On the Identification of Production Functions: How Heterogeneous is Productivity?

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Abstract

We show that existing practices for estimating production functions suffer from a fundamental non-identification problem due to flexible inputs, such as intermediate inputs. Using a transformation of the firm's first order condition, we develop a new identification strategy and propose a simple nonparametric estimator for the production function and productivity. We show that the alternative of approximating the effects of intermediate inputs using a value-added production function does not solve the identification problem. Applying our approach to plant-level data from Colombia and Chile, we find that a gross output production function implies fundamentally different patterns of productivity heterogeneity than a value-added specification.

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1 Introduction

The identification and estimation of production functions using data on inputs and output is among the oldest empirical problems in economics. As first pointed out by Marschak and Andrews (1944), a key challenge for identification arises because a firm's productivity is transmitted to the firm's optimal choice of inputs, giving rise to an endogeneity issue known in the production function literature as "transmission bias" (see e.g., Griliches and Mairesse, 1998, henceforth GM). Standard econometric solutions to correct the transmission bias, i.e., using firm fixed effects or input prices as instruments, have proven to be both theoretically problematic and unsatisfactory in practice (see e.g., GM and Ackerberg et al., 2007 for a review and Section 4 for a discussion).

The more recent literature on production function estimation attempts to address transmission bias by placing assumptions on the economic environment to allow researchers to exploit lagged input decisions as instruments for current inputs. This strategy is fundamental to both of the main strands of structural estimation approaches, namely dynamic panel methods (Arellano and Bond, 1991; Blundell and Bond, 1998, 2000) as well as the proxy variable methods (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves, and Frazer, 2006, henceforth OP, LP, and ACF, respectively) that are now prevalent in the applied literature on production function estimation.

Our first contribution in this paper is to show that these structural estimation methods face a fundamental identification problem when the production function contains flexible inputs, i.e., inputs that are variable in each period and have no dynamic implications, which is how intermediate inputs (raw materials, energy, and services) are typically modeled. The reason is intuitive: because flexible inputs are static decision variables, lagged input decisions do not directly affect current choices.

Although the idea that flexible inputs may pose an identification problem has been suggested previously (see Mendershausen, 1938; Marschak and Andrews, 1944; Bond and Söderbom, 2005; and ACF), the exact nature of the problem has not been formalized. In this paper we provide a formal proof of nonparametric non-identification under the standard model of production used in the literature.[1] As we preview below, the resulting problems for empirical work are much more severe than has been previously appreciated.

Our second contribution is that we present a new nonparametric identification strategy that solves the problem associated with flexible inputs in the production function. The key to our approach is that we exploit the information about the production function that is contained in the firm's first order condition for a flexible input. In parametric examples, such as Cobb-Douglas production functions, this basic idea dates back to at least Klein (1953) and Solow (1957).[2] Our key innovation is that we show that the information in the first order condition can be used in a completely nonparametric way, i.e, without making functional form assumptions on the production function. Thus we provide the first nonparametric identification result for the production function (of which we are aware) under the standard model in the literature (as described in Section 2). There are two main steps in our result.

In the first step we transform the flexible input's first order condition to nonparametrically identify both the flexible input's elasticity of production and ex-post shocks to output from the observed revenue share of that input. These are both identified from the nonparametric regression of the intermediate input's revenue share on all inputs (labor, capital, and intermediate inputs). Intuitively, when forming the revenue share of the flexible input, we difference out the component of productivity responsible for endogeneity. Since this term effectively cancels out, this allows us to identify the effect of observed changes in the flexible input on output even without independent exogenous variation in the flexible input. This is a nonparametric analogue of revenue shares directly identifying the intermediate input coefficient in a Cobb-Douglas setting. Moreover, the residual from this regression identifies ex-post shocks to production, and is thus also analogous to the first stage regression in ACF, except that we

[1]The model described in Section 2 contains the key components of the model in OP that underlies much of the subsequent empirical work on production function and productivity estimation. It also shares major elements with the models of production in the economics of education literature, e.g., Hanushek (1971), Todd and Wolpin (2003), and Cunha, Heckman, and Schennach (2010).

[2]In the simplest case of a Cobb-Douglas production function, the revenue share of a flexible input identifies that input's coefficient in the production function.

not only identify ex-post shocks to output but also the flexible input's elasticity.

In the second step, we recognize that the flexible input elasticity defines a partial differential equation on the production function, which imposes nonparametric cross-equation restrictions with the production function itself. We can solve this partial differential equation to nonparametrically identify the part of the production function that depends on the flexible input. We then combine this with conditional moment restrictions based on lagged input decisions for the remaining inputs (as used in the structural estimation methods discussed above), in order to nonparametrically identify both the rest of the production function and productivity. Thus, our key contribution is to show that the first order condition implies nonparametric cross-equation restrictions that can be exploited in a way that does not rely on the researcher *a priori* knowing the form of the production function.

The common empirical practice in the literature of estimating a value-added production function is a potential reason for why the identification problem discussed above has gone unrecognized. In order to analyze the effects of just the primary inputs (capital and labor) on the output of the firm, value added involves subtracting the value of intermediate inputs from gross output and redefining the object of interest to be a *value-added* production function. Since, in theory, the remaining function only depends on capital and labor, there is seemingly no identification problem associated with intermediate inputs, and the structural estimation methods mentioned above can be employed.

