Integration

• Orders of Integration Terminology
  – A series with a unit root (a random walk) is said to be integrated of order one, or $I(1)$
  – A stationary series without a trend is said to be integrated of order 0, or $I(0)$
  – An $I(1)$ series is differenced once to be $I(0)$
  – In general, we say that a series is $I(d)$ if its $d$’th difference is stationary.
Integrated of order d

- A series is I(d) if
  
  \[(1 - L)^d y_t = z_t\]

  is stationary and without trend.

- Examples
  - I(0): \[y_t = z_t\]
  - I(1): \[(1 - L)y_t = z_t\]
  - I(2): \[(1 - L)^2 y_t = z_t\]

- Possible I(2) series are price levels and money supply
Fractional Integration

• **Advanced side note!**
• We said a series is $I(d)$ if
  $$(1 - L)^d y_t = z_t$$
• We did not require $d$ to be an integer
• We say that $y$ is **fractionally integrated** if $0 < d < 1$ or $-1 < d < 0$
• A fractionally integrated series is in between $I(0)$ and $I(1)$
• Strong dependence, slow autocorrelation decay
• Popular model for asset return volatility.
Fractional Differencing

• The fractional differencing operator is an infinite series

\[(1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k\]

\[= \sum_{k=0}^{\infty} \frac{\prod_{a=0}^{k-1} (d - a) (-B)^k}{k!}\]

\[= 1 - dB + \frac{d(d-1)}{2} B^2 - \cdots .\]
Co-Integration

• We say that two series are co-integrated if a linear combination has a lower level of integration

• If $y$ and $x$ are each $I(1)$, yet $z = y - \theta x$ is $I(0)$

• Example: Term Structure
  – We saw before that $T_3$ appears to have a unit root
  – But the spread $T_{12} - T_3$ was stationary
  – $T_3$ and $T_{12}$ are co-integrated!
Common Co-Integration Relations

• Interest Rates of different maturities
• Stock prices and dividends
  – (Campbell and Shiller)
• Aggregate consumption and income
  – (Campbell and Shiller)
• Aggregate output, consumption, and investment
  – King, Plosser, Stock and Watson
Cointegrating Equation

• We said that $y$ and $x$ are cointegrated if

$$z_t = y_t - \theta x_t$$

is stationary

• This is called the cointegrating equation

• $\theta$ is the cointegrating coefficient

• In some cases, $\theta$ is known from theory
  – often $\theta=1$
Great Ratios

• If the aggregate variables $Y$ and $X$ are proportional in the long run, then

$$Z_t = \frac{Y_t}{X_t}$$

is stationary.

• Then

$$\log(Z_t) = \log(Y_t) - \log(X_t)$$

and

$$z_t = y_t - x_t$$

where $y=\log(Y)$ and $x=\log(X)$

• In this case, the logs $y$ and $x$ are cointegrated with coefficient 1.
Equilibrium Error

• The difference $z_t = y_t - \theta x_t$
  
is sometimes called the equilibrium error, as it measures the deviation of $y$ and $x$ from the long-term cointegrating relationship
Simulated Example
Scatter plot
Variables stay close to cointegration line
Granger Representation Theory

• If $y$ and $x$ are I(1) and cointegrated, then the optimal regression for $y$ takes the form

$$\Delta y_t = \mu + \gamma z_{t-1} + \alpha_1 \Delta y_{t-1} + \cdots + \alpha_p \Delta y_{t-p} + \beta_1 \Delta x_{t-1} + \cdots + \beta_q \Delta x_{t-q} + e_t$$

$$z_{t-1} = y_{t-1} - \theta x_{t-1}$$

• A dynamic regression in first differences, plus the error correction term $z$. 
Answer to spurious regression

• The reaction to spurious regression was:
  – If the series are I(1), then do regressions in differences

• Cointegration says:
  – Add the error correction $z$!

