

Seasonality

- Recall that we said that it can be useful to describe the mean of a time series as the sum of components

$$\mu_t = T_t + S_t + C_t$$

where S_t is the seasonal component.

- The seasonal component S_t is a repetitive cycle over the calendar year
- Seasonality S_t can be deterministic (predictable) or stochastic

Seasonality – Examples

- Gasoline consumption rises in summer due to increased auto travel
- International airline prices rise in summer due to increased tourism
- Natural gas consumption and prices rise in winter due to heating
- Electricity consumption increases in summer due to air conditioning
- Construction activity and jobs decrease in winter in the Midwest
- Consumer spending increases in November and December due to holiday shopping

Deterministic vs Stochastic Seasonality

- If the seasonal pattern repeats year after year, it is deterministic and predictable.
 - Christmas is always in December
- If the seasonal pattern roughly repeats itself, but evolves over the years, it is stochastic and only partially predictable
 - Holiday shopping as a percentage of income is not a fixed constant
- Seasonal patterns can change dramatically as the economy evolves
 - The spread of air conditioning shifted the seasonal pattern of residential electricity consumption from winter to summer

Seasonal Adjustment

- Most economic indicators reported by the government are **seasonally adjusted**.
- Roughly, the component S_t is estimated, and then what is reported is

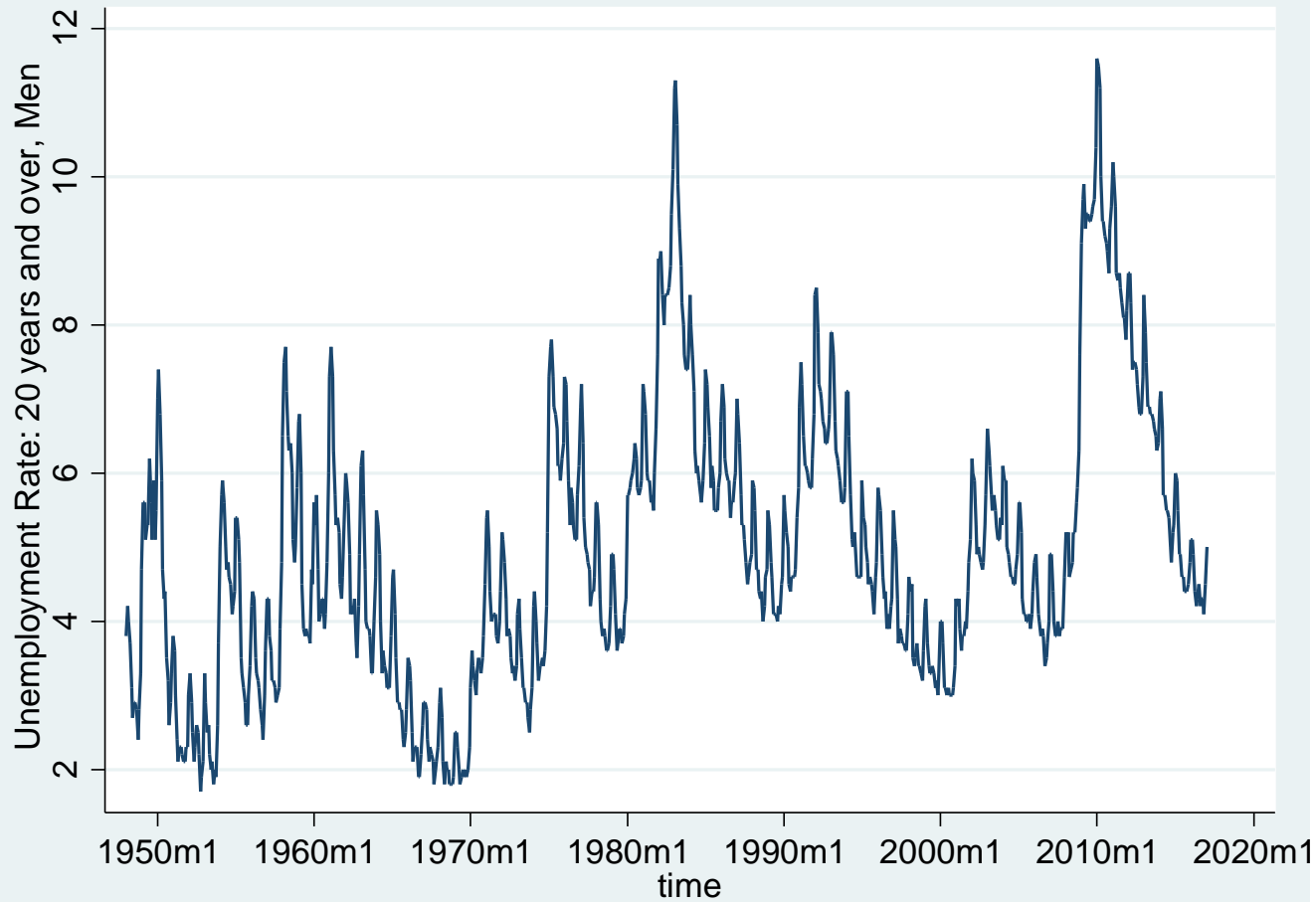
$$\begin{aligned}y_t^* &= y_t - S_t \\ &= T_t + C_t\end{aligned}$$

- The idea is that seasonality distracts from the main reporting purpose
 - Seasonally adjusted data allows users to focus on trend and business cycle movements
- Seasonal adjustment by central statistical agencies is sophisticated, allowing for evolving seasonal patterns.

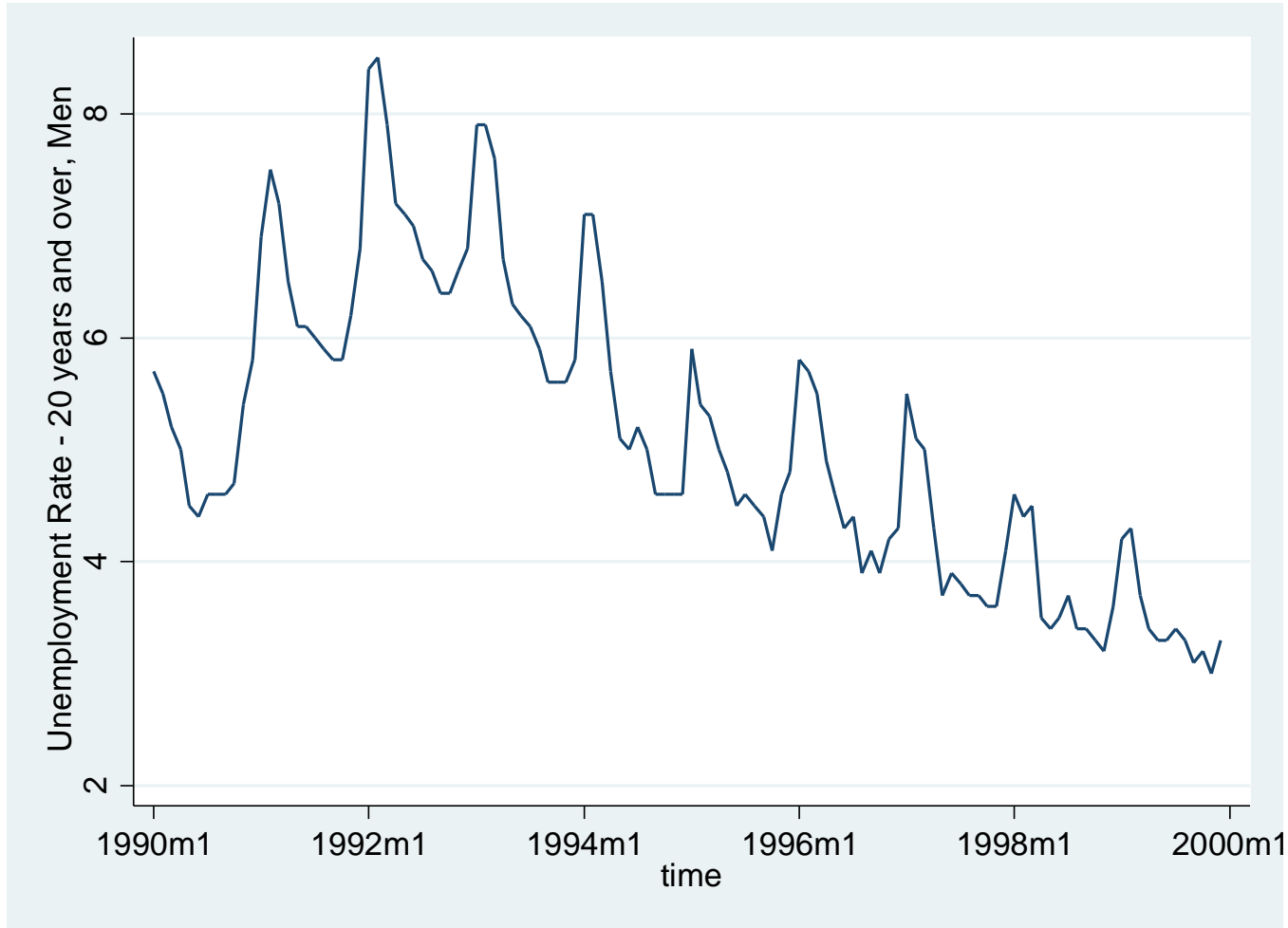
Examples of Seasonal Time Series

- Example 1:
- U.S. Unemployment Rate
 - Men, 20+ years
 - 1948-2009
 - Not seasonally adjusted

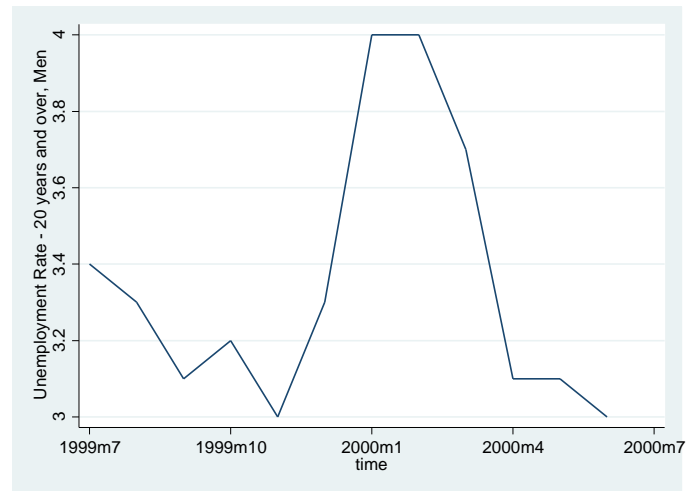
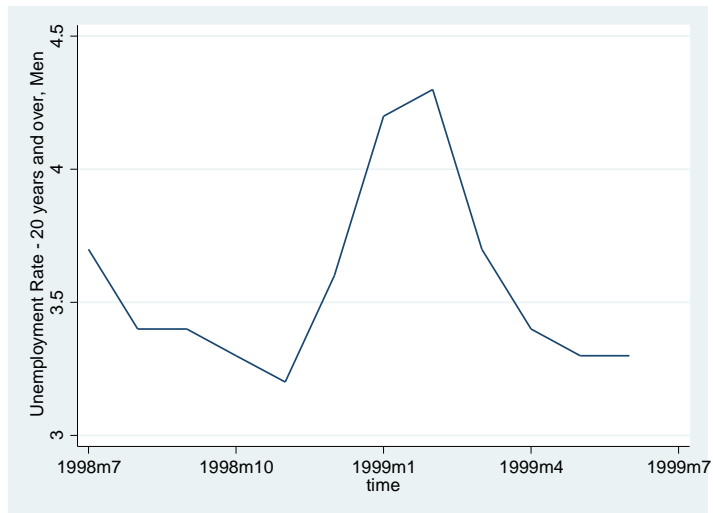
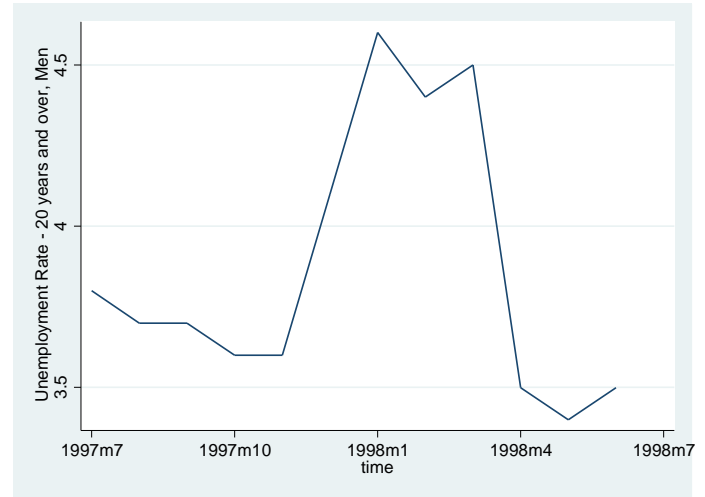
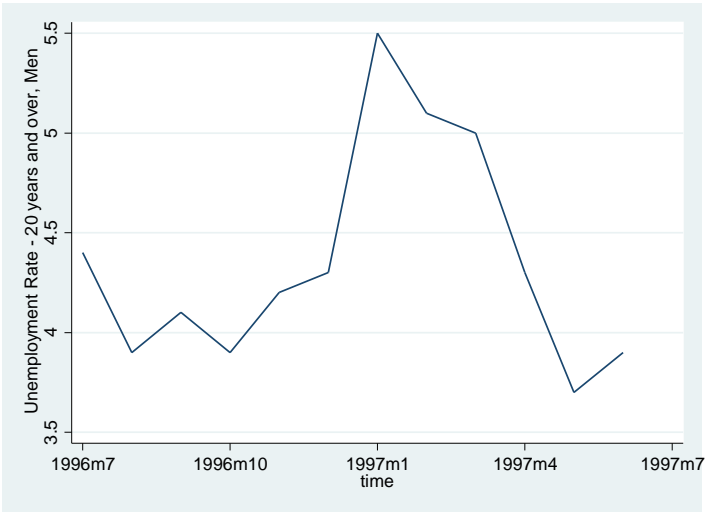
U.S. Unemployment Rate Men, 20+ years, 1948-current



