Econometrics 710
Final Exam, Spring 2016

Be sure and write complete answers. Be specific about estimators and covariance matrix estimators.

1. Consider the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(e_i|x_i) &= 0 \end{aligned}$$

with y_i scalar and x_i a k vector. You have a random sample $(y_i, x_i : i = 1, ..., n)$. You are interested in estimating the regression function $m(x) = E(y_i|x_i = x)$ at a fixed vector x and constructing a 95% confidence interval.

   (a) Write the standard estimator of $m(x)$.

   (b) Write out the standard asymptotic confidence interval for $m(x)$.

   (c) Describe the percentile bootstrap confidence interval for $m(x)$.

   (d) Describe the percentile-t bootstrap confidence interval for $m(x)$.

2. Consider the model

$$\begin{aligned} y_{it} &= x_{it}'\beta + u_i + e_{it} \\ E(z_{it}e_{it}) &= 0 \end{aligned}$$

for $i = 1, ..., n$ and $t = 1, ..., T$. The individual effect u_i is treated as fixed. Assume x_{it} and z_{it} are $k \times 1$ vectors.

Write out an appropriate estimator for β. You do not need to examine its distributional (e.g. asymptotic) properties.

3. Take the model

$$\begin{aligned} y_i &= \pi_i\beta + e_i \\ \pi_i &= E(x_i|z_i) = \gamma' z_i \\ E(e_i|z_i) &= 0 \end{aligned}$$

where y_i, x_i and π_i are scalars, and z_i is a k vector. β is scalar and γ is $k \times 1$. The sample is $(y_i, x_i, z_i : i = 1, ..., n)$ with π_i unobserved.

Consider the estimator $\widehat{\beta}$ for β by OLS of y_i on $\widehat{\pi}_i = z_i'\widehat{\gamma}$ where $\widehat{\gamma}$ is the OLS coefficient from the regression of x_i on z_i

   (a) Show that $\widehat{\beta}$ is consistent for β

   (b) Find the asymptotic distribution $\sqrt{n}\left(\widehat{\beta} - \beta\right)$ as $n \to \infty$ assuming that $\beta = 0$.

   (c) Why is the assumption $\beta = 0$ an important simplifying condition in part (b)?

   (d) Using the result in (c), construct an appropriate asymptotic test for the hypothesis $H_0 : \beta = 0$

4. You are at a seminar where a colleague presents a simulation study of a test of a hypothesis H_0 with nominal size 5%. Based on $B = 100$ simulation replications under H_0 the estimated size is 7%. Your colleague says: "Unfortunately the test over-rejects."

   (a) Do you agree or disagree with your colleague? Explain. Hint: Use an asymptotic (large B) approximation.

   (b) Suppose the number of simulation replications were $B = 1000$ yet the estimated size is still 7%. Does your answer change?

5. Consider the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(z_i e_i) &= 0 \qquad (1) \\ R'\beta &= 0 \qquad (2) \end{aligned}$$

   with y_i scalar, x_i a k vector and z_i an ℓ vector with $\ell > k$. The matrix R is $k \times q$ with $1 \leq q < k$. You have a random sample $(y_i, x_i, z_i : i = 1, ..., n)$.

   For simplicity, assume the "efficient" weight matrix $W = \left(E\left(z_i z_i' e_i^2\right)\right)^{-1}$ is known.

   (a) Write out the GMM estimator $\widehat{\beta}$ of β given the moment conditions (1) but ignoring constraint (2).

   (b) Write out the GMM estimator $\widetilde{\beta}$ of β given the moment conditions (1) and constraint (2).

   (c) Find the asymptotic distribution of $\sqrt{n}\left(\widetilde{\beta} - \beta\right)$ as $n \to \infty$ under the assumption that (1) and (2) are correct.