Econometrics 710
Midterm Exam
March 23, 2000

1. The model is
$$y_i = x_i\beta + e_i \qquad E(e_i \mid x_i) = 0$$
where x_i, β and e_i are scalar. We consider the estimator
$$\tilde{\beta} = \frac{\overline{y}}{\overline{x}} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}.$$
We assume that x_i and e_i have finite fourth moments and that $\{y_i, x_i\}$ are a random sample (iid).

   (a) Find $E\left(\tilde{\beta} \mid X\right)$.

   (b) Find $Var\left(\tilde{\beta} \mid X\right)$.

   (c) Show that $\tilde{\beta} \to_p \beta$ as $n \to \infty$. Does this require any additional assumptions?

   (d) Find the asymptotic distribution of $\sqrt{n}\left(\tilde{\beta} - \beta\right)$ as $n \to \infty$.

   (e) Without imposing any additional assumptions, is $\tilde{\beta}$ necessarily less efficient than OLS? (By efficiency, I mean lower asymptotic variance.)

2. Take the linear regression $Y = X\beta + e$ with $E(e_i \mid x_i) = 0$. Let $\theta = 1/\beta_1$ where β_1 is the first element of β. Let $\hat{\beta}$ be the OLS estimator of β and $\hat{V}$ be the estimator of $Var\left(\hat{\beta}\right)$. Find an asymptotically valid 95% confidence interval for θ. (Give the explicit formula as a function of $\hat{\beta}$ and $\hat{V}$.)

3. In the linear regression $Y = X\beta + e$ with $E(e_i \mid x_i) = 0$, it is known that the true β satisfies the restriction
$$R\beta = 0$$
where R is a $q \times k$ matrix with $q < k$. Consider the estimator
$$\tilde{\beta} = \hat{\beta} - (X'X)^{-1} R' \left[R (X'X)^{-1} R'\right]^{-1} R\hat{\beta}.$$

   (a) Show that $R\tilde{\beta} = 0$.

   (b) Find $E\left(\tilde{\beta} \mid X\right)$.

   (c) Find $Var\left(\tilde{\beta} \mid X\right)$. [Hint: First write $\tilde{\beta}$ as a linear function of $\hat{\beta}$.]

(d) Give an expression for a valid standard error for the elements of $\tilde{\beta}$. You do not need to give a proof of validity.

4. Take the linear regression $Y = X\beta + e$ with $E(e_i \mid x_i) = 0$. For one particular value of x, the object of interest is the conditional mean

$$E(y_i \mid x_i = x) = g(x).$$

Describe how you would use the percentile-t bootstrap to construct a confidence interval for $g(x)$.