Unemployment Rate, 1990-1999



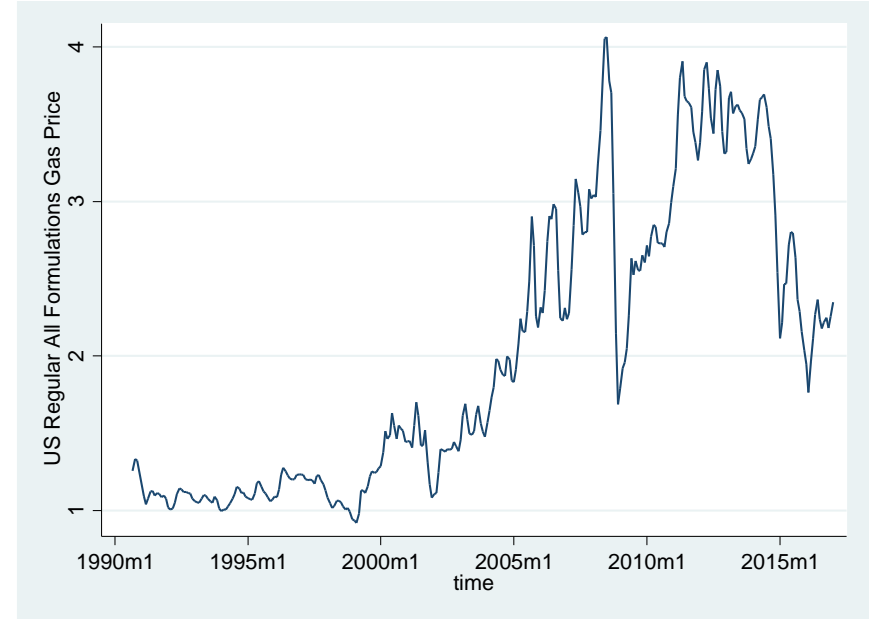
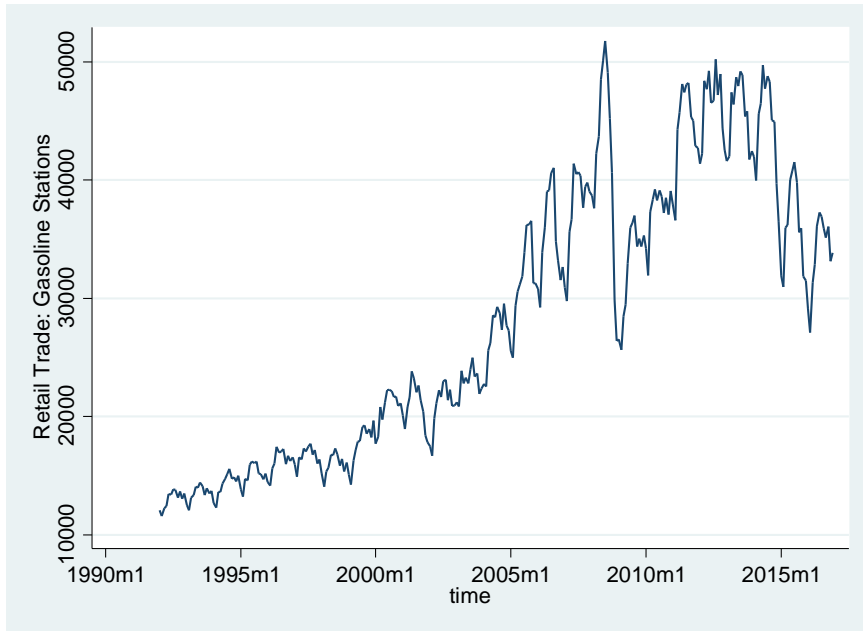
Unemployment Rate, by year



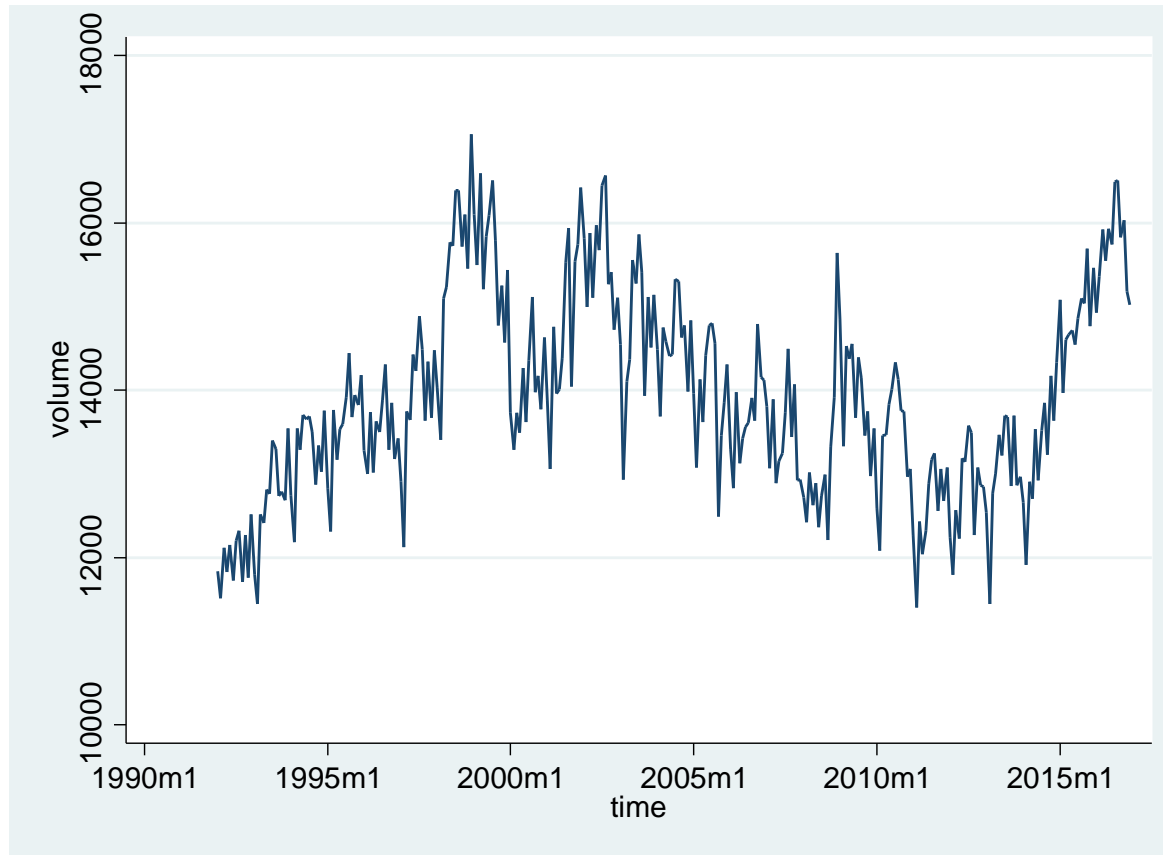
Examples of Seasonal Time Series

- Example 2:
- U.S. Gasoline Sales
 - RSGASSN, retail sales, \$million
 - GASREGM, price (\$) per gallon
 - volume= $RSGASSN / GASREGM$ (millions of gallons)
 - Monthly, 1992-current

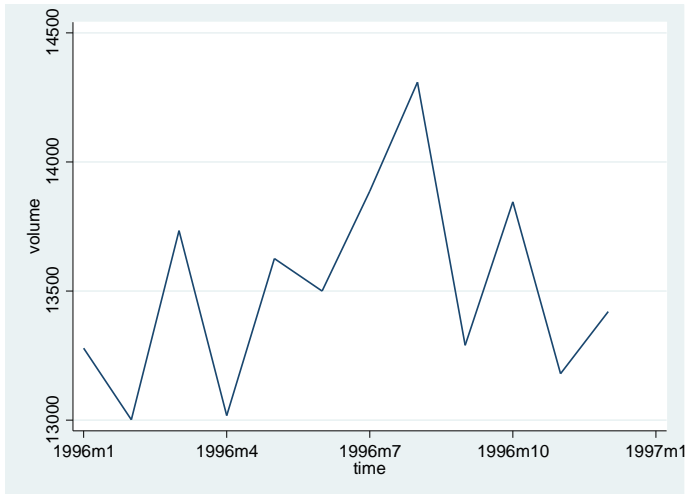
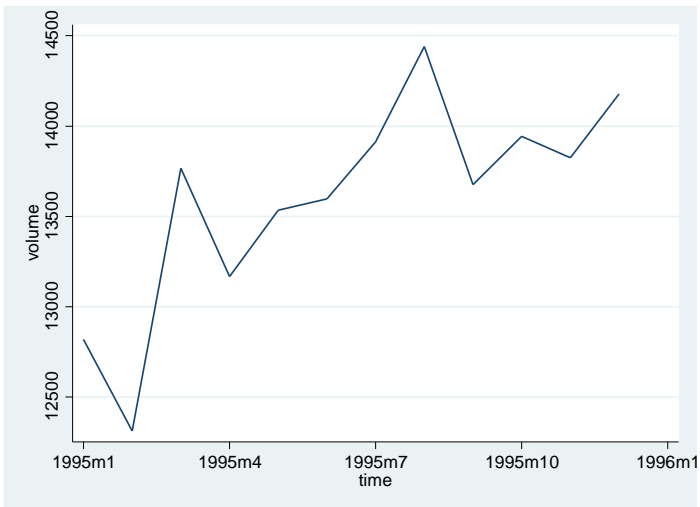
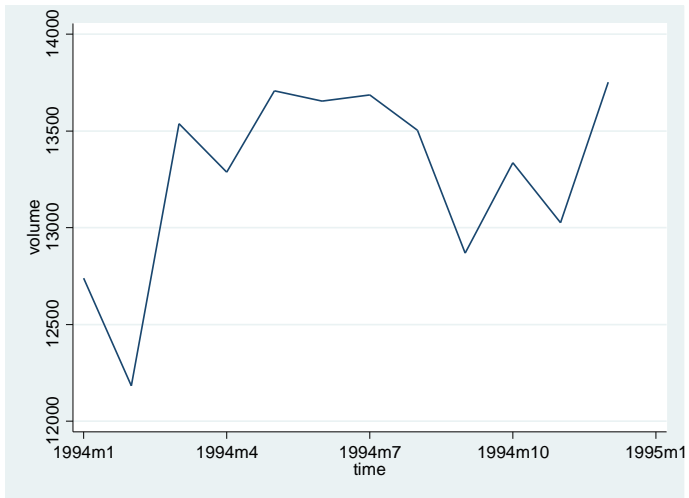
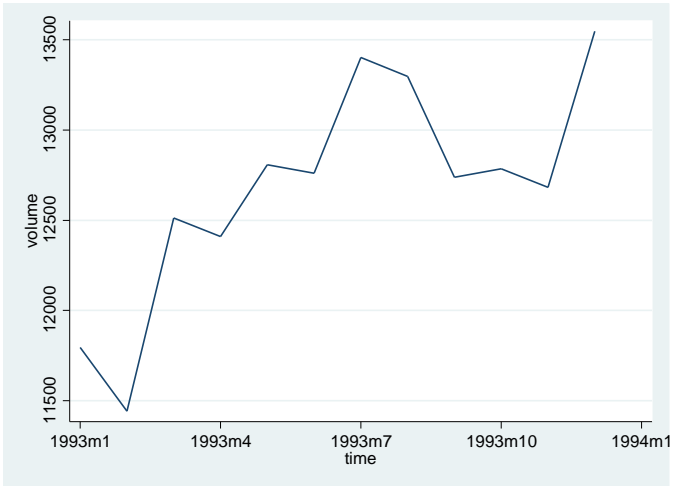
U.S. Gasoline Retail Sales (\$) & Price per Gallon



Gasoline Sales Volume (million of gallons)