Our third contribution is to show that this logic is not correct. We first show that the use of value added does not solve the fundamental identification problem associated with intermediate inputs. While value added is a well-defined concept, it cannot be used to identify features of interest (including productivity) from the underlying gross output production function under the standard model of production described in Section 2. Determining the empirical relevance of the misspecification arising from using value added is ultimately an empirical question, and a significant one,[3] since recovering productivity at the firm level is critical to addressing a

[3]There is a large literature that has generated several stylized facts about heterogeneity of productivity at the firm level. Among these are the general understanding that even narrowly defined industries exhibit "massive"

wide of range of economic policy issues.[4]

In order to investigate this further, we then apply our identification strategy to plant-level data from Colombia and Chile to study the underlying patterns of productivity under gross output compared to value-added specifications. We find that productivity differences become orders of magnitude smaller and sometimes even change sign when we analyze the data via gross output rather than value added. For example, the standard 90/10 productivity ratio taken among all manufacturing firms in Chile is roughly 9 under value added (meaning that the 90th percentile firm is 9 times more productive than the 10th percentile firm), whereas under our gross output estimates this ratio falls to 2. Moreover, these dispersion ratios exhibit a remarkable degree of stability across industries and across the two countries when measured via gross output, but exhibit much larger cross-industry and cross-country variance when measured via value added. We further show that value added mismeasures in an economically significant way the productivity premium of firms that export, firms that import, firms that advertise, and higher wage firms as compared to gross output.

Our findings illustrate the empirical importance of the misspecification introduced by using value added and emphasize the empirical relevance of our identification strategy for gross output production functions. Our empirical results suggest that the bias introduced from using value added is at least as important, if not more so, than the transmission bias that has been the main focus of the production function estimation literature to date.

The rest of the paper is organized as follows. In Section 2 we describe the model and characterize the identification problem caused by flexible inputs. In Section 3 we present our nonparametric identification and estimation strategy. Section 4 compares our approach to the related literature. Section 5 shows that the use of value added does not solve the identification

unexplained productivity dispersion (Dhrymes, 1991; Bartelsman and Doms, 2000; Syverson, 2004; Collard-Wexler, 2010; Fox and Smeets, 2011), and that productivity is closely related to other dimensions of firm-level heterogeneity, such as importing (Kasahara and Rodrigue, 2008), exporting (Bernard and Jensen, 1995, Bernard and Jensen, 1999, Bernard et al., 2003), wages (Baily, Hulten, and Campbell, 1992), etc. See Syverson (2011) for a review of this literature.

[4]Examples are the impact of trade liberalization on productivity (Pavcnik, 2002), the importance of the misallocation of resources (Hsieh and Klenow, 2009), and the effect of R&D and innovation (Doraszelski and Jaumandreu, 2013), to name just a few.

problem. In Section 6 we describe the Colombian and Chilean data and show the results comparing gross output to value added for productivity measurement. In particular, we show evidence of large differences in unobserved productivity heterogeneity suggested by value added relative to gross output. Section 7 concludes with an example of the policy relevance of our results.

2 The Identification Problem

2.1 The Model

For the sake of concreteness we adopt the economic model of production used in the structural "proxy variable" approach to estimating production functions (OP/LP/ACF), which has become a widely used approach to estimating production functions and productivity in applied work. One of the main points in this paper is to demonstrate that despite the ubiquity of these approaches in empirical work, they are insufficient for identification of the production function and productivity.[5] The economic model consists of three basic components: 1) the structure of the production function 2) the evolution of productivity and 3) the timing of input decisions.

We observe a panel consisting of firms $j = 1, \ldots, J$ over periods $t = 1, \ldots, T$.[6] The firm's output, labor, capital, and intermediate inputs will be denoted by $(Y_{jt}, L_{jt}, K_{jt}, M_{jt})$ respectively. Their log values will be denoted by $(y_{jt}, l_{jt}, k_{jt}, m_{jt})$. We assume that the relationship between output and inputs is determined by an underlying production function F_t, which is allowed to vary over time, and a Hicks neutral productivity shock ν_{jt}.

[5] The identification problem we isolate also applies to the dynamic panel approach to production function estimation following Arellano and Bond (1991); Blundell and Bond (1998, 2000). We draw an explicit comparison to the dynamic panel literature in Section 4.3.

[6] We assume a balanced panel both for notational simplicity and because the empirical literature has found little evidence that selection due to attrition makes much of a difference in practice (see OP, GM, and LP).

Assumption 1. *The relationship between output and the inputs takes the form*

$$Y_{jt} = F_t\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\nu_{jt}}.$$

The Hick's neutral productivity shock ν_{jt} is decomposed as $\nu_{jt} = \omega_{jt} + \varepsilon_{jt}$. The next assumption discusses the properties of each and their economic significance. Let $\mathcal{I}_{jt}$ denote the information available to the firm for making period t decisions.

Assumption 2. $\omega_{jt} \in \mathcal{I}_{jt}$ *is a persistent productivity shock. In particular,* ω_{jt} *is Markovian so that its distribution can be written as* $P_\omega\left(\omega_{jt} \mid \mathcal{I}_{jt-1}\right) = P_\omega\left(\omega_{jt} \mid \omega_{jt-1}\right)$. *The ex-post shock* $\varepsilon_{jt} \notin \mathcal{I}_{jt}$ *is drawn from a distribution* $P_\varepsilon\left(\varepsilon_{jt} \mid \mathcal{I}_{jt}\right) = P_\varepsilon\left(\varepsilon_{jt}\right)$.

It follows that we can express $\omega_{jt} = h(\omega_{jt-1}) + \eta_{jt}$, where η_{jt} satisfies $E\left[\eta_{jt} \mid \omega_{jt-1}\right] = 0$ and can be interpreted as the "innovation" to the firm's persistent productivity ω_{jt} in period t. It also follows that η_{jt} is independent of $\mathcal{I}_{jt-1}$ conditional on ω_{jt-1}, and this conditional independence property will play an important role in what follows.[7] Without loss of generality, we can normalize $E\left[\varepsilon_{jt} \mid \mathcal{I}_{jt}\right] = E\left[\varepsilon_{jt}\right] = 0$, which is in units of log output. The expectation of the ex-post shock, in units of the level of output, thus becomes a free parameter which we denote as $\mathcal{E} = E\left[e^{\varepsilon_{jt}} \mid \mathcal{I}_{jt}\right] = E\left[e^{\varepsilon_{jt}}\right]$.[8]

Following GM, we distinguish inputs based upon how they are adjusted by the firm over time. We classify inputs as flexible inputs or quasi-fixed inputs. Quasi-fixed inputs are subject to adjustment frictions (e.g., time-to-build, fixed investment costs, hiring/firing costs).[9] Flexible inputs are static inputs that the firm can freely adjust in each period. They have no dynamic implications, i.e., their period t levels do not affect the firm's profit in future periods.

