of the regressors. The model is thus estimated on the subsample for which $|x_i| \leq c$, for some fixed c. Let $\tilde{\beta}$ denote the OLS estimator on this subsample. It equals

$$\tilde{\beta} = \left(\sum_{i=1}^{n} x_i x_i 1\left(|x_i| \leq c\right)\right)^{-1} \left(\sum_{i=1}^{n} x_i y_i 1\left(|x_i| \leq c\right)\right)$$

where $1(\cdot)$ denotes the indicator function.

(a) Show that $\tilde{\beta} \rightarrow_p \beta$.

(b) Find the asymptotic distribution of $\sqrt{n}\left(\tilde{\beta} - \beta\right)$.

(c) Bonus Question: Suppose instead the model is

$$y_i = x_i'\beta + e_i \qquad E(x_i e_i) = 0.$$

Does result (a) change?

3. Let $(y_1, ..., y_n)$ be a real-valued random sample from distribution F with mean $\mu = E(y_i)$ and variance $\sigma^2 = Var(y_i)$. Let $\hat{\mu} = \overline{y}$ be the sample mean and let $T_n = \hat{\mu} - \mu$. Let

$$\tau_n = E(T_n) = \int T_n dF$$

be the bias of $\hat{\mu}$ for μ. Let

$$\tau_n^* = \int T_n dF_n$$

be the bootstrap estimate of bias, where $F_n(x)$ is the empirical distribution function of the data $(y_1, ..., y_n)$. (Note: This is distinct from $\hat{\tau}_n^*$, the simulation estimate of τ_n^*).

(a) Show that $\tau_n = 0$ and $\tau_n^* = 0$

(b) Now consider $\theta = \mu^2$. Let $\hat{\theta} = \hat{\mu}^2$, set $T_n = \hat{\theta} - \theta = \hat{\mu}^2 - \mu^2$. Find $\tau_n = E(T_n)$, the bias of $\hat{\theta}$ for θ.

(c) Bonus Question: Find $\tau_n^* = \int T_n dF_n$, the bootstrap estimate of bias.