• The difference is critical
  – The variable $z$ measures if $y$ is high or low relative to $x$
  – The error-correction coefficient $\gamma$ pushes $y$ back towards the cointegration relationship
Origin of Cointegration

- British econometricians
  - Davidson, Hendry, Srba and Yeo (1978)
  - Suggested $\ln(C_t)-\ln(Y_t)$ was a valuable predictor of consumption growth $\Delta \ln(C_t)$
  - This puzzled Clive Granger, as he knew that the variables were I(1), so should not be in a regression
Theory of Cointegration

• This led Clive Granger to develop the theory of cointegration and the Granger Representation Theorem.
• The most influential statement was a co-authored paper with Robert Engle (1987).
• Granger and Engle shared the Nobel Prize in economics in 2003.
Cointegration Development

• Much of the statistical theory was developed by Peter Phillips and his students at Yale

• A multivariate statistical method was developed by the statistician Soren Johansen (U Copenhagen)

• Some jointly with the economist Katarina Juselius (Copenhagen)

• Their methods are programmed in STATA as VECM (vector error-correction models)
Example: Term Structure

- Regress change in 3-month T-bill on lagged spread, lagged changes in 3-month and 10-year
- Positive error correction coefficient
- Short rate increases when long rate exceeds short

```
. reg d.t3 L.spread120 L(1/12).d.t3 L(1/12).d.t120,r
```

| D.t3   | Coef.  | Robust Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|--------|--------|------------------|-------|------|---------------------|
| L1     | .0304828 | .0186479        | 1.63  | 0.103| -.0061351 to .0671007 |
Regression for Long Rate

- Long Rate decreases when long rate exceeds short

```
. reg d.t120 L.spread120 L(1/12).d.t3 L(1/12).d.t120,r
```

Linear regression

| Coef. | Robust Std. Err. | t | P>|t| | [95% Conf. Interval] |
|-------|------------------|---|------|---------------------|
| D.t120 |      |      |      |                     |
| spread120 L1. | -.021608 | .0114718 | -1.88 | 0.060 | -.0441345 .0009185 |
Unknown Cointegrating Coefficient

- If the cointegrating coefficient is unknown, it can be estimated

- Simplest estimator
  - Least squares of y on x
  - Consistent (Stock, 1987), but inefficient
  - Standard errors meaningless

```
. reg t3 t120
```

```
<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 684</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4663.61118</td>
<td>1</td>
<td>4663.61118</td>
<td>F( 1, 682) = 3240.27</td>
</tr>
<tr>
<td>Residual</td>
<td>981.579369</td>
<td>682</td>
<td>1.43926594</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>5645.19055</td>
<td>683</td>
<td>8.2652863</td>
<td>R-squared = 0.8261</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Adj R-squared = 0.8259</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Root MSE = 1.1997</td>
</tr>
</tbody>
</table>

| t3       | Coef.    | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|----------|----------|-----------|-------|-----|---------------------|
| t120     | 0.9708703| 0.0170557 | 56.92 | 0.000| 0.9373823 1.004358  |
| _cons    | -1.220776| 0.1175367 | -10.39| 0.000| -1.451554 -.9899991|
```
Dynamic OLS

- Stock and Watson (1994) proposed a simple efficient estimator called dynamic OLS (DOLS)
- Regress y on x and leads and lags of Dx
- Use Newey-West standard errors
  - Lag M = 0.75 * T^{1/3}
- newey t3 t120 L(-12/12).d.t120, lag(6)
Interest Rate Cointegration

```
.newey t3 t120 L(-12/12).d.t120, lag(6)
```

Regression with Newey-West standard errors
maximum lag: 6

| t3  | Coef.   | Newey-West Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|-----|---------|----------------------|-------|------|----------------------|
| t120 |        |                      |       |      |                      |
| ---  | .9514564 | .0377427            | 25.21 | 0.000 | .8773402            | 1.025573 |

- The estimated cointegrating coefficient is 0.95
- The confidence interval contains our expected value of 1
- So in this case using the value 1 is recommended.
Estimated Cointegrating Coefficient

• Otherwise, the regression can use the estimated equilibrium error

\[ z_{t-1} = y_{t-1} - \hat{\theta}x_{t-1} \]
Johansen VECM Method

- Alternatively, you can estimate the full VECM
- vec t3  t120, trend(constant) lags(12)
- This estimates a Vector Error Correction model with the variables T3 and T120, including a constant, and 12 lags of the variables
- This estimates equations for both variables, plus the cointegrating coefficient
Cointegrating Estimate

| beta   | Coef.  | Std. Err.  | z     | P>|z|  | [95% Conf. Interval] |
|--------|--------|------------|-------|-------|---------------------|
| _ce1   |        |            |       |       |                     |
| t3     | 1      |            |       |       |                     |
| t120   | -.9629871 | .0747933   | -12.88 | 0.000 | -1.109579 - .8163949 |
| _cons  | 1.198716 |            |       |       |                     |

