Econometrics 710
Final Exam, Spring 2015

1. Consider the model

$$\begin{aligned} y_i &= \alpha + \beta x_i + e_i \\ E(e_i) &= 0 \\ E(x_i e_i) &= 0 \end{aligned}$$

with both y_i and x_i scalar. Assuming $\alpha > 0$ and $\beta < 0$, suppose the parameter of interest is the area under the regression curve (e.g. consumer surplus), which is $A = -\alpha^2/2\beta$.

Let $\hat{\theta} = (\hat{\alpha}, \hat{\beta})'$ be the least-squares estimates of $\theta = (\alpha, \beta)'$ so that $\sqrt{n}\left(\hat{\theta} - \theta\right) \to_d N(0, V_\theta)$ and let $\hat{V}_\theta$ be a standard consistent estimate for V_θ. You do not need to write out these estimators.

(a) Given the above, describe an estimator of A

(b) Construct an asymptotic $(1-\eta)$ confidence interval for A

(c) Describe how to construct a bootstrap $(1-\eta)$ percentile interval for A

2. Consider the structural equation

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + e_i \tag{1}$$

with x_i treated as endogenous so that $E(x_i e_i) \neq 0$. Assume y_i and x_i are scalar. Suppose we also have a scalar instructment z_i which satisfies

$$E(e_i | z_i) = 0$$

so in particular $E(e_i) = 0$, $E(z_i e_i) = 0$ and $E\left(z_i^2 e_i\right) = 0$.

(a) Should x_i^2 be treated as endogenous or exogenous?

(b) Suppose we have a scalar instrument z_i which satisfies

$$x_i = \gamma_0 + \gamma_1 z_i + u_i \tag{2}$$

with u_i independent of z_i and mean zero.

Consider using $(1, z_i, z_i^2)$ as instruments. Is this a sufficient number of instruments? (Would this be just-identified, over-identified, or under-identified)?

(c) Write out the reduced form equation for x_i^2. Under what condition on the reduced form parameters (2) are the parameters in (1) identified?

3. Consider the structural equation and reduced form

$$\begin{aligned} y_i &= \beta x_i^2 + e_i \\ x_i &= \gamma z_i + u_i \end{aligned}$$

with x_i^2 treated as endogenous so that $E\left(x_i^2 e_i\right) \neq 0$. For simplicity we assume no intercepts. Assume y_i, z_i, and x_i are scalar, and assume $\gamma \neq 0$. Consider the following estimator. First, estimate γ by OLS of x_i on z_i and construct the fitted values $\hat{x}_i = \hat{\gamma} z_i$. Second, estimate β by OLS of y_i on $\hat{x}_i^2$. [Added after the exam: Assume that $E\left(z_i e_i\right) = 0$ and $E\left(z_i u_i\right) = 0$ and consider adding extra conditions if helpful to answer the questions.]

(a) Write out this estimator $\hat{\beta}$ explicitly as a function of the sample

(b) Find its probability limit as $n \to \infty$

(c) In general, is $\hat{\beta}$ consistent for β? Is there a reasonable condition under which $\hat{\beta}$ is consistent?

4. Consider the structural equation

$$\begin{aligned} y_i &= x_{1i}'\beta_1 + x_{2i}'\beta_2 + e_i \\ E\left(z_i e_i\right) &= 0 \end{aligned}$$

where x_{2i} is $k_2 \times 1$ and treated as endogenous. The variables $z_i = \left(x_{1i}, z_{2i}\right)$ are treated as exogenous, where z_{2i} is $\ell_2 \times 1$ and $\ell_2 \geq k_2$. You are interested in testing the hypothesis

$$H_0 : \beta_2 = 0.$$

Consider the reduced form equation for y_i

$$y_i = x_{1i}'\lambda_1 + z_{2i}'\lambda_2 + v_i \tag{3}$$

Show how to test H_0 using only the OLS estimates of (3).

Hint: This will require an analysis of the reduced form equations and their relation to the structural equation.