The significance of this separation is two-fold. First, quasi-fixed inputs generate a source

[7] It is straightforward to allow the distribution of $P_\omega\left(\omega_{jt} \mid \mathcal{I}_{jt-1}\right)$ to depend upon other elements of $\mathcal{I}_{jt-1}$, such as firm export or import status, R&D, etc. In these cases ω_{jt} becomes a controlled Markov process from the firm point of view. See Kasahara and Rodrigue (2008) and Doraszelski and Jaumandreu (2013) for examples.

[8] See Goldberger (1968) for an early discussion of the implicit reinterpretation of results that arises from ignoring $\mathcal{E}$ (i.e., setting $\mathcal{E} \equiv E\left[e^{\varepsilon}\right] = 1$ while simultaneously setting $E\left[\varepsilon\right] = 0$) in the context of Cobb-Douglas production functions.

[9] See ACF and Ackerberg et al. (2007) for a more in-depth discussion of these frictions.

of exogenous variation based upon lagged decisions for these inputs. Second, flexible inputs can be used to proxy for productivity (as in LP and ACF). These two ingredients are central to the proposed identification strategy of the modern literature, and we capture them in the next two assumptions.

Assumption 3. *K_{jt} and L_{jt} are quasi-fixed and determined at or prior to period $t-1$. M_{jt} is determined flexibly at period t.*

Assumption 3 employs the "adjustment lag" or "time to build" approach for concreteness of exposition and because of its prominent role in the literature. Our analysis is amenable to many alternative assumptions about adjustment frictions that can be made, such as adjustment costs (e.g., hiring/firing costs).

Together, Assumptions 2 and 3 imply the following:

$$E\left[\eta_{jt} + \varepsilon_{jt} \mid L_{jt}, K_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}, ..., L_1, K_1, M_1\right] = 0. \tag{1}$$

These conditional moment restrictions have been used in the structural estimation literature to form moments for estimation.[10]

Assumption 4. *Intermediate inputs, M_{jt}, can be written as $M_{jt} = \mathbb{M}_t\left(L_{jt}, K_{jt}, \omega_{jt}\right)$, where $\mathbb{M}_t$ is strictly monotone in ω_{jt} for any (L_{jt}, K_{jt}).*

Although this assumption is usually justified by noting that it follows from the cost minimization/profit maximization of the firm under fairly weak conditions on the production function, it is typically invoked on its own (i.e., without direct reference to the optimizing behavior of the firm).

The influential insight of LP and the further development by ACF shows that, as a consequence of Assumption 3, $\mathbb{M}_t$ can be inverted to yield $\omega_{jt} = \mathbb{M}_t^{-1}\left(L_{jt}, K_{jt}, M_{jt}\right)$. As a result,

[10]Only capital was taken to be quasi-fixed in the original OP/LP articles. However, ACF demonstrated that, given the other assumptions of the model and absent other variation, it is necessary that labor also be subject to some adjustment friction for identification purposes.

observed inputs can be used to "proxy" for unobserved productivity ω_{jt}, which is the basis for the proxy variable terminology.[11]

Assumptions 1-4 now represent the standard restrictions used by the extensive literature on the estimation of production functions and productivity based on variants of the OP/LP/ACF methodology. These assumptions have been used to justify identification strategies for dealing with the correlation between observed inputs and unobserved productivity (transmission bias). In the next section we show that in the environment described by these assumptions, the production function f_t and productivity ν_{jt} are still nonparametrically not identified due to the presence of flexible inputs, M_{jt}.

2.2 The Identification Problem with Flexible Inputs

The presence of flexible inputs has been previously recognized in the literature as a potential source of identification problems (see Mendershausen, 1938; Marschak and Andrews, 1944; Bond and Söderbom, 2005; and ACF). Here we present a formal argument that the production function is nonparametrically not identified in the presence of flexible inputs under Assumptions 1-4.

Consider a population of firms where the joint distribution of the observables (Y_{jT},L_{jT}, K_{jT},M_{jT},Y_{jT-1},...,Y_{j1},...,M_{j1}) among firms in the population is identified in the data. The relationship between output and input can be expressed as

$$Q_{jt} = F_t(L_{jt}, K_{jt}, M_{jt})e^{\omega_{jt}}, \tag{2}$$

$$Y_{jt} = Q_{jt}e^{\varepsilon_{jt}}. \tag{3}$$

Expressed in logs, equation (3) becomes

$$y_{jt} = f_t(L_{jt}, K_{jt}, M_{jt}) + \omega_{jt} + \varepsilon_{jt}, \tag{4}$$

[11]OP employ an equivalent monotonicity assumption for investment in physical capital. The identification problem we study in Section 2.2 applies to both proxy variable approaches.

where $f_t = \ln F_t$. The empirical problem is to identify the function f_t, which allows us to identify the joint distribution of productivity $\omega_{jt} + \varepsilon_{jt} = y_{jt} - f_t(L_{jt}, K_{jt}, M_{jt})$ and the other firm observables.