- The estimate is .96, similar to DOLS (.95)
- The DOLS method is simpler, but many econometricians prefer the VECM estimate.
Evaluating Forecasts

• Are our forecasts good?
• How do we know?
• How do we assess a historical forecast?
• How do we compare competing forecasts?
Properties of Forecasts

• What are the properties of a good forecast?
• We start by examining optimal forecasts.
Linear Representation

• The Wold representation for $y$, $h$ steps out, is

$$ y_{n+h} = \mu + e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots $$

• The $h$-step-ahead optimal forecast is

$$ y_{n+h|n} = \mu + b_h e_n + b_{h+1} e_{n-1} + b_{h+2} e_{n-2} + \cdots $$

• The $h$-step-ahead optimal forecast error is

$$ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} $$
Optimal Forecast is Unbiased

• The forecast error is

\[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]

• It has expectation

\[ E(e_{n+h|n}) = 0 \]

• And thus the optimal forecast is unbiased
One-Step Errors are White Noise

• The one-step forecast error is

\[ e_{n+1|n} = e_{n+h} \]

• Which is unforecastable white noise

• Thus the optimal one-step-ahead forecast error is white noise and unforecastable
h-step-ahead errors are MA(h-1)

• The h-step forecast error is

\[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]

• This is a MA(h-1)

• Thus optimal h-step-ahead forecast errors are correlated, but at most a MA(h-1)
Forecast Variance

• The h-step forecast error is

\[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]

• Its variance is the forecast variance, and is

\[ \text{var}(e_{n+h|n}) = \left(1 + b_1^2 + b_2^2 + \cdots + b_{h-1}^2\right) \sigma^2 \]

• This is increasing in the forecast horizon \( h \)

• The variance of optimal forecasts increases with the forecast horizon
Unforecastable Errors

• The forecast errors should be unforecastable from all information available at the time of the forecast
• Not even the optimal forecast
• The coefficients should be zero in the regression

\[ e_{n+h|n} = \alpha + \beta y_{n+h|n} + \varepsilon_{n+h} \]
\[ \alpha = 0, \beta = 0 \]
Formal Comparison

• Since

\[ e_{n+h|n} = y_{n+h} - y_{n+h|n} \]

this implies

\[ y_{n+h} = \alpha + \beta y_{n+h|n} + e_{n+h|n} \]

\[ \alpha = 0, \beta = 1 \]

• The regression of the actual value on the ex-ante forecast should have a zero intercept and a coefficient of 1
Mincer-Zarnowitz Regression

• This is called a “Mincer-Zarnowitz” regression, proposed in a paper
  – “The evaluation of economic forecasts”
• Jacob Mincer (1922-2006)
  – Father of modern labor economics
• Victor Zarnowitz (1919-2009)
  – Leading figure in business cycle dating
Mincer-Zarnowitz Test

• Estimate the simple regression

\[ y_{n+h} = \alpha + \beta y_{n+h|n} + e_{n+h|h} \]

• Test the joint hypothesis

\[ \alpha = 0, \beta = 1 \]

• If the coefficients are different, it indicates systematic bias in the historical forecasts
Summary:
Properties of Optimal Forecasts

• Unbiased
• 1-step-ahead errors are white noise
• h-step-ahead errors are at most MA(h-1)
• Variance of h-step-ahead error is increasing in h
• Forecast errors should be unforecastable
Forecasting Average Growth

• When we are forecasting future growth, we may be interested in total future growth out to h periods

• For example, the growth rate of GDP during 2014

• This is the average of the growth rates during the four quarters 2014Q1, ..., 2014Q4
Average Growth

• If $y_t$ is the growth rate in period $t$, then the average future $h$-step growth is

$$
y_{n+1:n+h} = \frac{y_{n+1} + \cdots + y_{n+h}}{h}
$$

• The forecast of the average growth is

$$
y_{n+1:n+h|n} = \frac{y_{n+1|n} + \cdots + y_{n+h|n}}{h}
$$

• What are its properties?
Average Forecast Error

• The error of the average forecast is

\[
e_{n+1:n+h|n} = y_{n+1:n+h|n} - y_{n+1:n+h} \\
= \frac{y_{n+1|n} + \cdots + y_{n+h|n}}{h} - \frac{y_{n+1} + \cdots + y_{n+h}}{h} \\
= \frac{\left( y_{n+1|n} - y_{n+1} \right) + \cdots + \left( y_{n+h|n} - y_{n+h} \right)}{h} \\
= \frac{e_{n+1|n} + \cdots + e_{n+h|n}}{h}
\]