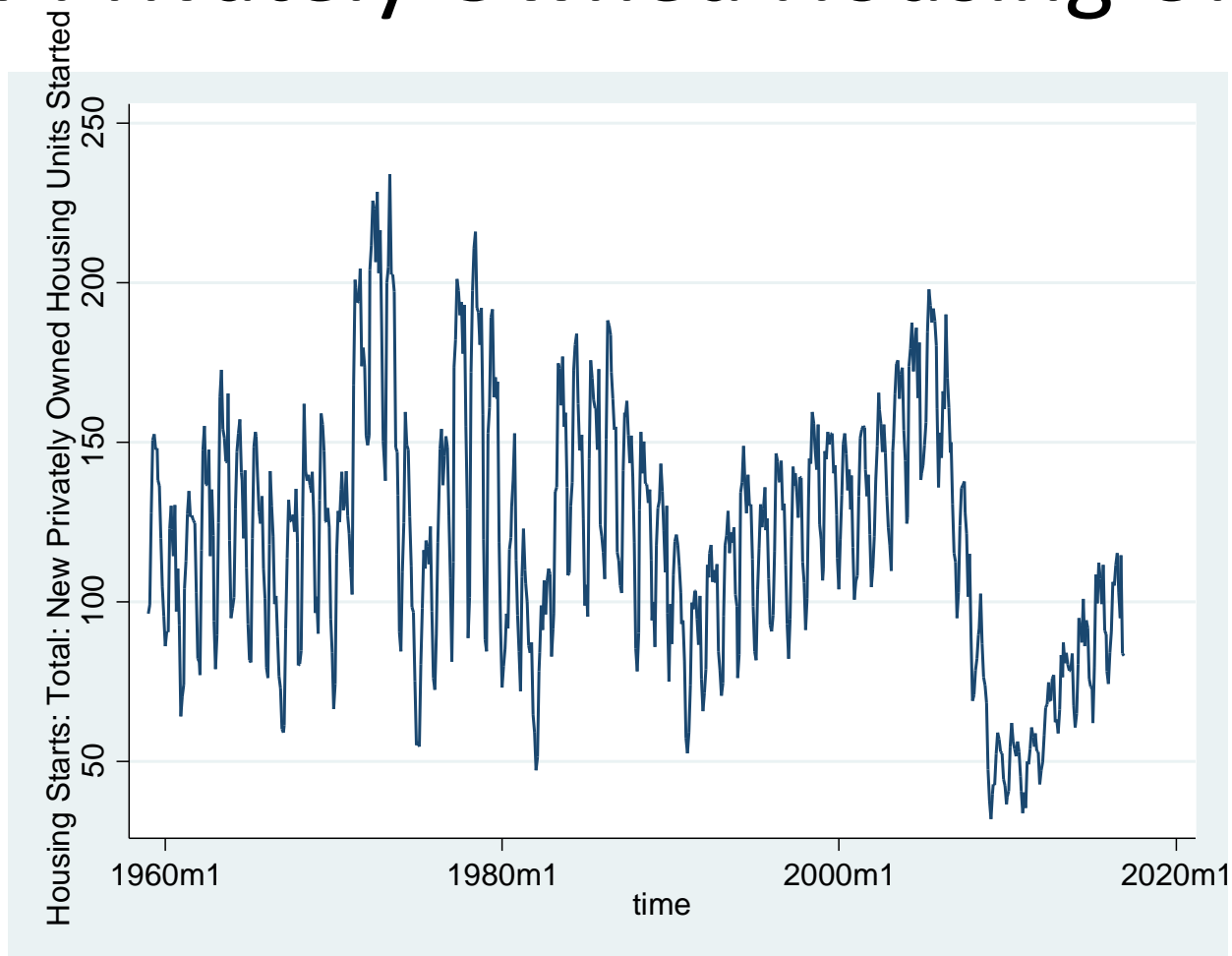
Gasoline Sales, by year



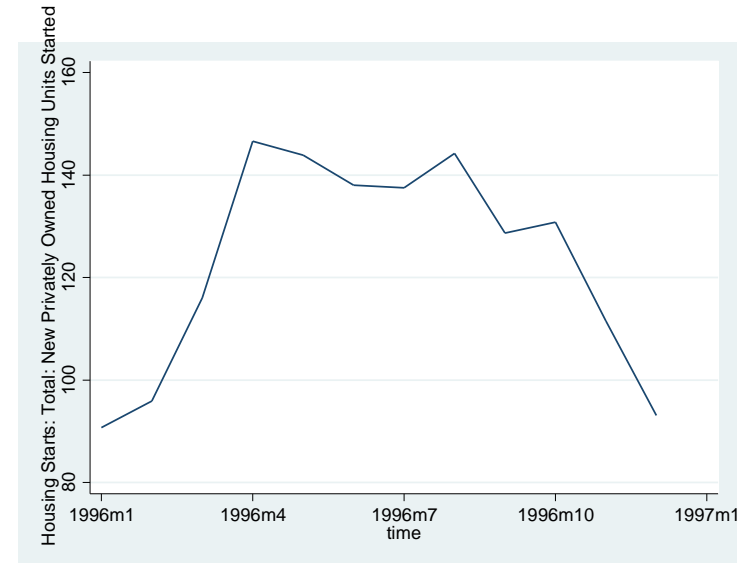
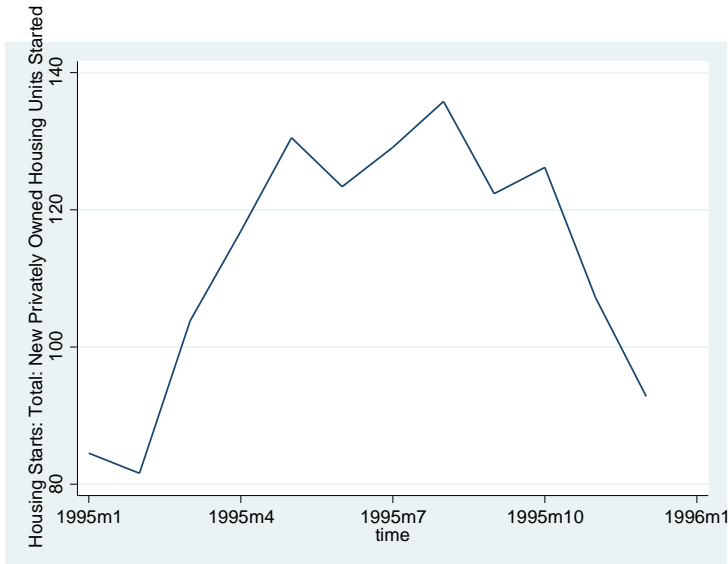
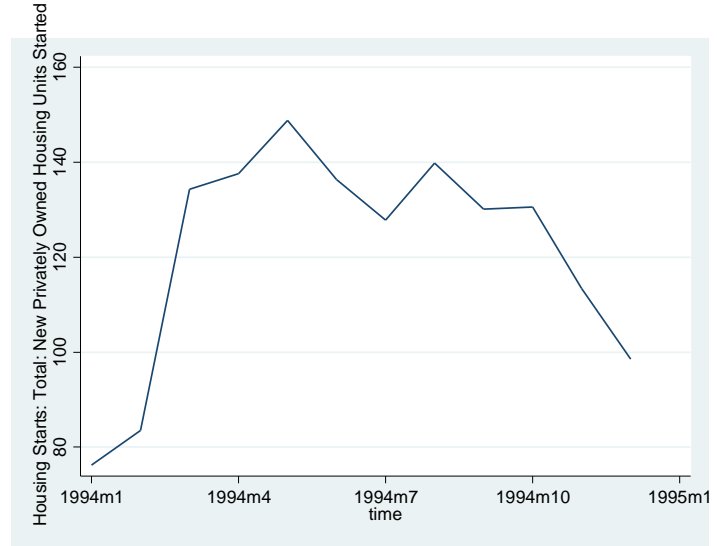
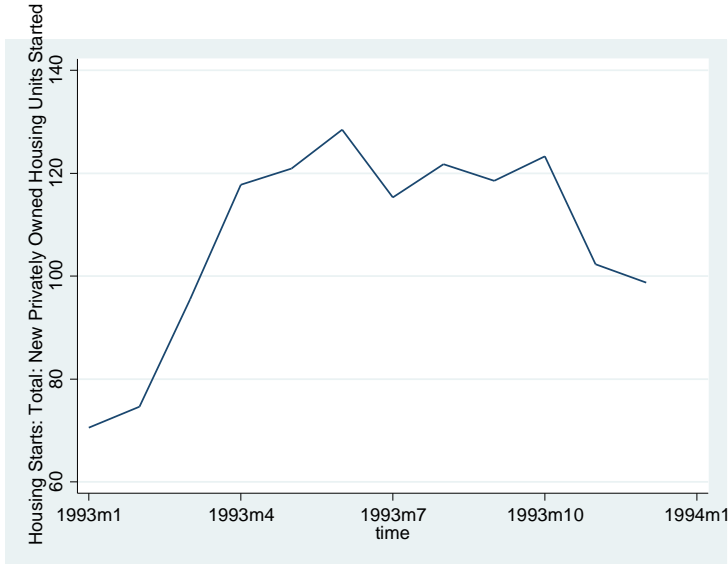
Example 3

U.S. Housing Starts

(New Privately Owned Housing Units)



Housing Starts, by year



Deterministic Seasonality

- If seasonality is constant and deterministic then S_t is simply a different constant for each period
- For example, for monthly data

$$S_t = \begin{cases} \gamma_1 & \text{if } t = \textit{January} \\ \gamma_2 & \text{if } t = \textit{February} \\ \vdots & \vdots \\ \gamma_{12} & \text{if } t = \textit{December} \end{cases}$$

- Seasonality is a constant which varies by the calendar period (quarter, month, week, day, or time of day)

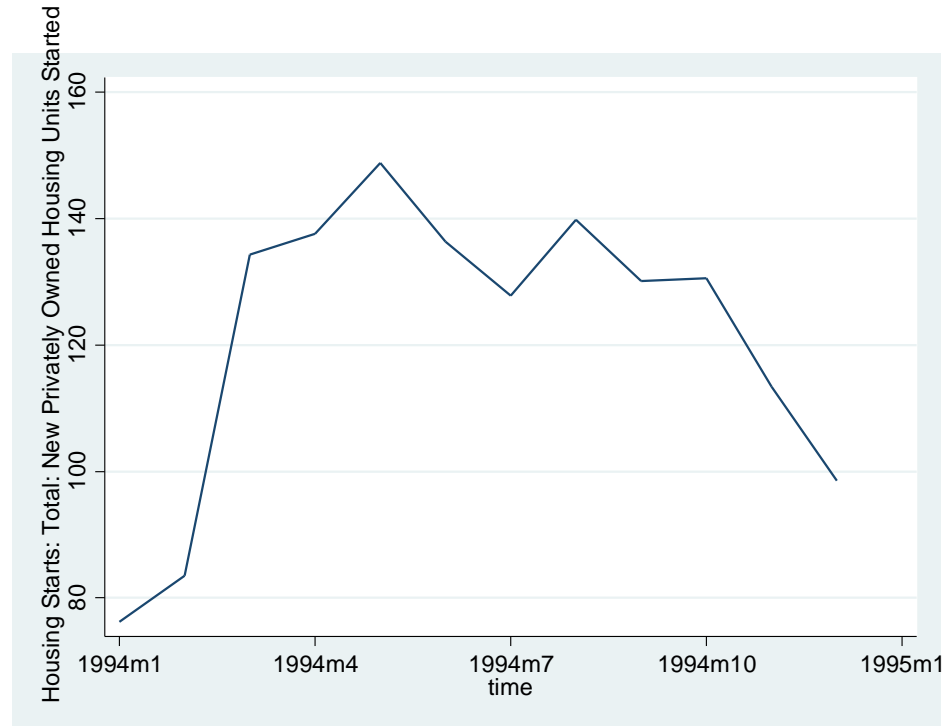
Fitted Values and Forecasts

Pure Deterministic Seasonality

- In the simple pure deterministic seasonality model, fitted values and forecasts are the simple seasonal pattern

Example – Housing Starts

| | |
|-----------|-----|
| January | 76 |
| February | 83 |
| March | 134 |
| April | 138 |
| May | 149 |
| June | 136 |
| July | 128 |
| August | 140 |
| September | 130 |
| October | 131 |
| November | 113 |
| December | 98 |



Seasonal Dummy Model

- Deterministic seasonality S_t can be written as a function of seasonal dummy variables
- Let s be the seasonal frequency
 - $s=4$ for quarterly
 - $s=12$ for monthly
- Let $D_{1t}, D_{2t}, D_{3t}, \dots, D_{st}$ be seasonal dummies
 - $D_{1t} = 1$ if s is the first period, otherwise $D_{1t} = 0$
 - $D_{2t} = 1$ if s is the second period, otherwise $D_{2t} = 0$
- At any time period t , one of the seasonal dummies $D_{1t}, D_{2t}, D_{3t}, \dots, D_{st}$ will equal 1, all the others will equal 0.

Seasonal Dummy Model

- Deterministic seasonality

$$S_t = \begin{cases} \gamma_1 & \text{if } t = \text{January} \\ \gamma_2 & \text{if } t = \text{February} \\ \vdots & \vdots \\ \gamma_{12} & \text{if } t = \text{December} \end{cases}$$
$$= \sum_{i=1}^s \gamma_i D_{it}$$

a linear function of the dummy variables

Estimation

- Least squares regression

$$y_{t+h} = \sum_{i=1}^s \gamma_i D_{it} + e_t$$
$$= \alpha + \sum_{i=1}^{s-1} \beta_i D_{it} + e_t$$

- You can either
 - Regress y on all the seasonal dummies, omitting the intercept, or
 - Regress y on an intercept and the seasonal dummies, omitting one dummy (one season, e.g. December)
- You cannot regress on both the intercept plus all seasonal dummies, for they would be collinear and redundant.

