Econometrics 710
Final Exam, Spring 2007

1. The observations are iid, $(y_{1i}, y_{2i}, x_i : i = 1, ..., n)$. The dependent variables y_{1i} and y_{2i} are real-valued. The regressor x_i is a k-vector. The model is the two-equation system

$$\begin{aligned} y_{1i} &= x_i'\beta_1 + e_{1i} \\ E(x_i e_{1i}) &= 0 \\ y_{2i} &= x_i'\beta_2 + e_{2i} \\ E(x_i e_{2i}) &= 0 \end{aligned}$$

   (a) What are the method-of-moments estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ for β_1 and β_2?
   (b) Find the joint asymptotic distribution of $\hat{\beta}_1$ and $\hat{\beta}_2$.
   (c) Describe a test for $H_0 : \beta_1 = \beta_2$.

2. The model is

$$\begin{aligned} y_i &= z_i\beta + x_i\gamma + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

   Thus z_i is potentially endogenous and x_i is exogenous. Assume that $z_i \in \mathbb{R}$ and $x_i \in \mathbb{R}$. Someone suggests estimating (β, γ) by GMM, using the pair (x_i, x_i^2) as the instruments. Is this feasible? Under what conditions, if any, (in additional to those described above) is this a valid estimator?

3. The observations are iid, $(y_i, x_i, q_i : i = 1, ..., n)$, where x_i is $k \times 1$ and q_i is $m \times 1$. The model is

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0 \\ E(q_i e_i) &= 0 \end{aligned}$$

   Find the efficient GMM estimator for β.

4. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0 \\ E\left(e_i^2\right) &= \sigma^2 \end{aligned}$$

   Describe the bootstrap percentile confidence interval for σ^2.

5. The parameter of β is defined in the model

$$y_i = x_i^*\beta + e_i$$

   where e_i is independent of x_i^*, $Ee_i = 0$, $Ee_i^2 = \sigma^2$. The observables are (y_i, x_i) where

$$x_i = x_i^* v_i$$

   and $v_i > 0$ is random measurement error. Assume that v_i is independent of x_i^* and e_i. Also assume that x_i and x_i^* are non-negative and real-valued. Consider the least-squares estimator $\hat{\beta}$ for β.

   (a) Find the plim of $\hat{\beta}$, expressed in terms of β and moments of (x_i, v_i, e_i)
   (b) Can you find a non-trivial condition under which $\hat{\beta}$ is consisent for β? (By non-trivial, I mean something other than $v_i = 1$.)