Econometrics 710
Final Exam, Spring 2013

1. Take the linear instrumental variables equation

$$\begin{aligned} y_i &= x_i\beta_1 + z_i\beta_2 + e_i \\ E\left(e_i|z_i\right) &= 0 \end{aligned}$$

where for simplicity both x_i and z_i are scalar 1×1.

(a) Can the coefficients (β_1, β_2) be estimated by 2SLS using z_i as an instrument for x_i? Why or why not?

(b) Can the coefficients (β_1, β_2) be estimated by 2SLS using z_i and z_i^2 as instruments?

(c) For the 2SLS estimator suggested in (b), what is the implicit exclusion restriction?

(d) In (b), what is the implicit assumption about instrument relevance?
[Hint: Write down the implied reduced form equation for x_i.]

(e) In a generic application, would you be comfortable with the assumptions in (c) and (d)?

2. Take the linear homoskedastic CEF

$$\begin{aligned} y_i^* &= x_i'\beta + e_i \qquad (1) \\ E(e_i|x_i) &= 0 \\ E(e_i^2|x_i) &= \sigma^2 \end{aligned}$$

and suppose that y_i^* is measured with error. Instead of y_i^*, we observe y_i which satisfies

$$y_i = y_i^* + u_i$$

where u_i is measurement error. Suppose that e_i and u_i are independent and

$$\begin{aligned} E(u_i|x_i) &= 0 \\ E(u_i^2|x_i) &= \sigma_u^2(x_i) \end{aligned}$$

(a) Derive an equation for y_i as a function of x_i. Be explicit to write the error term as a function of the structural errors e_i and u_i. What is the effect of this measurement error on the model (1)?

(b) Describe the effect of this measurement error on OLS estimation of β in the feasible regression of the observed Y on X.

(c) Describe the effect (if any) of this measurement error on appropriate standard error calculation for $\hat{\beta}$.

3. Take a linear equation with endogeneity and a just-identified linear reduced form

$$
\begin{aligned}
y_i &= x_i\beta + e_i && (2)\\
x_i &= \gamma z_i + u_i && (3)
\end{aligned}
$$

where both x_i and z_i are scalar 1×1. Assume that

$$
\begin{aligned}
E(z_i e_i) &= 0\\
E(z_i u_i) &= 0
\end{aligned}
$$

(a) Write down the standard 2SLS estimator $\hat{\beta}_{2SLS}$ for β using z_i as an instrument for x_i.

(b) Find the asymptotic distribution for $\hat{\beta}_{2SLS}$.
Write the asymptotic variance as a function of $\Omega = E(z_i^2 e_i^2)$, $Q = E(z_i^2)$, and γ

4. In the context of model (2)-(3) from question 3:

(a) Derive the reduced form equation

$$y_i = z_i\lambda + v_i. \tag{4}$$

Show that $\beta = \lambda/\gamma$ if $\gamma \neq 0$, and that

$$E(z_i v_i) = 0$$

(b) Let $\hat{\lambda}$ denote the OLS estimate from linear regression of Y on Z, and let $\hat{\gamma}$ denote the OLS estimate from linear regression of X on Z. Write $\theta = (\lambda, \gamma)'$ and let $\hat{\theta} = (\hat{\lambda}, \hat{\gamma})'$. Define the error vector $\xi_i = \begin{pmatrix} v_i \\ u_i \end{pmatrix}$. Write $\sqrt{n}\left(\hat{\theta} - \theta\right)$ using a single expression as a function of the error ξ_i.

(c) Show that $E(z_i \xi_i) = 0$

(d) Derive the joint asymptotic distribution of $\sqrt{n}\left(\hat{\theta} - \theta\right)$ as $n \to \infty$. Hint: Define $\Omega_\xi = E\left(z_i^2 \xi_i \xi_i'\right)$

(e) Using the previous result and the Delta Method, find the asymptotic distribution of the Indirect Least Squares estimator $\hat{\beta} = \hat{\lambda}/\hat{\gamma}$

(f) Bonus: Is the answer in (c) the same as the asymptotic distribution of the 2SLS estimator from question 3? [Hint: Show that $\begin{pmatrix} 1 & -\beta \end{pmatrix} \xi_i = e_i$ and $\begin{pmatrix} 1 & -\beta \end{pmatrix} \Omega_\xi \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \Omega$]