Econometrics 710
Final Exam
Spring 2003

1. Take the model

$$
\begin{aligned}
y_i &= x_i'\beta + e_i \\
E(x_i e_i) &= 0 \\
\beta &= Q\theta
\end{aligned}
$$

where β is $k \times 1$, Q is $k \times m$ with $m < k$, and Q is known. Assume that the observations (y_i, x_i) are iid across $i = 1, ..., n$.

Under these assumptions, what is the efficient estimator of θ?

2. Take the model

$$
\begin{aligned}
y_i &= x_{1i}\beta_1 + x_{2i}\beta_2 + e_i \\
E(x_i e_i) &= 0 \\
\theta &= \frac{\beta_1}{\beta_2}
\end{aligned}
$$

Assume that the observations (y_i, x_i) are iid across $i = 1, ..., n$. Describe how you would construct the percentile-t bootstrap confidence interval for θ.

3. For a sample $\{y_1, ..., y_n\}$ let $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} 1(y_i \leq x)$ be the empirical distribution function. Let y^* be a random variable with distribution function F_n.

Calculate $Var(y^*)$.

4. Take the simple panel data model

$$
\begin{aligned}
y_{it} &= \mu_i + e_{it} \\
E\left(e_{it}^2\right) &= \sigma^2
\end{aligned}
$$

where the e_{it} are iid with $E(\mu_i e_{it}) = 0$, $i = 1, ..., n$, $t = 1, ..., T$.

(a) Show that the GMM estimator for μ_i is $\overline{y}_i$, the fixed-effects estimator.
(b) Find the GMM estimator $\hat{\sigma}^2$ for σ^2.
(c) Find the probability limit of this estimator as $n \to \infty$ but T remains constant.
(d) Is $\hat{\sigma}^2$ consistent for σ^2?

5. Take the stationary AR(1) model

$$\begin{aligned} y_t &= \rho y_{t-1} + e_t \\ |\rho| &< 1 \end{aligned}$$

where for simplicity we assume that $E(y_t) = 0$ so there is no intercept. When the error e_t is a MDS, so that

$$\begin{aligned} E\left(e_t \mid I_{t-1}\right) &= 0 \\ I_{t-1} &= \left(y_{t-1}, y_{t-2}, ...\right) \end{aligned}$$

we know that the OLS estimator satisfies

$$\sqrt{T}\left(\hat{\rho} - \rho\right) \rightarrow N\left(0, V\right) \quad (1)$$

$$V = \frac{E\left(y_{t-1}^2 e_t^2\right)}{\left(E\left(y_{t-1}^2\right)\right)^2}. \quad (2)$$

Explain whether or not the MDS assumption is important for (1)-(2).

In particular, (1)-(2) still hold under the less restrictive assumption

$$E\left(y_{t-1} e_t\right) = 0$$

Explain why or why not.

6. Take the model

$$\begin{aligned} y_i &= z_i \beta + e_i \\ E\left(z_i e_i\right) &\neq 0 \end{aligned}$$

where the observations (y_i, z_i) are iid across $i = 1, ..., n$, z_i is scalar (1×1) and $E\left(e_i\right) = 0$.

(a) Do we say that z_i is "exogenous" or "endogenous" for β?

(b) Is the OLS estimator

$$\hat{\beta} = \frac{\sum_{i=1}^{n} z_i y_i}{\sum_{i=1}^{n} z_i^2}$$

consistent for β?

(c) Consider an alternative estimator

$$\tilde{\beta} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} z_i}$$

Is there a condition (other than $E\left(z_i e_i\right) = 0$) under which $\tilde{\beta}$ is consistent for β?

(d) Explain your finding in (c) by showing that you can write $\tilde{\beta}$ as a valid IV estimator. Explain the identifying restriction.