The literature has proceeded to estimate production functions seemingly on the basis that Assumptions 2-4 generate enough sources of exogenous variation to identify the production function f_t with instrumental variables. Let $\Gamma_{jt} = (L_{jt}, K_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}, \ldots, L_{j1}, K_{j1}, M_{j1})$ denote the $3\,(t-1)+2$ available exogenous variables (with respect to $\eta_{jt}+\varepsilon_{jt}$). Despite this apparent abundance of instruments, the structural model f_t is not identified. We formally now state

Theorem 1. *Given Assumptions 1-4, for any production function f_t there exists an alternative production function $\bar{f}_t$ such that for any constant c, $\bar{f}_t \neq f_t + c$, and such that $\bar{f}_t$ generates the same reduced form $E\left[y_{jt} \mid \Gamma_{jt}\right]$ as f_t with positive probability. Hence, the production function f_t is not identified (up to a constant) from the reduced form.*

The formal proof of the theorem is included in Appendix A. The intuition for why the production function is not identified can be seen by considering both (4) alongside the intermediate inputs equation in Assumption 4, which gives us a (nonparametric) triangular simultaneous equation system. We can write $\omega_{jt-1} = \mathbb{M}_{t-1}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)$ to rewrite the production function as

$$y_{jt} = f_t\left(L_{jt}, K_{jt}, M_{jt}\right) + h\left(\underbrace{\mathbb{M}_{t-1}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)}_{\omega_{jt-1}}\right) + \eta_{jt} + \varepsilon_{jt}, \tag{5}$$

and form the (identifiable) conditional expectation of y_{jt} on Γ_{jt}

$$E\left[y_{jt}|\Gamma_{jt}\right] = E\left[f_t\left(L_{jt}, K_{jt}, M_{jt}\right)|\Gamma_{jt}\right] + h_t\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right),$$

where $h_t\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right) \equiv h\left(\mathbb{M}_{t-1}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)\right)$.

The key is that since η_{jt} is independent of Γ_{jt} as a consequence of Assumption 2,

$E\left[f_t\left(L_{jt}, K_{jt}, M_{jt}\right)|\Gamma_{jt}\right]$ will just depend on $\left(L_{jt}, K_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}\right)$. Consequently, there are only five exogenous variables that can be used to vary each of the six coordinates of these functions (f_t and h_t) independently.

In other words, since $M_{jt} = \mathbb{M}_t\left(L_{jt}, K_{jt}, h\left(\mathbb{M}_{t-1}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)\right) + \eta_{jt}\right)$, conditional on $L_{jt}, K_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}$, the only source of variation left for M_{jt} is the unobservable η_{jt} which is also in the residual of the production function. Thus the flexible input lacks an instrument from outside of the production function. Its only sources of exogenous variation are inputs that are included on the right hand side of equation (5). It is this lack of an exclusion restriction that creates the fundamental identification problem.[12] The proof of the theorem is based on a formalization of these ideas.[13,14]

In light of Theorem 1, estimating the model described by Assumptions 1-4 by using "flexible parametric approximations" to the production function is not a valid procedure. The parametric assumptions employed cannot be seen as flexible approximations but rather structural assumptions on the shape of the economic primitives, and researchers will generally have little basis for imposing these restrictions. Our Theorem 1 implies that Assumptions 1-4 alone do not allow researchers to identify the shape of the production function from data. In so far as the imposed parametric restrictions do not hold, and there is little *a priori* reason

[12]Intermediate input prices have been suggested as a potential external instrument to address this identification problem. In Section 4.1 we provide a more detailed discussion of the use of prices as exclusion restrictions, as well as the potential problems associated with this approach.

[13]It may be possible to achieve identification in the absence of exclusion restrictions by imposing additional restrictions. One example is using heteroskedasticity restrictions (see e.g., Rigobon, 2003; Klein and Vella, 2010; and Lewbel, 2012), although these approaches require explicit restrictions on the form of the error structure. We thank an anonymous referee for pointing this out. We are not aware of any applications of these ideas in the production function setting.

[14]Note that the procedure proposed by Wooldridge (2009) that is now widely used in the literature – under the premise that the moments it exploits solves the identification problem raised by ACF – is in fact nonparametrically not identified. The estimating equation used in Wooldridge is

$$y_{jt} = \alpha l_{jt} + \beta k_{jt} + \gamma m_{jt} + h\left(\underbrace{\mathbb{M}_{t-1}^{-1}\left(l_{jt-1}, k_{jt-1}, m_{jt-1}\right)}_{\omega_{jt-1}}\right) + \eta_{jt} + \varepsilon_{jt}.$$

where (l_{jt}, k_{jt}) and lagged values of $(l_{jt-k}, m_{jt-k}, k_{jt-k})$ for $k \geq 1$ are used as instruments. In this parametric example, the problem is in identifying γ. As our Theorem 1 shows, m_{jt} lacks any exogenous variation after conditioning on $(l_{jt}, k_{jt}, l_{jt-1}, k_{jt-1}, m_{jt-1})$ and the model is nonparametrically (semi-parametrically in this example) not identified.

they should, the estimator will no longer be consistent and can generate misleading inferences about the production function and productivity (see Manski, 2003; Roehrig, 1988; and Matzkin, 2007 for more detail). Furthermore, as we show in Appendix B, for the case of the commonly-employed Cobb-Douglas form, even imposing structural parametric assumptions is not necessarily sufficient to solve the identification problem.

In the next section, we show that this fundamental identification problem can be solved by exploiting the full power of the firm's optimizing behavior. While Assumption 4 is one implication of the optimizing behavior of the firm, the economics of the firm's problem implies a first-order condition for flexible inputs, which contains important nonparametric information about the production function, not only about the unobservables (ω and ε).

3 Nonparametric Identification via First Order Conditions

We focus attention in the main body on the classic case of perfect competition in the intermediate input and output markets. The perfect competition case makes our proposed solution to the identification problem caused by intermediate inputs particularly evident. In Online Appendix O1, we show that under imperfect competition the same identification problem arises and our approach can be extended to handle this case.