• Which is the average of the 1-step through h-step errors
Average Forecast Error Variance

• Since the average forecast error is the average of forecast errors, it has a smaller variance than the h-step variance

\[
\text{var}(e_{n+1:n+h|n}) = \text{var}\left(\frac{e_{n+1|n} + \cdots + e_{n+h|n}}{h}\right) \leq \text{var}(e_{n+h|n})
\]

• So multi-period growth rate forecasts will have smaller variance than h-step ahead growth forecasts
  – The forecasted average growth rate for 2010 has a smaller variance than the forecasted growth rate for 2010Q4
Evaluating Forecasts

• Suppose we have a sequence of real forecasts
• Perhaps they are our own forecasts
• How can we evaluate the forecasts?
Measures of Forecast Performance

• Form the historical sequence of forecasts and actual values.
• Construct the forecast error as the difference
Example

• CBO’s Economic Forecasting Record: 2009 Update

• Economic forecasts made by
  – Congressional budget office (CBO)
  – U.S. Administration
  – Private forecasters
    • Blue Chip average
  – CBO regularly assesses their forecasts
CBO Comparison

- Real Output
- Nominal Output
- Inflation
- 3-month T-Bill rate
- 10-year Treasury note rate
- Difference between CPI and GDP inflation
- Both 2-year and 5-year forecasts
### Table 4.

Comparison of CBO's, *Blue Chip*'s, and the Administration's Forecasts of Two-Year Average Growth Rates for Nominal Output

(Percent, by calendar year)

<table>
<thead>
<tr>
<th></th>
<th>CBO Actual</th>
<th>CBO Forecast</th>
<th>CBO Error²</th>
<th>Blue Chipb Forecast</th>
<th>Blue Chipb Error²</th>
<th>Administration Forecast</th>
<th>Administration Error²</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1976–1977</td>
<td>11.5</td>
<td>13.1</td>
<td>1.7</td>
<td>*</td>
<td>*</td>
<td>12.3</td>
<td>0.8</td>
</tr>
<tr>
<td>1977–1978</td>
<td>12.1</td>
<td>10.8</td>
<td>-1.3</td>
<td>*</td>
<td>*</td>
<td>11.2</td>
<td>-1.0</td>
</tr>
<tr>
<td>1978–1979</td>
<td>12.5</td>
<td>10.9</td>
<td>-1.6</td>
<td>*</td>
<td>*</td>
<td>11.2</td>
<td>-1.3</td>
</tr>
<tr>
<td>1979–1980</td>
<td>10.4</td>
<td>11.0</td>
<td>0.5</td>
<td>*</td>
<td>*</td>
<td>10.4</td>
<td>-0.1</td>
</tr>
<tr>
<td>1980–1981</td>
<td>10.4</td>
<td>9.7</td>
<td>-0.7</td>
<td>*</td>
<td>*</td>
<td>9.5</td>
<td>-0.8</td>
</tr>
<tr>
<td>1981–1982</td>
<td>8.0</td>
<td>12.1</td>
<td>4.1</td>
<td>*</td>
<td>*</td>
<td>11.9</td>
<td>4.0</td>
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<tr>
<td>1982–1983</td>
<td>6.3</td>
<td>9.7</td>
<td>3.4</td>
<td>9.5</td>
<td>3.2</td>
<td>9.8</td>
<td>3.5</td>
</tr>
<tr>
<td>1983–1984</td>
<td>9.8</td>
<td>8.2</td>
<td>-1.6</td>
<td>9.0</td>
<td>-0.9</td>
<td>8.0</td>
<td>-1.8</td>
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<tr>
<td>1984–1985</td>
<td>9.0</td>
<td>9.9</td>
<td>0.9</td>
<td>9.6</td>
<td>0.6</td>
<td>9.6</td>
<td>0.6</td>
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<tr>
<td>1985–1986</td>
<td>6.2</td>
<td>7.6</td>
<td>1.3</td>
<td>7.4</td>
<td>1.2</td>
<td>8.2</td>
<td>1.9</td>
</tr>
<tr>
<td>1986–1987</td>
<td>5.8</td>
<td>7.1</td>
<td>1.3</td>
<td>6.7</td>
<td>0.9</td>
<td>7.7</td>
<td>1.8</td>
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<tr>
<td>1987–1988</td>
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<td>-0.5</td>
<td>6.4</td>
<td>-0.5</td>
<td>6.9</td>
<td>-0.1</td>
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<tr>
<td>1988–1989</td>
<td>7.6</td>
<td>6.3</td>
<td>-1.3</td>
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<td>-1.5</td>
<td>6.8</td>
<td>-0.9</td>
</tr>
<tr>
<td>1989–1990</td>
<td>6.7</td>
<td>6.8</td>
<td>0.1</td>
<td>6.6</td>
<td>-0.1</td>
<td>7.1</td>
<td>0.4</td>
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<tr>
<td>1990–1991</td>
<td>4.6</td>
<td>6.1</td>
<td>1.5</td>
<td>6.0</td>
<td>1.4</td>
<td>7.1</td>
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<tr>
<td>1991–1992</td>
<td>4.4</td>
<td>5.7</td>
<td>1.3</td>
<td>5.2</td>
<td>0.8</td>
<td>5.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>
Comparison

