Econometrics 710

Final Exam, Spring 2009

1. The observed data is $\{y_i, x_i\} \in \mathbb{R} \times \mathbb{R}^k$, $k > 1$, $i = 1, ..., n$. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0 \\ \mu_3 &= E(e_i^3) \end{aligned}$$

   (a) Write down an estimator for μ_3
   (b) Explain how to use the Efron percentile method to construct a 90% confidence interval for μ_3 in this specific model.

2. An economist reports a set of parameter estimates, including the coefficient estimates $\hat{\beta}_1 = 1.0$, $\hat{\beta}_2 = 0.8$, and standard errors $s(\hat{\beta}_1) = 0.07$ and $s(\hat{\beta}_2) = 0.07$. The author writes "The estimates show that β_1 is larger than β_2."

   (a) Write down the formula for an asymptotic 95% confidence interval for $\theta = \beta_1 - \beta_2$, expressed as a function of $\hat{\beta}_1$, $\hat{\beta}_2$, $s(\hat{\beta}_1)$, $s(\hat{\beta}_2)$ and $\hat{\rho}$, where $\hat{\rho}$ is the estimated correlation between $\hat{\beta}_1$ and $\hat{\beta}_2$.
   (b) Can $\hat{\rho}$ be calculated from the reported information?
   (c) Is the author correct? Does the reported information support the author's claim?

3. The observed data is $\{y_i, x_i, z_i\} \in \mathbb{R} \times \mathbb{R}^k \times \mathbb{R}^\ell$, $k > 1$ and $\ell > k > 1$, $i = 1, ..., n$. The model is

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(z_i e_i) &= 0 \end{aligned} \qquad (1)$$

   (a) Given a weight matrix $W > 0$, write down the GMM estimator for $\hat{\beta}$.
   (b) Suppose model (1) is misspecified in that

$$\begin{aligned} e_i &= \delta n^{-1/2} + u_i \\ E(u_i \mid z_i) &= 0 \end{aligned} \qquad (2)$$

   with $\mu_z = Ez_i \neq 0$ and $\delta \neq 0$.
   Show that (2) implies (1) is false.
   (c) Express$\sqrt{n}\left(\hat{\beta} - \beta\right)$ as a function of W, n, δ, and the variables $(x_i,\ z_i,\ u_i)$.
   (d) Find the asymptotic distribution of $\sqrt{n}\left(\hat{\beta} - \beta\right)$ under Assumption (2).

4. The observed data is $\{y_i, x_i, z_i\} \in \mathbb{R} \times \mathbb{R}^k \times \mathbb{R}^\ell$, $k > 1$ and $\ell > 1$, $i = 1, ..., n$. An econometrician first estimates

$$y_i = x_i'\hat{\beta} + \hat{e}_i \qquad (3)$$

   by least squares. The econometrician next regresses the residual $\hat{e}_i$ on z_i, which can be written as

$$\hat{e}_i = z_i'\tilde{\gamma} + \tilde{u}_i \qquad (4)$$

   (a) Define the population γ being estimated in the (4).
   (b) Find the probability limit for $\tilde{\gamma}$.
   (c) Suppose the econometrician constructs a Wald statistic W_n for $H_0 : \gamma = 0$ from the regression (4), ignoring regression (3). Write down the formula for W_n.
   (d) Assuming $E(z_i x_i') = 0$, find the asymptotic distribution for W_n under $H_0 : \gamma = 0$.
   (e) If $E(z_i x_i') \neq 0$ will your answer to (d) change?