Let ρ_t denote the intermediate input price and P_t denote the output price facing all firms in period t. Since capital and labor are determined prior to period t and the choice of intermediate inputs does not have any dynamic implications, the prices of capital and labor are not relevant for the choice of intermediate inputs M_{jt}. Thus $(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t)$ is the vector of variables relevant for the firm's choice of M_{jt}. The first order condition with respect to M_{jt} yields,

$$P_t F_{M,t}\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}} \mathcal{E} = \rho_t, \tag{6}$$

where $F_{M,t}$ denotes the partial derivative of F_t with respect to M and $\mathcal{E} = E\left[e^{\varepsilon_{jt}}\right]$. Thus M_{jt}

is an implicit function of $(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t)$, i.e.,

$$M_{jt} = \widetilde{\mathbb{M}}_t(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t) = \mathbb{M}_t(L_{jt}, K_{jt}, \omega_{jt}). \tag{7}$$

Recall that the source of the identification problem is that the intermediate inputs demand, equation (7), does not include any source of variation excluded from the production function. By just imposing Assumption 4, we are failing to use all the information provided by the economics of the problem via the firm's first order condition in equation (6). Our solution to this problem is based on recognizing that despite this fundamental lack of variation, $\mathbb{M}_t$ is actually an implicit function of the elasticity of intermediate inputs (and hence of the production function itself) as we show below. We make this implicit relationship an explicit one by transforming the first order condition to identify the intermediate inputs elasticity and the ex-post shock ε_{jt} nonparametrically, which fills the void left by the lack of an exclusion restriction in Theorem 1. The additional information that we derive from the underlying economics of the intermediate inputs equation (which the proxy variable approach does not exploit) can be combined with the standard timing assumptions (Assumption 3), and the implied conditional moment restrictions in equation (1) to give full nonparametric identification of the production function f_t.

The key idea behind our identification strategy is to recognize that the production function (4) and the first order condition (6) form a system of equations

$$\begin{aligned} \ln \rho_t &= \ln P_t + \ln F_{M,t}\left(L_{jt}, K_{jt}, M_{jt}\right) + \ln \mathcal{E} + \omega_{jt} \\ y_{jt} &= f_t\left(L_{jt}, K_{jt}, M_{jt}\right) + \omega_{jt} + \varepsilon_{jt}, \end{aligned}$$

which we can difference to remove the persistent productivity shock ω_{jt}. Adding m_{jt} to each side and re-arranging terms gives

$$s_{jt} = \ln G_t\left(L_{jt}, K_{jt}, M_{jt}\right) + \ln \mathcal{E} - \varepsilon_{jt}, \tag{8}$$

where $G_t(L_{jt}, K_{jt}, M_{jt}) \equiv \frac{F_{M,t}(L_{jt}, K_{jt}, M_{jt}) M_{jt}}{F_t(L_{jt}, K_{jt}, M_{jt})}$ is the elasticity of the production function $F_t(L_{jt}, K_{jt}, M_{jt})$ with respect to intermediate inputs, and $s_{jt} = \ln \frac{\rho_t M_{jt}}{P_t Y_{jt}}$ is the log of the nominal share of intermediate inputs. We observe s_{jt} in the data and hence the nonparametric regression of s_{jt} on (L_{jt}, K_{jt}, M_{jt}) identifies both $G_t(L_{jt}, K_{jt}, M_{jt}) \mathcal{E}$ and the ex-post shock ε_{jt}.[15] We thus can recover $\mathcal{E} = E[e^{\varepsilon_{jt}}]$ and hence the elasticity $G_t(L_{jt}, K_{jt}, M_{jt})$ from the nonparametric regression (8). We will refer to this nonparametric regression as the share regression.

Given that $\frac{G_t(L_{jt}, K_{jt}, M_{jt})}{M_{jt}} = \frac{\partial}{\partial M_{jt}} \ln F_t(L_{jt}, K_{jt}, M_{jt})$, by the fundamental theorem of calculus[16]

$$\int \frac{G_t(L_{jt}, K_{jt}, M_{jt})}{M_{jt}} dM_{jt} = \ln F_t(L_{jt}, K_{jt}, M_{jt}) + \mathscr{C}_t(L_{jt}, K_{jt}). \tag{9}$$

The share regression allows us to nonparametrically identify the production function $f_t(L_{jt}, K_{jt}, M_{jt}) = \ln F_t(L_{jt}, K_{jt}, M_{jt})$ up to a constant of integration $\mathscr{C}_t(L_{jt}, K_{jt})$ that only depends upon the quasi-fixed inputs.[17] To identify this constant and hence the production function as a whole, we can now use the standard moments of equation (1) for quasi-fixed inputs. In particular, by differencing the production function and equation (9), we have

$$y_{jt} - \int \frac{G_t(L_{jt}, K_{jt}, M_{jt})}{M_{jt}} dM_{jt} - \varepsilon_{jt} = -\mathscr{C}_t(L_{jt}, K_{jt}) + \omega_{jt}. \tag{10}$$

[15]In Online Appendix O1 we also show that, when the production function does not vary over time, the revenue share of a flexible input allows us to nonparametrically recover the pattern of industry markups over time, a new and potentially useful result.

[16]We have used indefinite integral notation here for notational ease, but strictly speaking the operation of integrating up the function G identified from the data should be done using a definite integral,

$$\int_{\mathcal{L}_{jt}}^{M_{jt}} \frac{G_t(L_{jt}, K_{jt}, m)}{m} dm = \ln F_t(L_{jt}, K_{jt}, M_{jt}) + \mathscr{C}_t(L_{jt}, K_{jt}),$$

where $\mathcal{L}_{jt} \leq M_{jt}$ is any lower bound such that the support of intermediate inputs conditional on (K_{jt}, L_{jt}) over the interval $[\mathcal{L}_{jt}, M_{jt}]$ is continuous (and hence the integral $\int_{\mathcal{L}_{jt}}^{M_{jt}} \frac{G_t(L_{jt}, K_{jt}, m)}{m} dm$ can be recovered in the data).

