Econometrics 710
Final Exam, Spring 2012

1. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ \mathbb{E}(z_i e_i) &= 0 \end{aligned}$$

and consider the two-stage least-squares estimator. The first-stage estimate is

$$\begin{aligned} \widehat{X} &= Z\widehat{\Gamma} \\ \widehat{\Gamma} &= \left(Z'Z\right)^{-1} Z'X \end{aligned}$$

and the second-stage is LS of y_i on $\widehat{x}_i$:

$$\widehat{\beta} = \left(\widehat{X}'\widehat{X}\right)^{-1} \widehat{X}'Y$$

with LS residuals

$$\widehat{e} = Y - \widehat{X}\widehat{\beta}.$$

Consider $\widehat{\sigma}^2 = \dfrac{1}{n}\widehat{e}'\widehat{e}$ as an estimator for $\sigma^2 = \mathbb{E}e_i^2$. Is this appropriate? If not, propose an alternative estimator.

2. You have two independent iid samples $(y_{1i}, x_{1i}, z_{1i} : i = 1, ..., n)$ and $(y_{2i}, x_{2i}, z_{2i} : i = 1, ..., n)$. The dependent variables y_{1i} and y_{2i} are real-valued. The regressors x_{1i} and x_{2i} and instruments z_{1i} and z_{2i} are k-vectors. The model is standard just-identified linear instrumental variables

$$\begin{aligned} y_{1i} &= x_{1i}'\beta_1 + e_{1i} \\ E(z_{1i}e_{1i}) &= 0 \\ y_{2i} &= x_{2i}'\beta_2 + e_{2i} \\ E(z_{2i}e_{2i}) &= 0 \end{aligned}$$

For concreteness, sample 1 are women and sample 2 are men. You want to test $H_0 : \beta_1 = \beta_2$, that the two samples have the same coefficients.

   (a) Develop a test statistic for H_0
   (b) Derive the asymptotic distribution of the test
   (c) Describe (in brief) the testing procedure

3. Take the model

$$\begin{aligned} y_i &= x_{1i}\beta_1 + x_{2i}\beta_2 + e_i \\ \mathbb{E}(x_i e_i) &= 0 \end{aligned}$$

with both $\beta_1 \in \mathbb{R}$ and $\beta_2 \in \mathbb{R}$, and define the parameter

$$\theta = \beta_1\beta_2$$

(a) What is the appropriate estimator $\widehat{\theta}$ for θ?

(b) Find the asymptotic distribution of $\widehat{\theta}$ under standard regularity conditions.

(c) Show how to calculate an asymptotic 95% confidence interval for θ

(d) Describe how to use the percentile bootstrap to calculate a 95% confidence interval for θ

4. You have a friend who wants to estimate β in the model

$$\begin{aligned} y_i &= x_i\beta + e_i \\ \mathbb{E}(e_i \mid z_i) &= 0 \end{aligned}$$

with both $x_i \in \mathbb{R}$ and $z_i \in \mathbb{R}$, and z_i is continuously distributed. Your friend wants to treat the reduced form equation for x_i as nonparametric

$$\begin{aligned} x_i &= g(z_i) + u_i \\ \mathbb{E}(u_i \mid z_i) &= 0 \end{aligned}$$

Your friend asks you for advice and help to construct an estimator $\widehat{\beta}$ of β. Describe an appropriate estimator. You do not have to develop the distribution theory, but try to be sufficiently complete with your advice so your friend can compute $\widehat{\beta}$.