Econometrics 710
Final Exam
Spring 2001
May 17, 2001

1. (20 points) Take the model

$$\begin{aligned} y_i &= \theta + e_i \\ E(x_i e_i) &= 0 \end{aligned}$$

with (y_i, x_i) a random sample. y_i is real-valued and x_i is $k \times 1$, $k > 1$.

   (a) Find the efficient GMM estimator of θ.
   (b) Is this model over-identified or just-identified?
   (c) Find the GMM test statistic for over-identification.

2. (20 points) Recall the simple RESET specification test for nonlinearity in a random sample:

   The null hypothesis is a linear regression

$$\begin{aligned} y_i &= x_i \beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

   The parameter β is estimated by OLS yielding predicted values $\hat{y}_i$. Then using a second-stage LS regression

$$y_i = x_i' \tilde{\beta} + (\hat{y}_i)^2 \tilde{\gamma} + \tilde{e}_i$$

   The RESET test statistic R is the squared t-ratio on $\tilde{\gamma}$.

   A colleague suggests obtaining the critical value for the test using the bootstrap. He proposes the following bootstrap implementation

   (a) Draw n observations (y_i^*, x_i^*) randomly from the observed sample pairs (y_i, x_i) to create a bootstrap sample.
   (b) Compute the statistic R^* on this bootstrap sample as described above.
   (c) Repeat this 999 times. Sort the bootstrap statistics R^*, take number 950 (the 95% percentile) and use this as the critical value.
   (d) Reject the null hypothesis if R exceeds this critical value, otherwise do not reject.

   Is this procedure a correct implementation of the bootstrap in this context? If not, propose a modified bootstrap.

3. (20 points) Let y_t be a strictly stationary and ergodic time series which follows the following nonlinear autoregressive model

$$\begin{aligned} y_t &= \alpha y_{t-1}^2 + e_t \\ E(e_t \mid I_{t-1}) &= 0 \end{aligned}$$

Let α be estimated by the OLS regression of y_t on y_{t-1}^2.

(a) Show that the OLS estimator $\hat{\alpha}$ is consistent.

(b) Find its asymptotic distribution as $T \to \infty$

4. (30 points) Let π_t be inflation between time t and time $t+1$. Let

$$z_t = E(\pi_t \mid I_{t-1})$$

be the expected inflation rate. A structural model implies the following relationship between economic activity in period t, y_t, and inflationary expectations z_t

$$\begin{aligned} y_t &= z_t\beta + e_t \\ E(e_t \mid I_{t-1}) &= 0 \end{aligned}$$

Inflation expectations z_t is a function of observables

$$z_t = x_t'\theta$$

where x_t is elements of I_{t-1}.

We propose estimation of the structural parameter β as follows.

First, regress π_t on x_t to get OLS coefficient $\hat{\theta}$ and estimated inflation expectations $\hat{z}_t = x_t'\hat{\theta}$.

Second, regress y_t on $\hat{z}_t$ to get OLS coefficient $\hat{\beta}$.

Questions: You may assume that (y_t, π_t, x_t) are strictly stationary and ergodic.

(a) Is $\hat{\theta}$ consistent for θ?

(b) Find the asymptotic distribution for $\hat{\theta}$.

(c) Is $\hat{\beta}$ consistent for β?

(d) Find the asymptotic distribution for $\hat{\beta}$. [Note: This is tricky – be careful.]

5. (10 points) In the model of question 4, show that the estimator $(\hat{\theta}, \hat{\beta})$ can be written as an exactly-identified GMM estimator of the parameter (θ, β). Hint: from the first-order conditions for $\hat{\theta}$ and $\hat{\beta}$, find the moment conditions which these estimators are solving.