[17]See Houthakker (1950) for a similar solution to the related problem of how to recover the utility function from the demand functions.

Note that the left hand side of this equation

$$\mathcal{Y}_{jt} \equiv y_{jt} - \int \frac{G_t\left(L_{jt}, K_{jt}, M_{jt}\right)}{M_{jt}} dM_{jt} - \varepsilon_{jt} \tag{11}$$

is an observable random variable because it is based upon primitives identifiable from the data. Since $\omega_{jt} = \mathcal{Y}_{jt} + \mathscr{C}_t\left(L_{jt}, K_{jt}\right)$, the remainder of the model can be expressed as

$$\mathcal{Y}_{jt} + \mathscr{C}_t\left(L_{jt}, K_{jt}\right) = h\left(\mathcal{Y}_{jt-1} + \mathscr{C}_t\left(L_{jt-1}, K_{jt-1}\right)\right) + \eta_{jt}. \tag{12}$$

It follows that because Assumptions 2 and 3 imply that $E\left[\eta_{jt} \mid L_{jt}, K_{jt}, \mathcal{Y}_{jt-1}, L_{jt-1}, K_{jt-1}\right] = 0$, $\mathscr{C}_t\left(L_{jt}, K_{jt}\right)$ is identified from the following nonparametric regression:[18]

$$\mathcal{Y}_{jt} = -\mathscr{C}_t\left(L_{jt}, K_{jt}\right) + \tilde{h}_t\left(\mathcal{Y}_{jt-1}, L_{jt-1}, K_{jt-1}\right) + \eta_{jt}.$$

Since we can recover $\mathscr{C}_t\left(L_{jt}, K_{jt}\right)$, it is clear from equation (9) that the production function, and hence productivity, is identified.

3.1 Estimation

In this section we show how to obtain a simple nonparametric estimator of the production function that is an analogue of our identification proof. For notational simplicity we drop the time index from the estimation.[19]

We use standard sieve series estimators as analyzed by Chen (2007), and propose a finite-dimensional truncated linear series given by a complete polynomial of degree r for the share regression. Given the observations $\left\{\left(Y_{jt}, L_{jt}, K_{jt}, M_{jt}\right)\right\}_{t=1}^{T}$ for the firms $j = 1, \ldots, J$ sam-

[18]Under alternative models of adjustment frictions, such as allowing capital and labor to be flexible but subject to adjustment costs, the appropriate moment condition involves lagging capital and labor one period, i.e., $E\left[\eta_{jt} \mid L_{jt-1}, K_{jt-1}, \mathcal{Y}_{jt-1}, L_{jt-2}, K_{jt-2}\right] = 0$. It is straightforward to replace our assumptions about the evolution of the quasi-fixed inputs for these alternative models.

[19]That is, the parameters we use to define the series estimators below, (γ, τ), should be indexed by time.

pled in the data, we propose to use a polynomial (in logs):

$$G_r\left(L_{jt}, K_{jt}, M_{jt}\right)\mathcal{E} = \sum_{r_l+r_k+r_m \leq r} \gamma'_{r_l,r_k,r_m} l_{jt}^{r_l} k_{jt}^{r_k} m_{jt}^{r_m}, \text{ with } r_l, r_k, r_m \geq 0 \tag{13}$$

and we use the sum of squared residuals, $\sum_{jt} \varepsilon_{jt}^2$, as our objective function. For a complete polynomial of degree two, our estimator would solve:

$$\min_{\gamma'} \sum_{j,t} \left\{ s_{jt} - \ln \left(\begin{array}{c} \gamma'_0 + \gamma'_l l_{jt} + \gamma'_k k_{jt} + \gamma'_m m_{jt} + \gamma'_{ll} l_{jt}^2 + \gamma'_{kk} k_{jt}^2 \\ +\gamma'_{mm} m_{jt}^2 + \gamma'_{lk} l_{jt} k_{jt} + \gamma'_{lm} l_{jt} m_{jt} + \gamma'_{km} k_{jt} m_{jt} \end{array} \right) \right\}^2.$$

The solution to this problem is an estimator $\hat{G}_r\left(L_{jt}, K_{jt}, M_{jt}\right)\mathcal{E}$ (i.e., of the elasticity up to the constant $\mathcal{E}$) as well as the residual $\hat{\varepsilon}_{jt}$ corresponding to the ex-post shocks to production.[20] Since we can estimate $\hat{\mathcal{E}} = \frac{1}{JT}\sum_{j,t} e^{\hat{\varepsilon}_{jt}}$, we can recover $\gamma \equiv \frac{\gamma'}{\mathcal{E}}$, and thus estimate $\hat{G}_r\left(L_{jt}, K_{jt}, M_{jt}\right)$ from equation (13), free of the constant.