Interpreting Coefficients

- In the model

$$S_t = \alpha + \sum_{i=1}^{s-1} \beta_i D_{it}$$

the intercept $\alpha = \gamma_s$ is the seasonality in the omitted season.

- The coefficients $\beta_i = \gamma_i - \gamma_s$ are the difference in the seasonal component from the s 'th period.

STATA Programming

- If the time index is *time* and is formatted as a time index, you can determine the period using the commands

```
generate m=month(dofm(time))
```

```
generate q=quarter(dofq(time))
```

for monthly and quarterly data, respectively

(See dates and times in STATA Data manual)

Creating Dummies

- If m is the month (1 for January, 2 for February, etc.), then
 - **generate m1=(m==1)**
 - This creates a dummy variable “m1” for January
 - Then
 - **regress y m1 m2 m3 m4 m5 m6 m7 m8 m9 m10 m11**
 - or
 - **regress y m1 m2 m3 m4 m5 m6 m7 m8 m9 m10 m11 m12, noconstant**
- Easier
 - Type “b12.m” in the regressor list
 - **regress y b12.m**
 - This includes dummies for months 1 through 11, omits 12
 - Same as mechanically listing the eleven dummies, but easier.
 - It is important that “m” be the numerical month (1 for January, 2 for February, etc.)

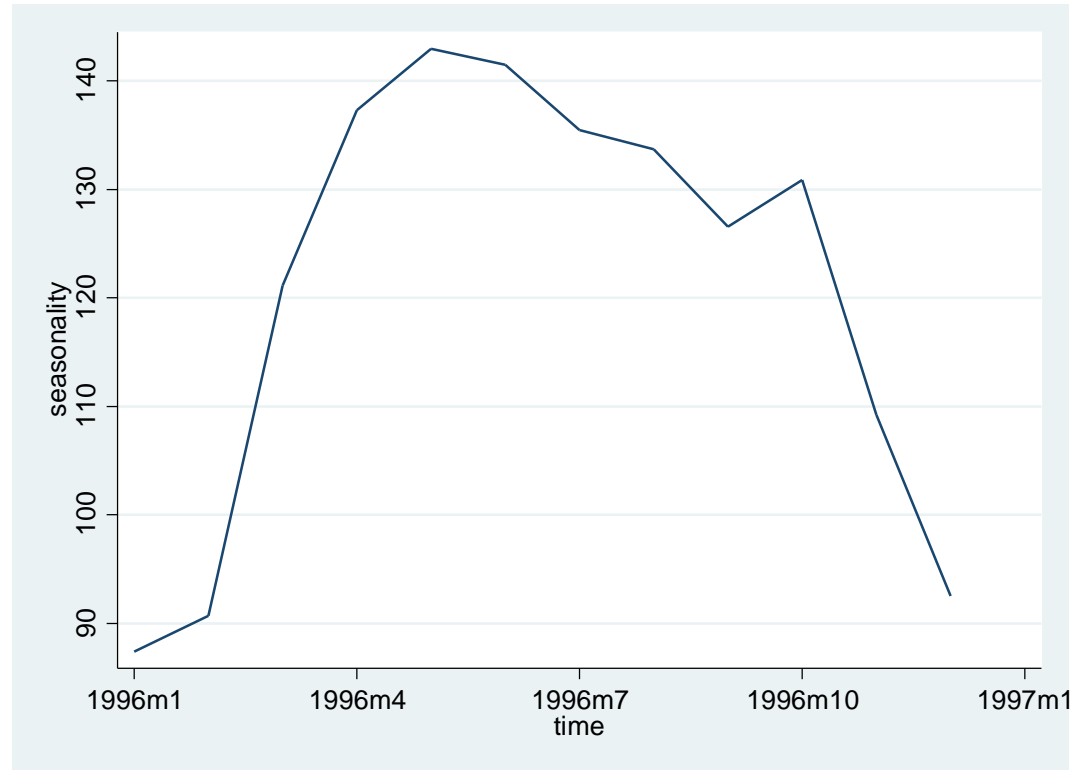
Example 3: Housing Starts

- `gen m = month(dofm(time))`
- `regress starts b12.m`

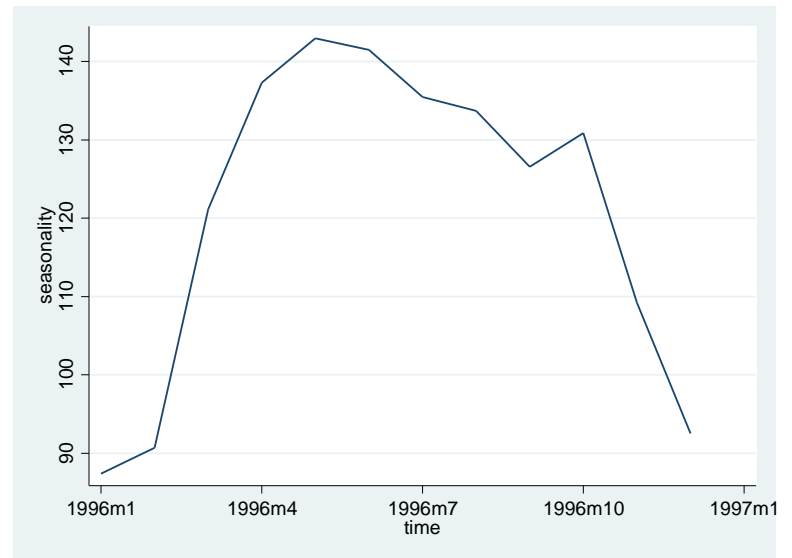
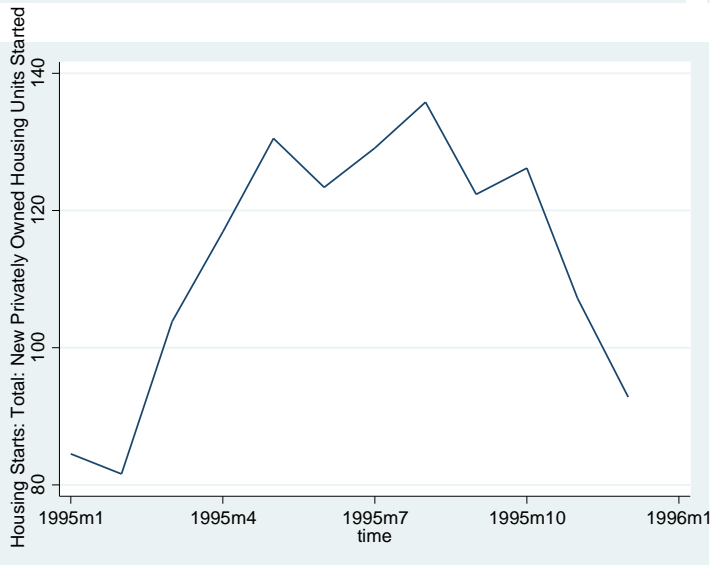
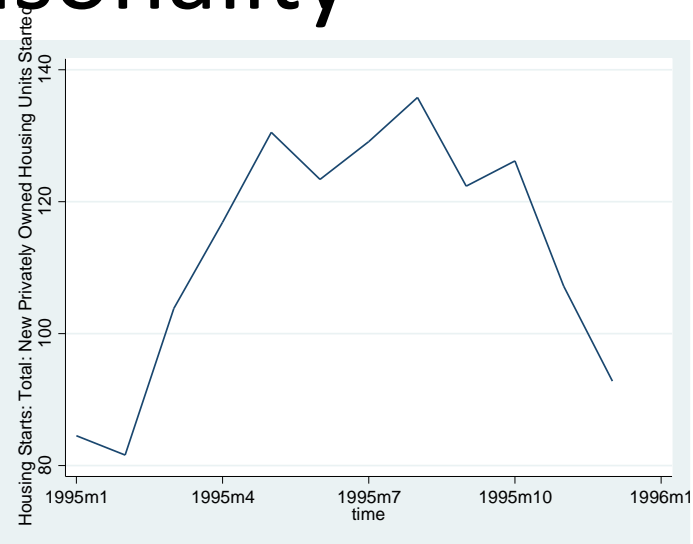
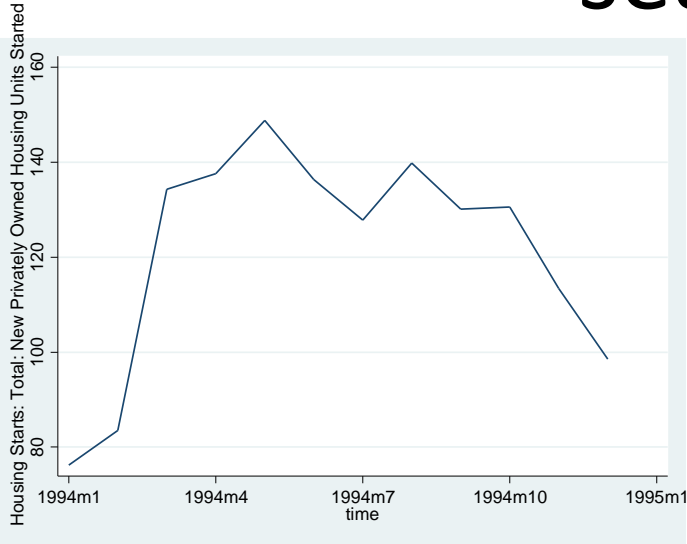
| starts | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|--------|-----------|-----------|-------|-------|----------------------|
| m | | | | | |
| 1 | -5.125 | 6.515521 | -0.79 | 0.432 | -17.91865 7.668648 |
| 2 | -1.841071 | 6.515521 | -0.28 | 0.778 | -14.63472 10.95258 |
| 3 | 28.59464 | 6.515521 | 4.39 | 0.000 | 15.80099 41.38829 |
| 4 | 44.76429 | 6.515521 | 6.87 | 0.000 | 31.97064 57.55793 |
| 5 | 50.42857 | 6.515521 | 7.74 | 0.000 | 37.63492 63.22222 |
| 6 | 48.99821 | 6.515521 | 7.52 | 0.000 | 36.20457 61.79186 |
| 7 | 42.95179 | 6.515521 | 6.59 | 0.000 | 30.15814 55.74543 |
| 8 | 41.19286 | 6.515521 | 6.32 | 0.000 | 28.39921 53.98651 |
| 9 | 34.03929 | 6.515521 | 5.22 | 0.000 | 21.24564 46.83293 |
| 10 | 38.35893 | 6.515521 | 5.89 | 0.000 | 25.56528 51.15258 |
| 11 | 16.71964 | 6.515521 | 2.57 | 0.011 | 3.925994 29.51329 |
| _cons | 92.52143 | 4.607169 | 20.08 | 0.000 | 83.47495 101.5679 |

Estimated Seasonality – Housing Starts

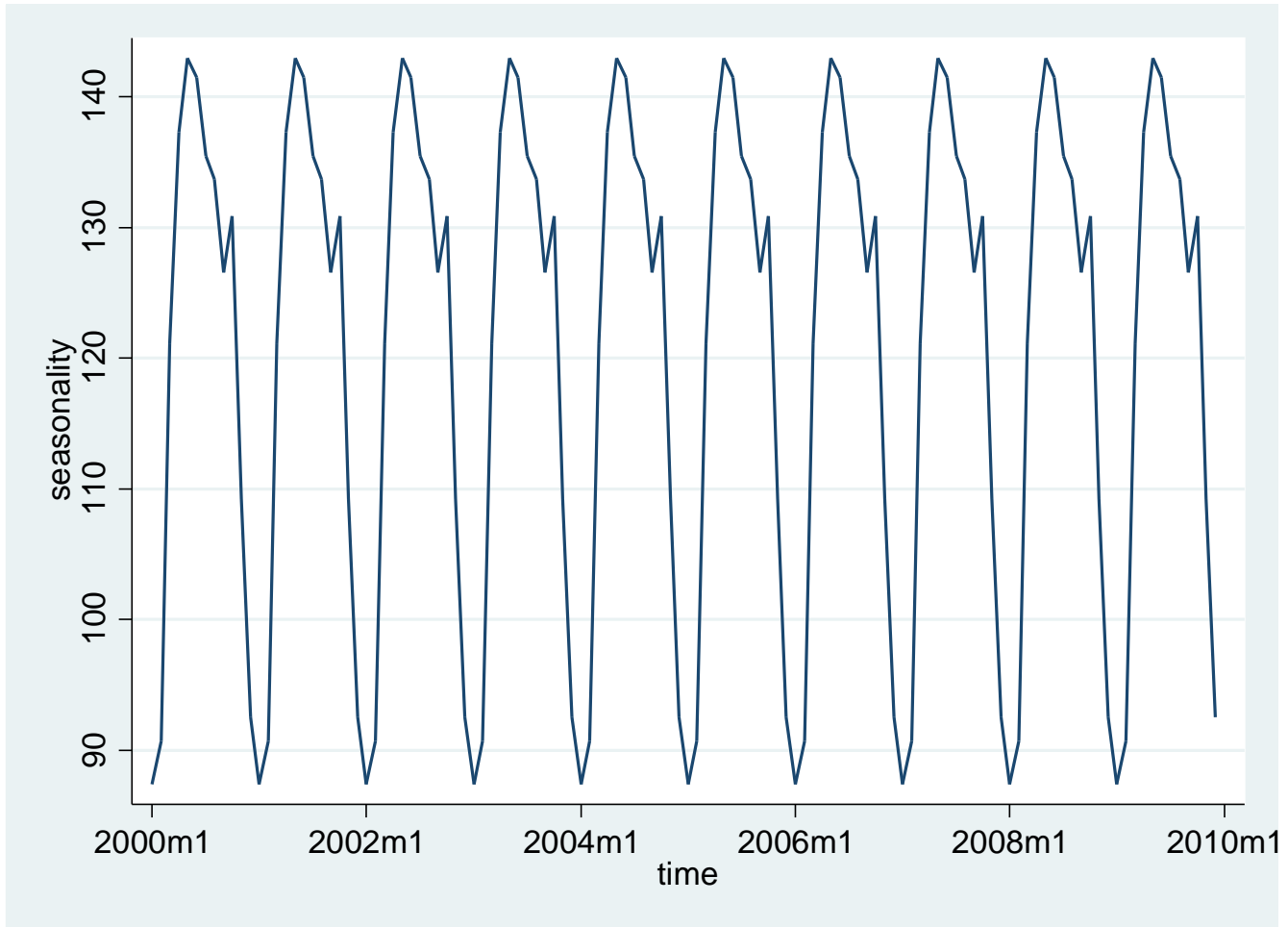
| | |
|-----------|-----|
| January | 87 |
| February | 91 |
| March | 121 |
| April | 137 |
| May | 143 |
| June | 142 |
| July | 135 |
| August | 134 |
| September | 127 |
| October | 131 |
| November | 109 |
| December | 93 |



