Econometrics 710
Final Exam
Spring 2002
Friday, May 17

1. (20 points) Take the linear model with iid observations (y_i, x_i), $i = 1, ..., n$

$$
\begin{aligned}
y_i &= x_i'\beta + e_i \\
E(x_i e_i) &= 0.
\end{aligned}
$$

The variables y_i and e_i are scalars, x_i is $k \times 1$.

   (a) Given the information, is e_i homoskedastic or heteroskedastic?
   (b) Write out the efficient GMM estimator of β in this model.
   (c) Briefly, in what sense is this estimator efficient?
   (d) Is β just-identified or over-identified?

2. (40 points) Take the linear conditionally homoskedastic simultaneous equations model with iid observations (y_i, x_i, z_i), $i = 1, ..., n$,

$$
\begin{aligned}
y_i &= z_i'\beta + e_i \\
E(e_i \mid x_i) &= 0. \\
E\left(e_i^2 \mid x_i\right) &= \sigma^2
\end{aligned}
$$

z_i is $k \times 1$ and x_i is $l \times 1$ with $l > k$. The k-class estimator of β is

$$
\begin{aligned}
\hat{\beta} &= \left(Z'\left((1-\lambda) I_k + \lambda P_X\right) Z\right)^{-1} \left(Z'\left((1-\lambda) I_k + \lambda P_X\right) Y\right) \\
P_X &= X\left(X'X\right)^{-1} X'
\end{aligned}
$$

using the standard matrix notation, where λ is a non-negative scalar.

   (a) Show that $\hat{\beta}$ equals the OLS estimator when $\lambda = 0$.
   (b) Show that $\hat{\beta}$ equals the 2SLS estimator when $\lambda = 1$.
   (c) Define

$$
\begin{aligned}
Q &= E\left(x_i z_i'\right) \\
M &= E\left(x_i x_i'\right) \\
S &= E\left(z_i z_i'\right) \\
\mu &= E(z_i e_i)
\end{aligned}
$$

   Let

$$
\beta^* = \operatorname*{plim}_{n \to \infty} \hat{\beta}
$$

   Assuming $\mu \neq 0$, find β^*. (Note: It may depend on λ.)

   (d) Find the asymptotic distribution of $\sqrt{n}\left(\hat{\beta} - \beta^*\right)$. [Note: this may take some work.]

3. (40 points) Take the simple AR(1) model

$$y_t = \rho y_{t-1} + e_t$$

with e_t iid, $Ee_t = 0$, $Ee_t^2 = \sigma^2$, and $|\rho| < 1$.

The *long-run variance* of y_t for the AR(1) is

$$\omega^2 = \frac{\sigma^2}{(1-\rho)^2}. \tag{1}$$

The definition can be motivated using two alternative expressions. First,

$$\omega^2 = Ey_t^2 + 2\sum_{k=1}^{\infty} E\left(y_t y_{t-k}\right). \tag{2}$$

Second,

$$\omega^2 = \lim_{T\to\infty} E\left(\sum_{t=1}^{T} y_t\right)^2. \tag{3}$$

(a) Show that equations (1) and (2) are equivalent.
(Show that the solution to the right-hand-side of (2) is the expression in (1).)

(b) Show that equations (2) and (3) are equivalent, and thus (1)-(2)-(3) are equivalent.
(Show that the right-hand-side of (3) can be written like the right-hand-side of (2).)

(c) Describe joint (GMM?) estimation of the parameters (ρ, σ^2) and ω^2.
Be explicit.

(d) Suppose you want to test the hypothesis

$$H_0 : \omega^2 = \omega_0^2$$

(a specific number). Describe an appropriate test of H_0. Write out the test statistic and describe the test procedure explicitly.