Econometrics 710
Final Exam
Spring 2000
May 15, 2000

You have 150 minutes for the exam. Complete answers get full credit. Incomplete answers get partial credit.

Be careful to write full and complete answers where possible. At the same time, you should exercise judgement and only write down the important (and of course correct) points.

1. The model is

$$\begin{aligned} y_i &= \beta_1 x_{1i} + \beta_2 x_{2i} + e_i \\ E(x_i e_i) &= 0. \end{aligned}$$

where $x_i = (x_{1i}, x_{2i})$. The parameter of interest is

$$\theta = \frac{\beta_1}{\beta_2}.$$

Show how to construct an asymptotic confidence interval for θ.

2. Let

$$Y = X_1\beta_1 + X_2\beta_2 + e$$

where X_1 is $n \times k_1$ and X_2 is $n \times k_2$. Let $(\hat{\beta}_1, \hat{\beta}_2)$ denote the 2SLS estimates of (β_1, β_2) when (X_1, Z) are used as instruments and Z is a $n \times k_2$ matrix of instruments. (Note: the estimator is just-identified.) Let $(\tilde{\alpha}_1, \tilde{\alpha}_2)$ be the OLS estimates from the regression

$$Y = X_1\alpha_1 + Z\alpha_2 + u.$$

Show that $\hat{\beta}_1 = \tilde{\alpha}_1$.

3. Suppose that $y_t = y_{t-1} + e_t$ with e_t iid, $E(e_t) = 0$, $E(e_t^2) = \sigma^2 < \infty$. Demonstrate that y_t is non-stationary.

4. The model is

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &\neq 0, \end{aligned}$$

so the regressor x_i is endogenous. We know that in this case, the OLS estimator $\hat{\beta}$ is biased for the parameter β. We also know that the non-parametric bootstrap is (generally) a good method to estimate bias, and thereby make bias-adjusted. Explain whether or not the non-parametric bootstrap can be used to estimate the bias of OLS in the above context.

5. The model is

$$
\begin{aligned}
y_i &= x_i'\beta + e_i \\
E\left(z_i e_i\right) &= 0
\end{aligned}
$$

where x_i is $k \times 1$ and z_i is $m \times 1$, $m \geq k$.

(a) Write down the (efficient) GMM estimator $\hat{\beta}$ of β for the above model.

(b) Find the asymptotic distribution of $\hat{\beta}$ in the special case that $z_i = (x_i \quad x_i^2)$ and $E\left(e_i^2 \mid x_i\right) = \sigma^2$.

6. An AR(1) model is

$$
\begin{aligned}
y_t &= \rho y_{t-1} + e_t \\
E\left(e_t \mid I_{t-1}\right) &= 0
\end{aligned}
$$

where $I_{t-1} = (y_{t-1}, y_{t-2}, ...)$. Suppose that you do the reverse OLS regression of y_{t-1} on y_t :

$$
y_{t-1} = \hat{\alpha} y_t + \hat{u}_{t-1}.
$$

Find the probability limit of $\hat{\alpha}$.

7. The Tobit model is

$$
\begin{aligned}
y_i^* &= x_i'\beta + e_i \\
e_i &\sim N\left(0, \sigma^2\right) \\
y_i &= y_i^* 1\left(y_i^* \geq 0\right)
\end{aligned}
$$

where $1(\cdot)$ is the indicator function.

(a) Find $E\left(y_i \mid x_i\right)$.
Note: You may use the fact that since $e_i \sim N\left(0, \sigma^2\right)$,

$$
E\left(e_i 1\left(e_i \geq -x\right)\right) = \sigma\lambda(x/\sigma) = \sigma\phi(x/\sigma)/\Phi(x/\sigma).
$$

(b) Use the result from part (a) to suggest a NLLS estimator for the parameter β given a sample $\{y_i, x_i\}$.