Housing Starts, by year and estimated seasonality

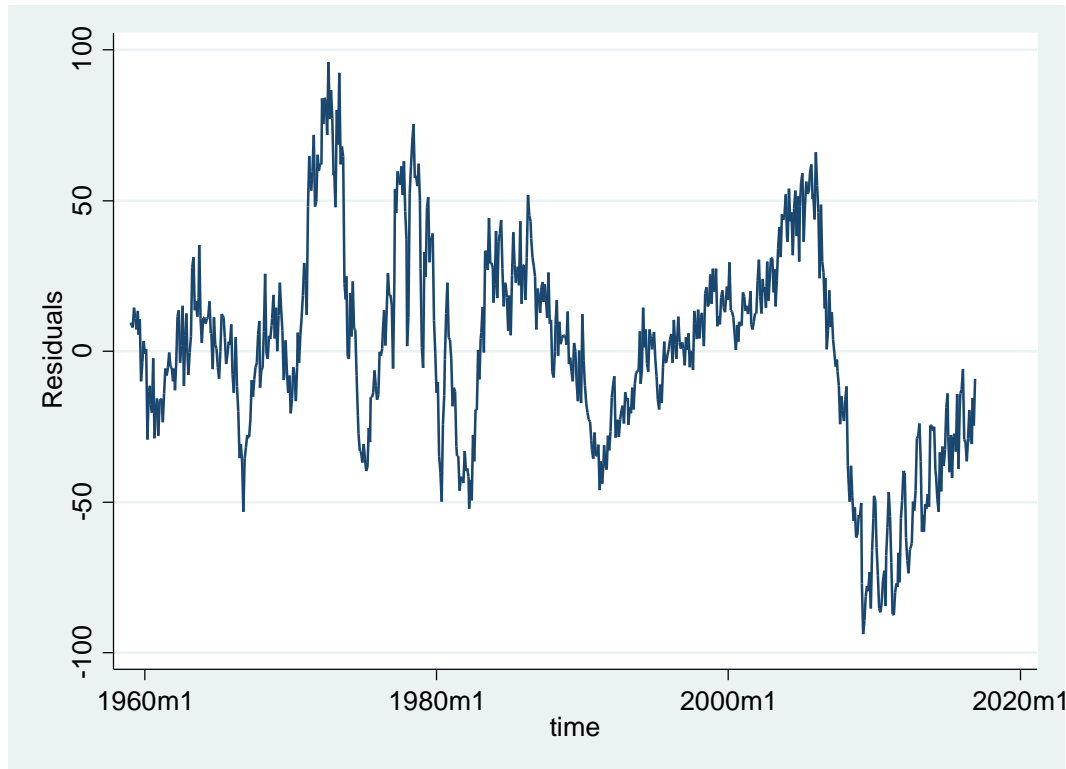


Predicted Values



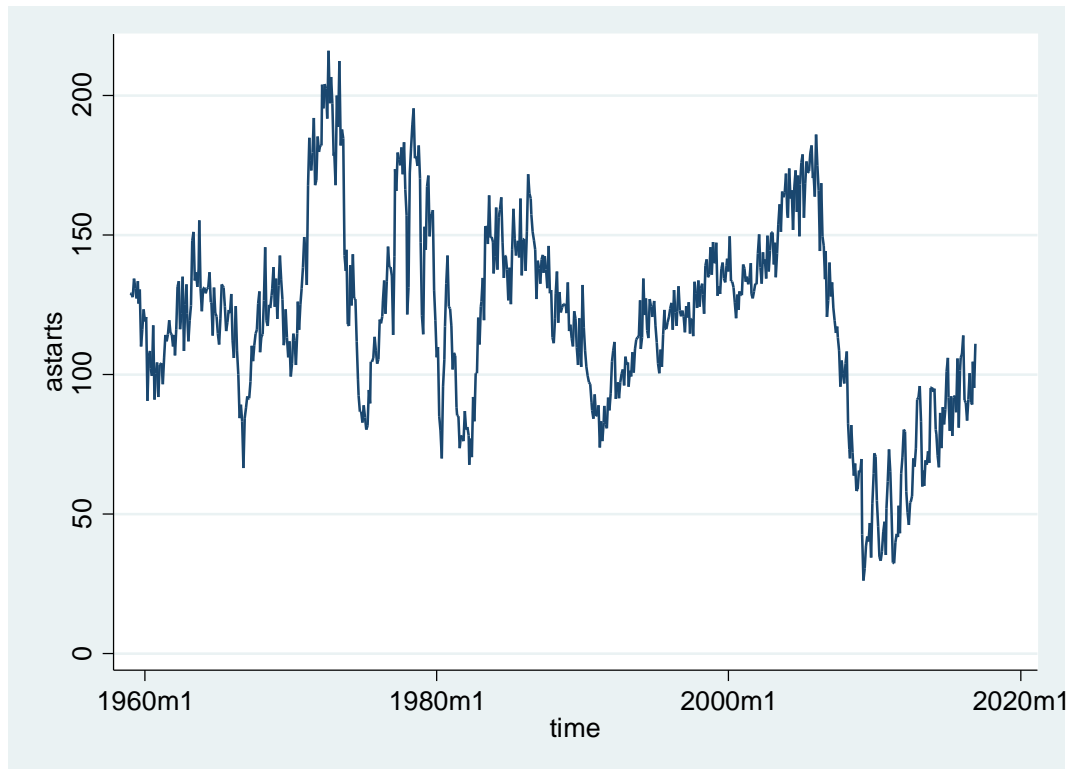
Residual Plots

- Plot the residuals after the seasonal dummy regression, to see if the seasonality has largely been modeled
- If seasonality has changed, the residuals will still display seasonality
 - predict e, residuals
 - tsline e



DeSeasonalized

- Original series less seasonal
- In seasonal dummy model, equals residual plus the sample mean
- $\text{egen mu} = \text{mean}(\text{starts})$
- $\text{gen astarts} = \text{mu} + e$
- tsline astarts



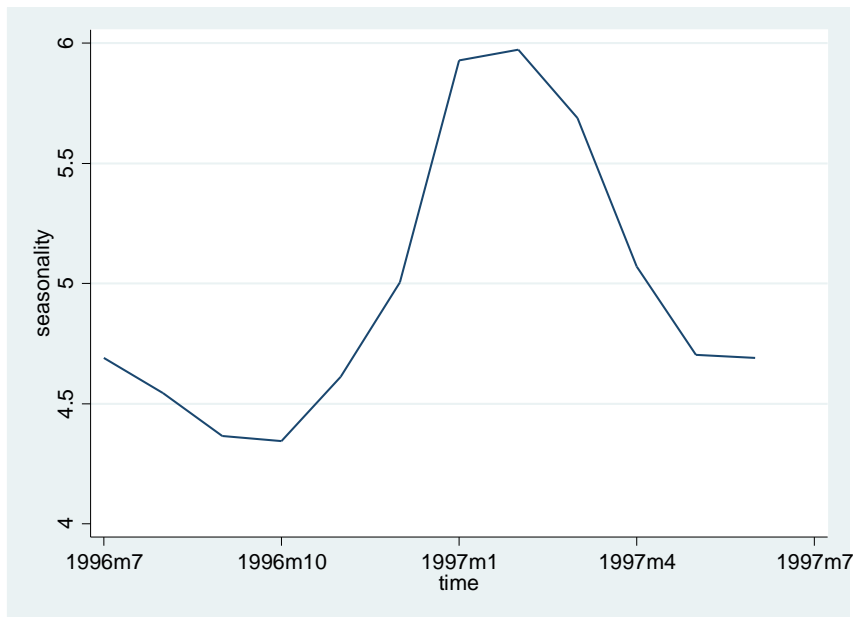
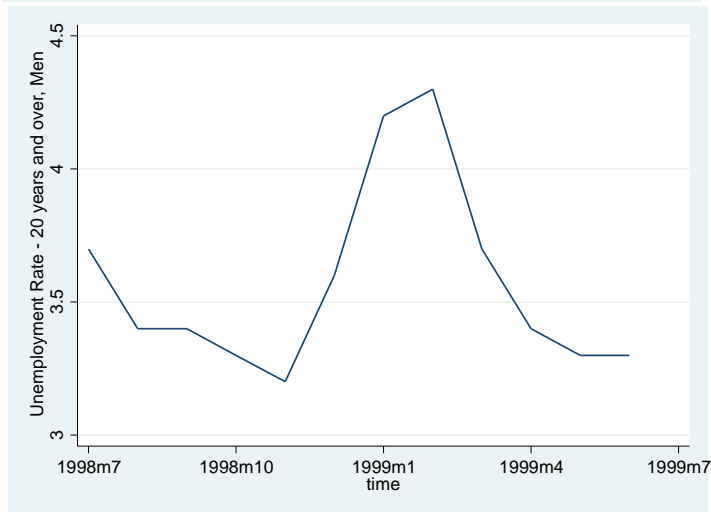
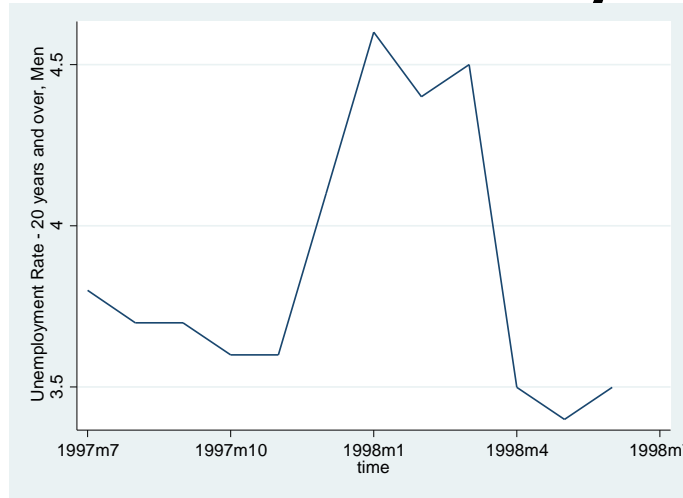
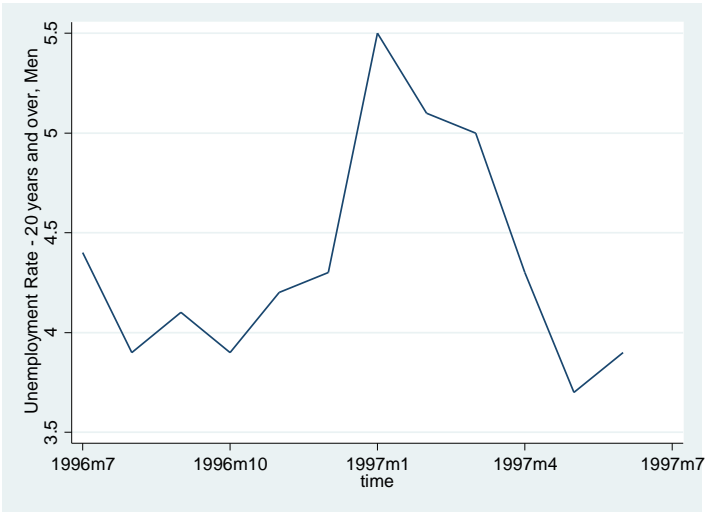
Example 1