Following our identification strategy, in the second step we calculate the integral in (9). One advantage of the polynomial sieve estimator we have selected is that this integral will have a closed-form solution:

$$\mathcal{G}_r\left(L_{jt}, K_{jt}, M_{jt}\right) \equiv \int \frac{G_r\left(L_{jt}, K_{jt}, M_{jt}\right)}{M_{jt}} dM_{jt} = \sum_{r_l+r_k+r_m \leq r} \frac{\gamma_{r_l,r_k,r_m}}{r_m+1} l_{jt}^{r_l} k_{jt}^{r_k} m_{jt}^{r_m+1}.$$

For a degree two estimator we would have

$$\hat{\mathcal{G}}_2\left(L_{jt}, K_{jt}, M_{jt}\right) = \left(\begin{array}{c} \gamma_0 + \gamma_k k_{jt} + \gamma_l l_{jt} + \frac{\gamma_m}{2} m_{jt} + \gamma_{ll} l_{jt}^2 + \gamma_{kk} k_{jt}^2 \\ +\frac{\gamma_{mm}}{3} m_{jt}^2 + \gamma_{lk} l_{jt} k_{jt} + \frac{\gamma_{lm}}{2} l_{jt} m_{jt} + \frac{\gamma_{km}}{2} k_{jt} m_{jt} \end{array} \right) m_{jt}.$$

With an estimate of ε_{jt} and of $\int \frac{G(L_{jt}, K_{jt}, M_{jt})}{M_{jt}} dM_{jt}$ in hand, we can form a sample analogue

[20] As with all nonparametric sieve estimators, the number of terms in the series increases with the number of observations. Under mild regularity conditions these estimators will be consistent and asymptotically normal for sieve M-estimators like the one we propose. See Chen (2007).

of (11): $\hat{\mathcal{Y}}_{jt} \equiv \ln\left(\frac{Y_{jt}}{e^{\hat{\varepsilon}_{jt}}e^{\hat{\mathcal{G}}_r\left(L_{jt},K_{jt},M_{jt}\right)}}\right)$. Finally, in order to recover the constant of integration in (12), we also use a similar complete polynomial series estimator.[21] That is, we use

$$\mathscr{C}_{\tau}\left(L_{jt}, K_{jt}\right) = \sum_{\tau_l+\tau_k\leq\tau} \alpha_{\tau_l,\tau_k} l_{jt}^{\tau_l} k_{jt}^{\tau_k}, \text{ with } \tau_l, \tau_k > 0$$

for some degree τ (that increases with the number of observations), to form $\omega_{jt}(\alpha) = \hat{\mathcal{Y}}_{jt} + \mathscr{C}_{\tau}\left(L_{jt}, K_{jt}\right)$. We then run a nonparametric regression of $\omega_{jt}(\alpha)$ on $\omega_{jt-1}(\alpha)$ to recover $\eta_{jt}(\alpha)$. This also produces an estimate of $h\left(\omega_{jt-1}\right)$, the Markovian process for ω, as a function of α. We then use the restrictions implied by the moment conditions $E\left[\eta_{jt} \mid k_{jt}, l_{jt}\right] = 0$ to form a standard sieve GMM criterion function to estimate α.[22]

3.2 Allowing for Fixed Effects

One benefit of our identification strategy is that it can easily incorporate fixed effects in productivity. With fixed effects, the production function in equation (4) can be written as

$$y_{jt} = f_t\left(L_{jt}, K_{jt}, M_{jt}\right) + \widetilde{\omega}_{jt} + \varepsilon_{jt}, \tag{14}$$

where $\widetilde{\omega}_{jt} \equiv a_j + \omega_{jt}$, and a_j is the fixed effect. From the firm's perspective, the optimal decision problem for intermediate inputs is the same as before, and the first part of our approach (the nonparametric share regression) is the same up through equation (11), with $\widetilde{\omega}_{jt}$ replacing ω_{jt}. The second half of our approach can be easily augmented to allow for the fixed effects. In particular, the equivalent of equation (12) is given by:

$$\mathcal{Y}_{jt} + \mathscr{C}_t\left(L_{jt}, K_{jt}\right) = a_j + \delta\left[\mathcal{Y}_{jt-1} + \mathscr{C}_t\left(L_{jt-1}, K_{jt-1}\right)\right] + \eta_{jt}.$$

[21] As is well known, it is not possible to separately identify a constant in the production function from mean productivity, $E\left[\omega_{jt}\right]$. In our context this means that we normalize $\mathscr{C}\left(L_{jt}, K_{jt}\right)$ so that it contains no constant.

[22] Asymptotic standard errors for the nonparametric estimates in this second stage require results from two step nonparametric sieve estimation recently provided by "Asymptotic Properties of Nonparametric Two-Step Sieve Estimates" (Xiaohong Chen, Jinyong Hahn, Zhipeng Liao, and Geert Ridder), which generalizes the existing semiparametric two step GMM estimation literature to allow for nonparametric functions in both steps.

Subtracting the counterpart for period $t-1$ eliminates the fixed effect. Re-arranging terms leads to a similar equation as the setup without fixed effects:

$$\mathcal{Y}_{jt} = -\mathscr{C}_t\left(L_{jt}, K_{jt}\right) + \widetilde{\widetilde{h}}_t\left(\mathcal{Y}_{jt-1}, L_{jt-1}, K_{jt-1}, \mathcal{Y}_{jt-2}, L_{jt-2}, K_{jt-2}\right) + \left(\eta_{jt} - \eta_{jt-1}\right).$$

Since $E\left[\eta_{jt} - \eta_{jt-1} \mid L_{jt-1}, K_{jt-1}, \mathcal{Y}_{jt-2}, L_{jt-2}, K_{jt-2}, \mathcal{Y}_{jt-3}, L_{jt-3}, K_{jt-3}\right] = 0$, this regression equation identifies $\mathscr{C}_t\left(L_{jt}, K_{jt}\right)$. The estimation strategy for the model with fixed effects is almost exactly the same as without fixed effects. The first stage, estimating $\hat{\mathcal{G}}\left(L_{jt}, K_{jt}, M_{jt}\right)$, is the same. We then form $\hat{\mathcal{Y}}_{jt}$ in the same way, and use the same series estimator for $\mathscr{C}_\tau\left(L_{jt}, K_{jt}\right)$. The difference is that instead of regressing $\omega_{jt}\left(\alpha\right)$ on $\omega_{jt-1}\left(\alpha\right)$ to recover $\eta_{jt}\left(\alpha\right)$, we regress $\left(\widetilde{\omega}_{jt} - \widetilde{\omega}_{jt-1}\right)\left(\alpha\right)$ on $\left(\widetilde{\omega}_{jt-1} - \widetilde{\omega}_{jt-2}\right)\left(\alpha\right)$ to recover $\left(\eta_{jt} - \eta_{jt-1}\right)\left(\alpha\right)$, where $\widetilde{\omega}_{jt}\left(\alpha\right) = \hat{\mathcal{Y}}_{jt} + a_j + \mathscr{C}_\tau\left(L_{jt}, K_{jt}\right)$. We then use the restrictions implied by the moment conditions $E\left[\eta_{jt} - \eta_{jt-1} \mid k_{jt-1}, l_{jt-1}\right] = 0$ to recover α using GMM, as above.

