Econometrics 710
Final Exam, Spring 2014

1. You have n iid observations (y_i, x_{1i}, x_{2i}), and consider two alternative regression models

$$\begin{aligned} y_i &= x_{1i}'\beta_1 + e_{1i} \\ E(x_{1i}e_{1i}) &= 0 \end{aligned} \tag{1}$$

$$\begin{aligned} y_i &= x_{2i}'\beta_2 + e_{2i} \\ E(x_{2i}e_{2i}) &= 0 \end{aligned} \tag{2}$$

where x_{1i} and x_{2i} have at least some different regressors. (For example, (1) is a wage regression on geographic variables and (2) is a wage regression on personal appearance measurements.) You want to know if model (1) or model (2) fits the data better. Define $\sigma_1^2 = E\left(e_{1i}^2\right)$ and $\sigma_2^2 = E\left(e_{2i}^2\right)$. You decide that the model with the smaller variance fit (e.g., model (1) fits better if $\sigma_1^2 < \sigma_2^2$.) You decide to test for this by testing the hypothesis of equal fit $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative of unequal fit $H_1 : \sigma_1^2 \neq \sigma_2^2$. For simplicity, suppose that e_{1i} and e_{2i} are observed.

   (a) Construct an estimate $\widehat{\theta}$ of $\theta = \sigma_1^2 - \sigma_2^2$.
   (b) Find the asymptotic distribution of $\sqrt{n}\left(\widehat{\theta} - \theta\right)$ as $n \to \infty$.
   (c) Find an estimator of the asymptotic variance of $\widehat{\theta}$.
   (d) Propose a test of asymptotic size α of H_0 against H_1.
   (e) Suppose the test accepts H_0. Briefly, what is your interpretation?

2. Take the linear instrumental variables equation

$$\begin{aligned} y_i &= x_{1i}'\beta_1 + x_{2i}'\beta_2 + e_i \\ E(z_i e_i) &= 0 \end{aligned}$$

where x_{1i} is $k_1 \times 1$, x_{2i} is $k_2 \times 1$, and z_i is $\ell \times 1$, with $\ell \geq k = k_1 + k_2$. The sample size is n. Assume that $Q_{zz} = E z_i z_i' > 0$ and $Q_{zx} = E z_i x_i'$ has full rank k.

Suppose that only (y_i, x_{1i}, z_i) are available, and x_{2i} is missing from the dataset.

Consider the 2SLS estimator $\widehat{\beta}_1$ of β_1 obtained from the misspecified IV regression, by regressing y_i on x_{1i} only, using z_i as an instrument for x_{1i}.

   (a) Find a stochastic decomposition $\widehat{\beta}_1 = \beta_1 + b_{1n} + r_{1n}$ where r_{1n} depends on the error e_i, and b_{1n} does not depend on the error e_i.
   (b) Show that $r_{1n} \to_p 0$ as $n \to \infty$.
   (c) Find the probability limit of b_{1n} and $\widehat{\beta}_1$ as $n \to \infty$.
   (d) Does $\widehat{\beta}_1$ suffer from "omitted variables bias"? Explain. Under what conditions is there no omitted variables bias?
   (e) Find the asymptotic distribution as $n \to \infty$ of

$$\sqrt{n}\left(\widehat{\beta}_1 - \beta_1 - b_{1n}\right).$$