Unemployment Rate

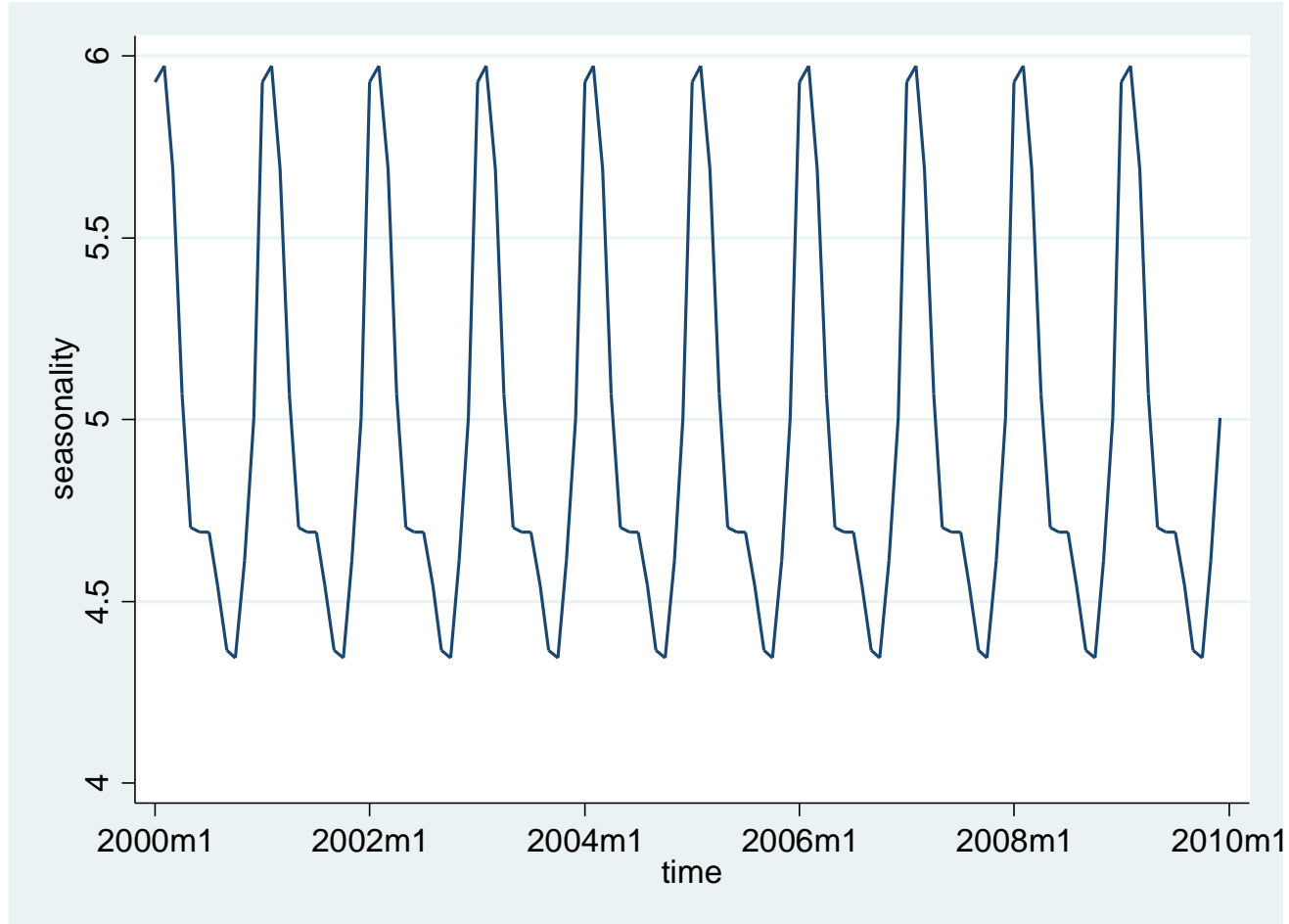
- `gen m = month(dofm(time))`
- `regress men b12.m`

| men | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|-------|-----------|-----------|-------|-------|----------------------|
| m | | | | | |
| 1 | .9238806 | .3142008 | 2.94 | 0.003 | .3071158 1.540645 |
| 2 | .9686567 | .3142008 | 3.08 | 0.002 | .351892 1.585421 |
| 3 | .6835821 | .3142008 | 2.18 | 0.030 | .0668173 1.300347 |
| 4 | .0656716 | .3142008 | 0.21 | 0.834 | -.5510931 .6824364 |
| 5 | -.3014925 | .3142008 | -0.96 | 0.338 | -.9182573 .3152722 |
| 6 | -.3134328 | .3142008 | -1.00 | 0.319 | -.9301976 .3033319 |
| 7 | -.3149254 | .3142008 | -1.00 | 0.317 | -.9316901 .3018394 |
| 8 | -.461194 | .3142008 | -1.47 | 0.143 | -1.077959 .1555707 |
| 9 | -.638806 | .3142008 | -2.03 | 0.042 | -1.255571 -.0220412 |
| 10 | -.6597015 | .3142008 | -2.10 | 0.036 | -1.276466 -.0429367 |
| 11 | -.3925373 | .3142008 | -1.25 | 0.212 | -1.009302 .2242274 |
| _cons | 5.004478 | .2221735 | 22.53 | 0.000 | 4.568359 5.440596 |

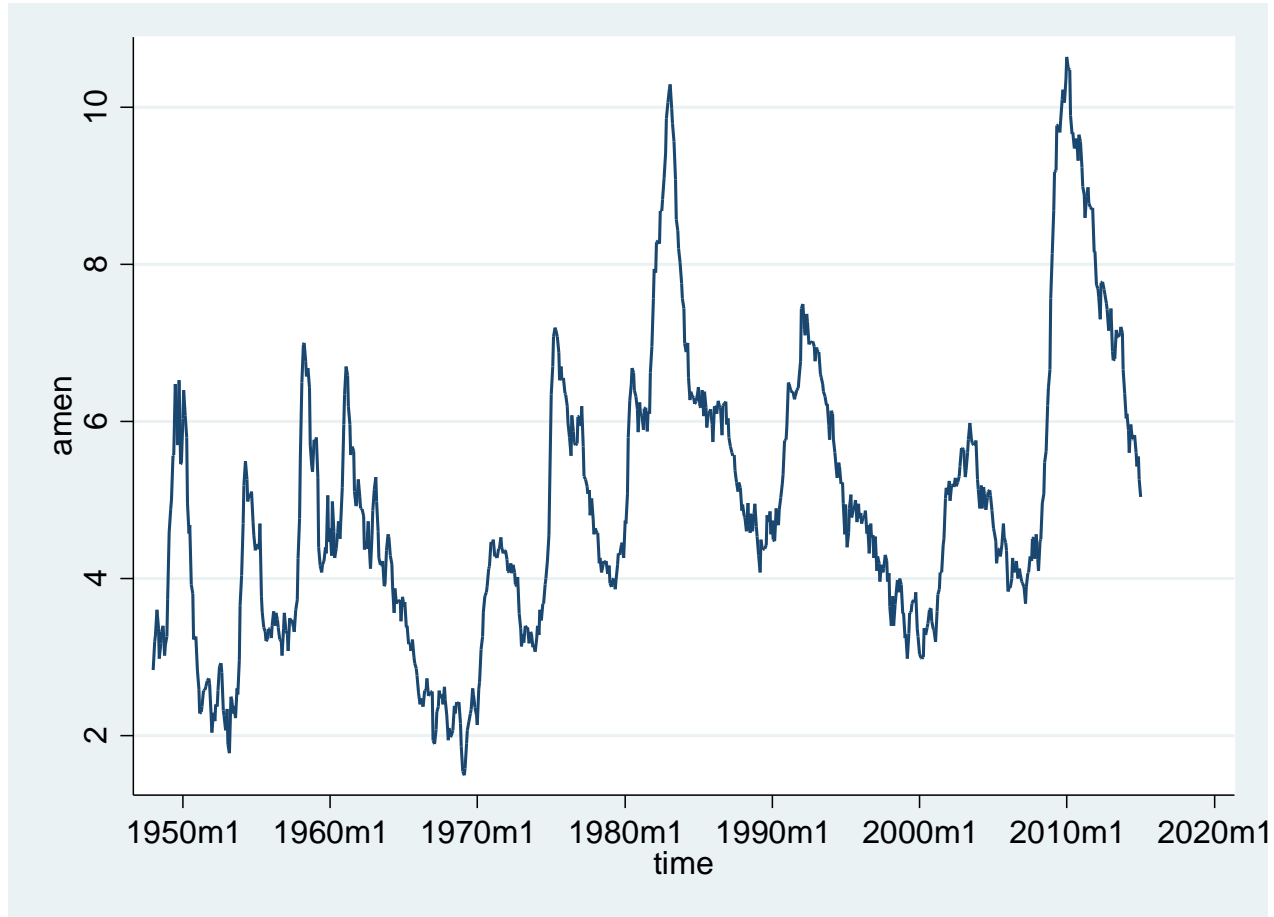
Unemployment Rate, by year, and estimated seasonality



Predicted Values



DeSeasonalized

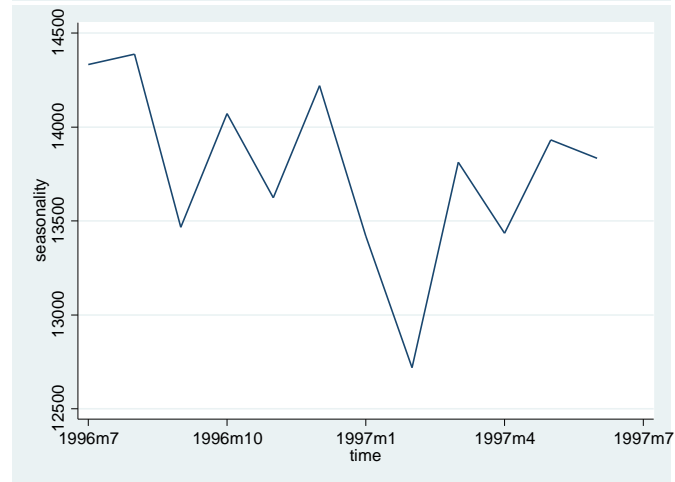
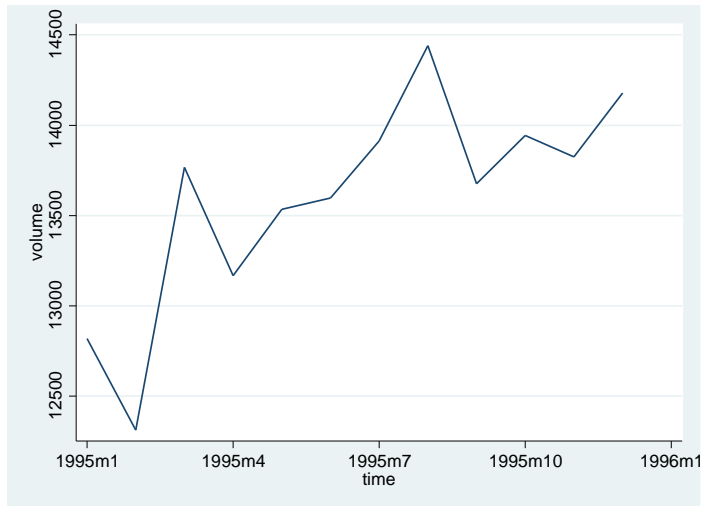
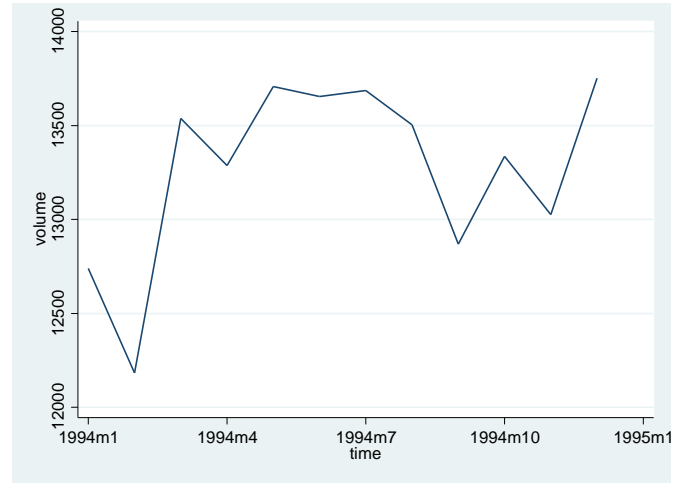
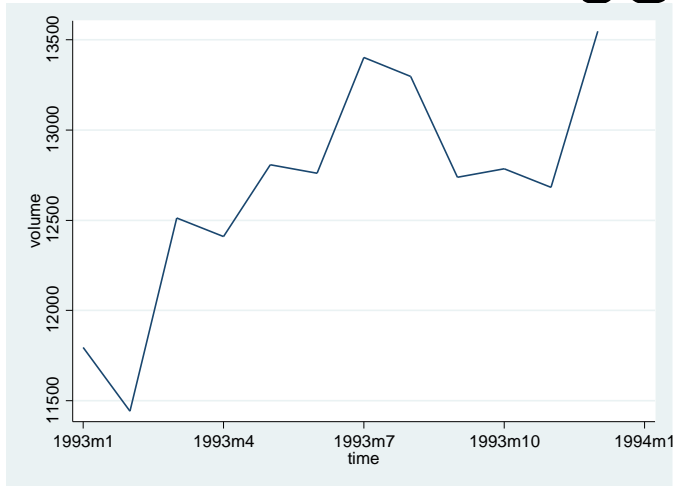