• By showing the actual, forecasts, and forecast errors side-by-side, we can informally see which forecast performs better
Formal Comparison

• The forecasts can be compared by estimating the **bias** and **risk** (expected loss) of the forecasts

• They are estimated from $R$ forecast errors:
  
  – Bias, Mean Absolute Error, Root Mean Squared Error

  \[
  \text{Bias} = \frac{1}{R} \sum_{n=1}^{R} e_{n+h|n} \\
  \text{MAE} = \frac{1}{R} \sum_{n=1}^{R} \left| e_{n+h|n} \right| \\
  \text{RMSE} = \left( \frac{1}{R} \sum_{n=1}^{R} e_{n+h|n}^2 \right)^{1/2}
  \]
# CBO Comparison

## Table 1.

**Summary Measures of Performance for Two-Year Average Forecasts**

(Percentage points)

<table>
<thead>
<tr>
<th></th>
<th>CBO</th>
<th>Blue Chip&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Administration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Growth Rate for Real Output (1982-2007)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>1.2</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>Growth Rate for Nominal Output (1982-2007)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>1.0</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>1.2</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Inflation in the Consumer Price Index (1982-2007)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>0.7</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>
### Table 2.

<table>
<thead>
<tr>
<th>Summary Measures of Performance for Five-Year Average Projections</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Percentage points)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Growth Rate for Real Output (1979-2004)</strong></td>
</tr>
<tr>
<td>Mean error</td>
</tr>
<tr>
<td>Mean absolute error</td>
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<tr>
<td>Root-mean-square error</td>
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<td><strong>Growth Rate for Nominal Output (1982-2004)</strong></td>
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<td>Mean error</td>
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<tr>
<td>Mean absolute error</td>
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<tr>
<td>Root-mean-square error</td>
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<td>Mean absolute error</td>
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Data Revision

• A major difficulty with forecast evaluation is that for many series, there are serious data revisions
• The data used for forecasting, and the series published today, are different
• The series forecasted, and the series reported today, are different
• Price series, and real series based on price levels, are rebased every few years
• These rebasing are not scale transformations, because the construction of real output is done at a disaggregate level, and then aggregated.
## Real Output

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Meese-Rogoff Puzzle

• The most influential paper using the method of forecast model comparison is
  – “Empirical exchange rate models of the seventies”
  – Richard Meese and Kenneth Rogoff
Meese-Rogoff

- Ken Rogoff (currently Harvard)
  - Recent book
  - *This Time is Different: Eight Centuries of Financial Folly*
- Dick Meese (formerly Berkeley, now Barclay Global Investors)
  - 1978 UW Ph.D.
  - Economics Dept Advisory Board
Meese-Rogoff paper

• They compare the RMSE and bias of 1-month, 6-month and 12-month forecasts of a set of exchange rates, using structural models
• They compare the performance of the economic models with the performance of a random walk
• They found the random walk beat the economic models
• Very influential paper
### Root mean square forecast errors.

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<th>Forward rate</th>
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<th>Vector autoregression</th>
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<th>Dornbusch-Frankel&lt;sup&gt;b&lt;/sup&gt;</th>
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Summary

• Evaluation of forecasts achieve by comparing the bias, MAE and RMSE of forecast errors
• Most influential paper is Meese-Rogoff, because they showed that naïve random walk model has lower forecast risk than structural economic models
• This established a challenge for economic modeling and forecasting.
  – Can we beat simple naïve models?!