Example 2: Gasoline Sales

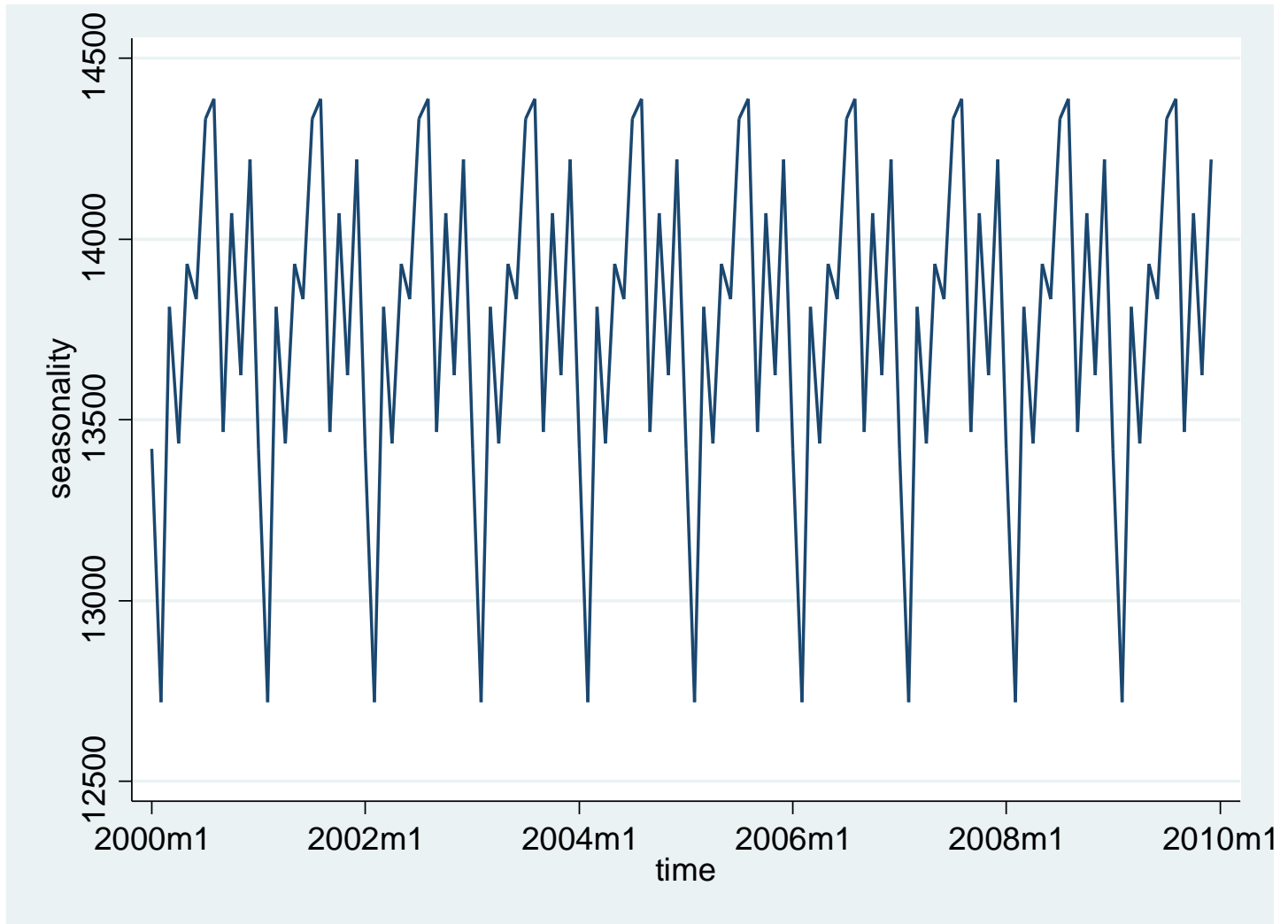
- `gen m = month(dofm(time))`
- `regress volume b12.m`

| volume | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|--------|-----------|-----------|-------|-------|----------------------|-----------|
| m | | | | | | |
| 1 | -800.4976 | 320.3884 | -2.50 | 0.013 | -1431.339 | -169.6559 |
| 2 | -1500.67 | 320.3884 | -4.68 | 0.000 | -2131.512 | -869.8283 |
| 3 | -407.3326 | 320.3884 | -1.27 | 0.205 | -1038.174 | 223.5091 |
| 4 | -784.7925 | 320.3884 | -2.45 | 0.015 | -1415.634 | -153.9508 |
| 5 | -288.0735 | 320.3884 | -0.90 | 0.369 | -918.9152 | 342.7682 |
| 6 | -386.9278 | 320.3884 | -1.21 | 0.228 | -1017.77 | 243.9139 |
| 7 | 113.6742 | 320.3884 | 0.35 | 0.723 | -517.1675 | 744.5159 |
| 8 | 167.2559 | 320.3884 | 0.52 | 0.602 | -463.5858 | 798.0976 |
| 9 | -752.7424 | 320.3884 | -2.35 | 0.020 | -1383.584 | -121.9006 |
| 10 | -149.3386 | 320.3884 | -0.47 | 0.642 | -780.1804 | 481.5031 |
| 11 | -597.5424 | 320.3884 | -1.87 | 0.063 | -1228.384 | 33.29936 |
| _cons | 14220.11 | 226.5488 | 62.77 | 0.000 | 13774.04 | 14666.18 |

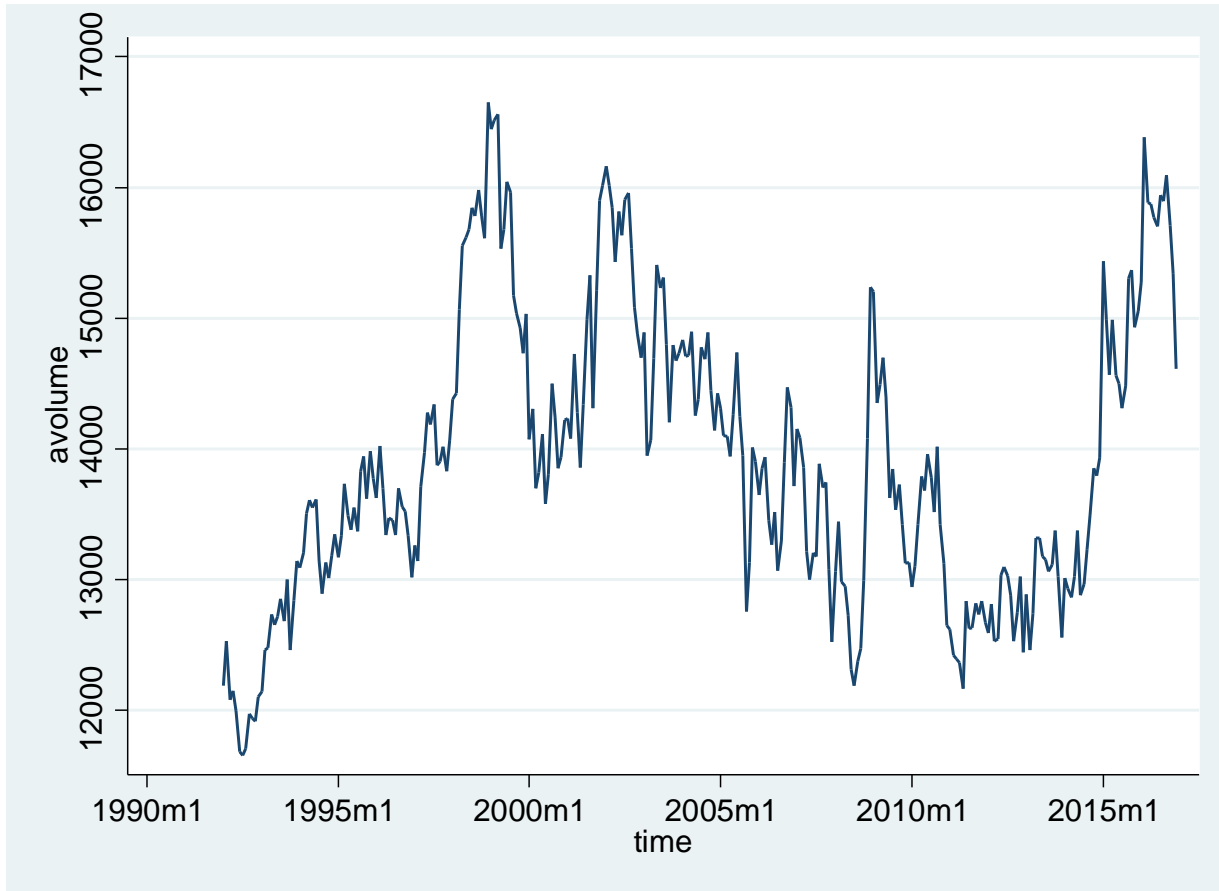
Gasoline Sales, by year, and estimated seasonality



Predicted Values



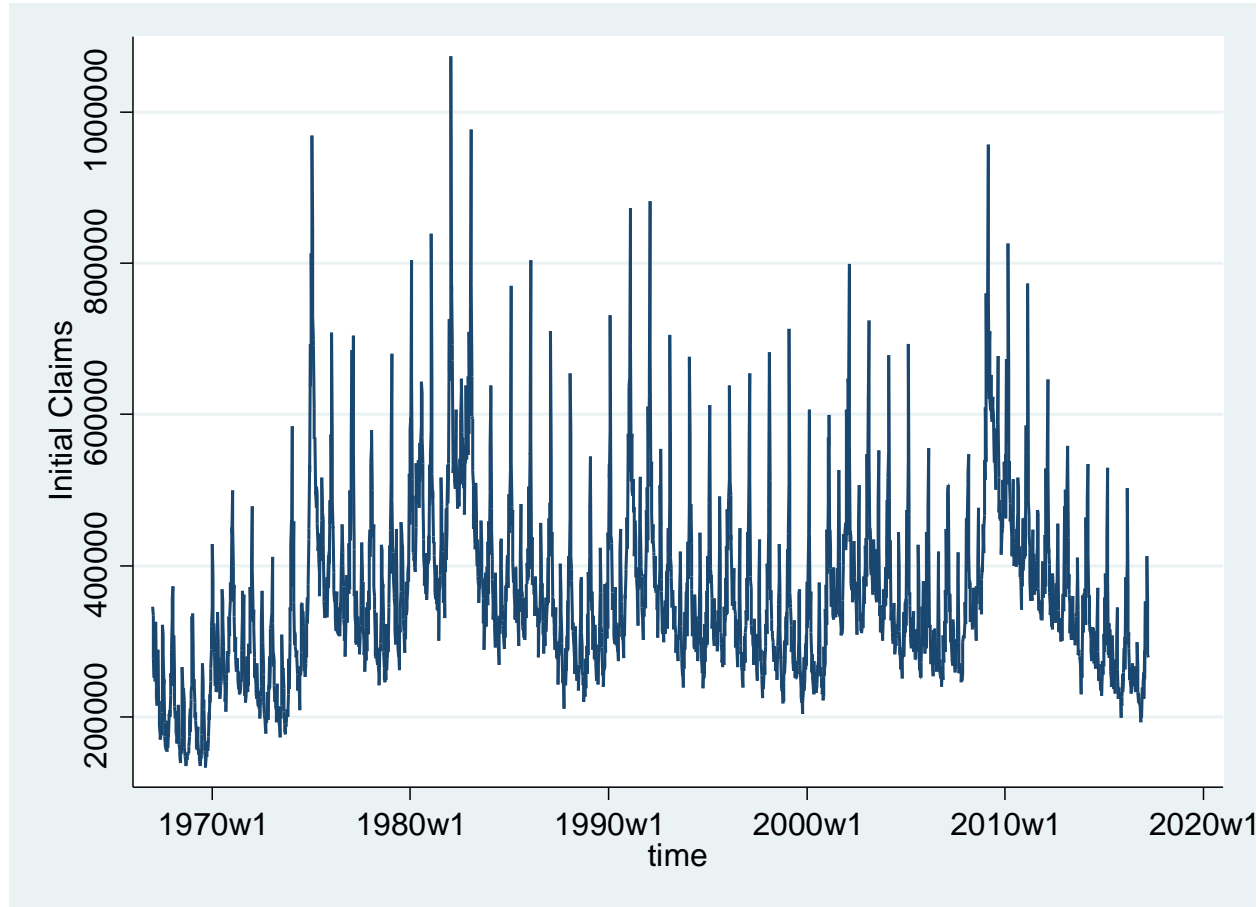
DeSeasonalized



Example 4

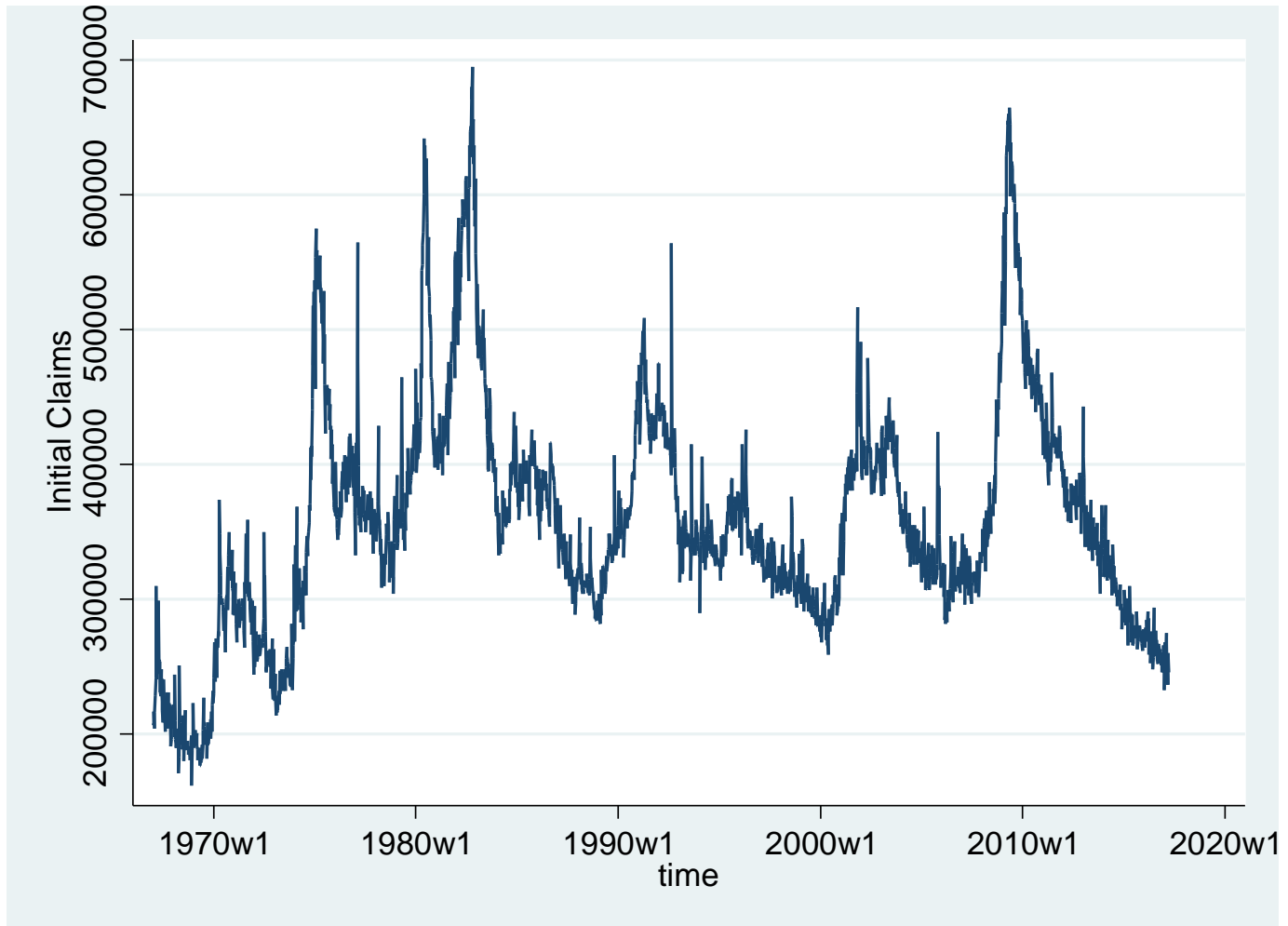
- Unemployment Insurance Claims
- Department of Labor
- Weekly
- Important indicator for unemployment

Unemployment Claims Not Seasonally Adjusted



Unemployment Claims

Official Seasonally Adjusted Series



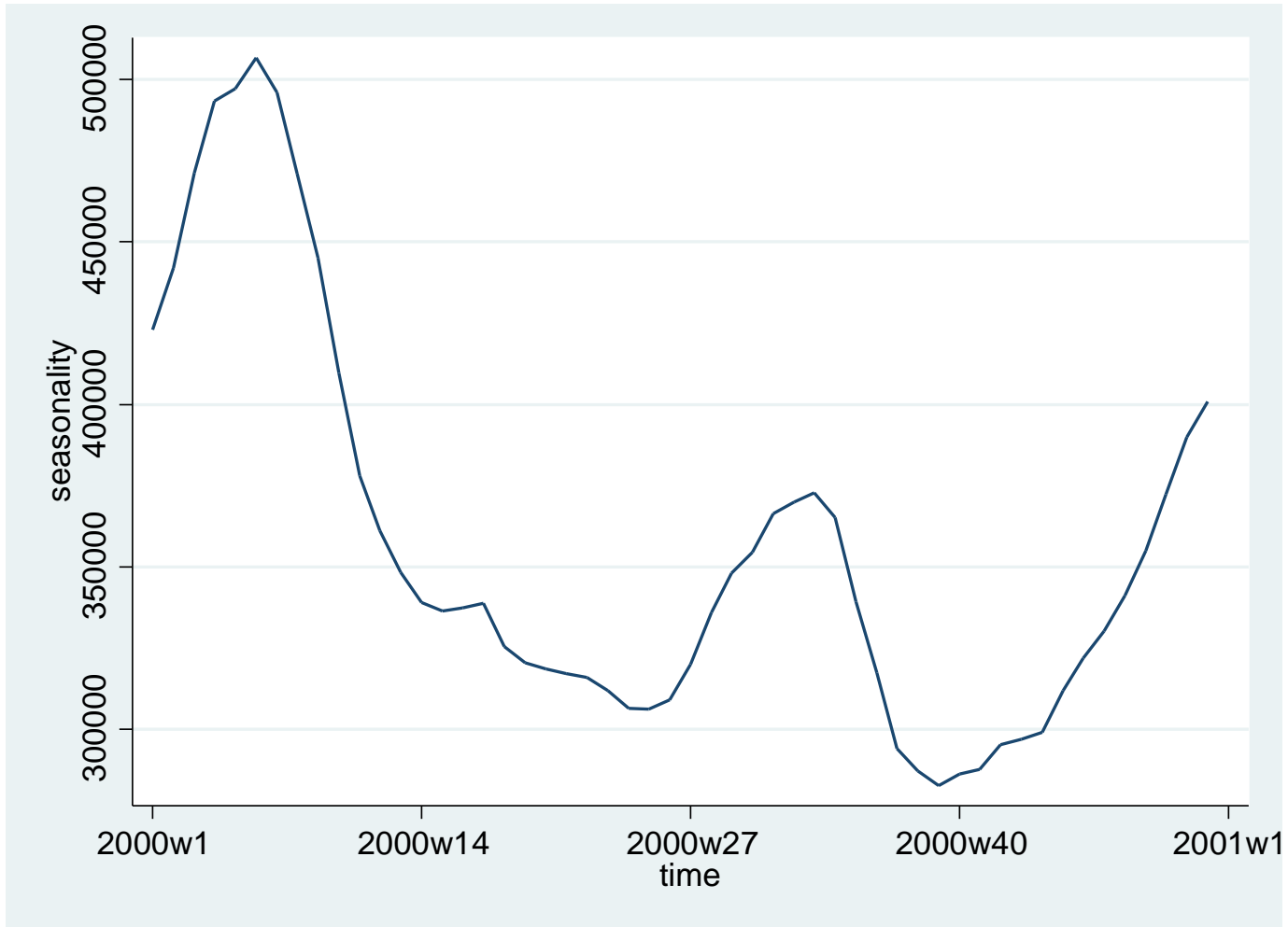
Estimation

. regress iclaims_nsa b52.w

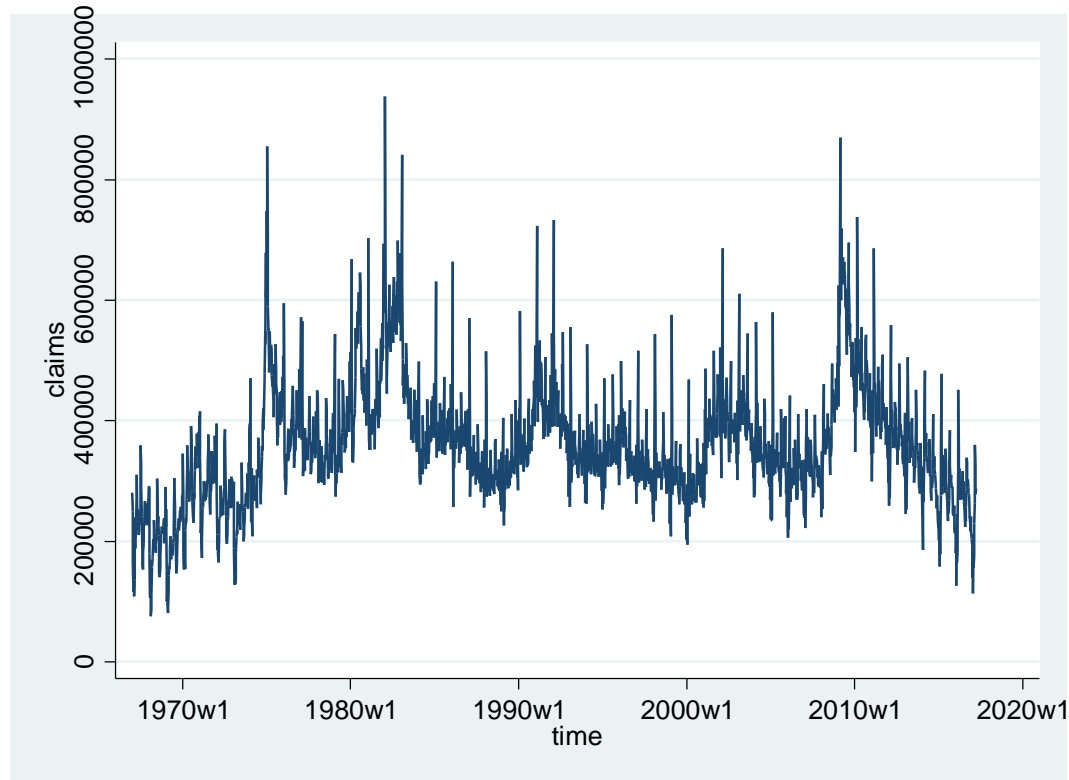
| Source | SS | df | MS | Number of obs = | 2238 |
|----------|------------|------|------------|-----------------|--------|
| Model | 1.2804e+13 | 51 | 2.5105e+11 | F(51, 2186) = | 29.09 |
| Residual | 1.8865e+13 | 2186 | 8.6297e+09 | Prob > F = | 0.0000 |
| Total | 3.1668e+13 | 2237 | 1.4157e+10 | R-squared = | 0.4043 |
| | | | | Adj R-squared = | 0.3904 |
| | | | | Root MSE = | 92896 |

| iclaims_nsa | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|-------------|-----------|-----------|-------|-------|----------------------|-----------|
| w | | | | | | |
| 1 | 35954.91 | 19920.38 | 1.80 | 0.071 | -3109.952 | 75019.78 |
| 2 | 168185.3 | 19920.38 | 8.44 | 0.000 | 129120.4 | 207250.2 |
| 3 | 11166.58 | 20034.54 | 0.56 | 0.577 | -28122.15 | 50455.32 |
| 4 | -71213.05 | 20034.54 | -3.55 | 0.000 | -110501.8 | -31924.31 |
| 5 | -77421.58 | 20034.54 | -3.86 | 0.000 | -116710.3 | -38132.85 |
| 6 | -70397.88 | 20034.54 | -3.51 | 0.000 | -109686.6 | -31109.15 |
| 7 | -125853.5 | 20034.54 | -6.28 | 0.000 | -165142.3 | -86564.8 |
| 8 | -148580.9 | 20034.54 | -7.42 | 0.000 | -187869.6 | -109292.2 |
| 9 | -142809.6 | 20034.54 | -7.13 | 0.000 | -182098.4 | -103520.9 |
| 10 | -142668.5 | 20034.54 | -7.12 | 0.000 | -181957.2 | -103379.8 |
| 11 | -167656.8 | 20034.54 | -8.37 | 0.000 | -206945.6 | -128368.1 |
| 12 | -178125.4 | 20034.54 | -8.89 | 0.000 | -217414.1 | -138836.7 |
| 13 | -187898.8 | 20034.54 | -9.38 | 0.000 | -227187.6 | -148610.1 |
| 14 | -157631.2 | 20034.54 | -7.87 | 0.000 | -196919.9 | -118342.5 |
| 15 | -144329.8 | 20034.54 | -7.20 | 0.000 | -183618.5 | -105041.1 |
| 16 | -171520.5 | 20034.54 | -8.56 | 0.000 | -210809.2 | -132231.8 |

Estimated Seasonal Process



DeSeasonalized



- Seasonality reduced, but not eliminated by dummy variables
- Indicates that seasonality has changed over time

Other types of seasonality

- Daily data
 - Day of the week
 - Handle by including dummy variables for each day
- High-frequency data
 - Include hourly or time-of-day indicators
- Holiday effects
 - Flower sales big on Valentines Day, Mothers Day, Easter, yet these days can move around
 - Trading-day/business-day variation
 - Number of trading days/business days varies across months
 - Can divide by number of trading days, or include as a regressor

Assignments

- Read Diebold through Chapter 6.4
- Problem Set # 4
 - Due Tuesday (2/14)
- Read Chapter 3 from *The Signal and the Noise*
 - Reading Reflection
 - Due Thursday (2/9)