4 Relationship to Literature

4.1 Price Variation as an Instrument

Recall that the identification problem with respect to intermediate inputs stems from insufficient variation in M_{jt} to identify their influence in the production function independently of the other inputs. However, by looking at the intermediate input demand equation, $M_{jt} = \widetilde{\mathbb{M}}_t(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t)$, it can be seen that if prices (P_t, ρ_t) were firm specific, they could potentially serve as a source of variation to address the identification problem. In fact, given enough sources of price variation, prices could be used directly as instruments to estimate the entire production function while controlling for the endogeneity of input decisions.

However, the validity of prices as instruments has been found to be problematic both in theory and practice (see GM and Ackerberg et al., 2007). First, in many firm-level production datasets, prices are not observed. Moreover, even if price variation is observed, in order to be useful as an instrument it must reflect differences due to firms being in separate markets, as opposed to differences due to market power or the quality of either inputs or output. If prices vary due to market power, it may no longer be the case that this variation is exogenous with respect to the innovation to productivity, η_{jt}. Furthermore, to the extent that input and output prices capture quality differences, prices should be included in the measure of the quantity of the input. For example, if wage differences across firms reflect differences in the average human capital of the workers at those firms, labor should be measured using the total wage bill, rather than the number of workers. This is because for firms with workforces that have low (high) average human capital, the number of workers overestimates (underestimates) the total productivity value of the workers.[23]

This suggests that price variation, even if it is observed, may not be a suitable source of exogenous variation. This is not to say that if one can isolate exogenous price variation, it should not be used to aid in identification. The point is that just observing price variation is not enough. The case must be made that price variation is indeed exogenous. An alternative approach may be to treat lagged prices as instruments for flexible inputs assuming that prices are serially correlated and vary by firm. Doraszelski and Jaumandreu (2013) explore this type of approach. However, their empirical strategy hinges critically on exploiting both the assumption that lagged price variation is exogenous (thereby imposing important restrictions on the joint evolution of prices and productivity) as well as the parametric form of the production function (since they employ a parametric version of LP). [24] Our approach offers an alternative identification strategy that can be employed even without external instruments, such as price variation, and it does not require parametric assumptions on the production function. Provided that the first-order condition for intermediate inputs still holds, our method can incorporate price variation, regardless of whether it is exogenous or not.

[23]Recent work has suggested that quality differences may be a key driver of price differences. As noted by GM, "Why do wages differ across firms at a point of time and within firms over time? The first is likely to be related to differences in the quality of labor..." In addition, Fox and Smeets (2011) finds that differences in wages paid by firms almost entirely measure differences in the human capital, and therefore quality, of their workers.

[24]See Appendix B for further discussion on the role of parametric assumptions in identification with price variation.

4.2 Exploiting First-Order Conditions

The use of first-order conditions for the estimation of production functions dates back to at least the work by Klein (1953) and Solow (1957),[25] who recognized that for a Cobb-Douglas production function, there is an explicit relationship between the parameters representing input elasticities and input cost or revenue shares. This observation forms the basis for index number methods (see e.g., Caves, Christensen, and Diewert, 1982) that are used to nonparametrically recover input elasticities and productivity.[26]

Griliches and Ringstad (1971) also exploit the relationship between the first order condition for a flexible input and the production function in a Cobb-Douglas parametric setting. They use the average revenue share of the flexible input to measure the output elasticity of flexible inputs. This combined with the log-linear form of the Cobb-Douglas production function allows them to then subtract out the term involving flexible inputs. Finally, under the assumption that the quasi-fixed inputs are predetermined and uncorrelated with productivity (not just the innovation), they estimate the coefficients for the quasi-fixed inputs.[27]

Our identification solution can be seen as a nonparametric generalization of the Griliches and Ringstad (1971) empirical strategy. Instead of using the Cobb-Douglas restriction, our share equation (8) instead uses revenue shares to estimate input elasticities in a fully nonparametric setting. In addition, rather than subtract out the effect of intermediate inputs from the production function, we instead integrate up the intermediate input elasticity and take advantage of the nonparametric cross-equation restrictions between the share equation and the production function. Furthermore, we allow for quasi-fixed inputs to be correlated with pro-

[25] Other examples of using first-order conditions to obtain identification include Stone (1954) on consumer demand, Heckman (1974) on labor supply, Hansen and Singleton (1982) on Euler equations and consumption, Paarsch (1992) and Laffont and Vuong (1996) on auctions, and Heckman, Matzkin, and Nesheim (2010) on hedonics.

[26] Index number methods are grounded in three important economic assumptions. First, all inputs are flexible and competitively chosen, which rules out quasi-fixed inputs. Second, the production technology exhibits constant returns to scale, which while not strictly necessary is typically assumed in order to avoid imputing a rental price of capital. Third, and most importantly for our comparison, there are no ex-post shocks to output. Allowing for ex-post shocks in the index number framework can only be relaxed by assuming that elasticities are constant across firms, i.e., by imposing the parametric structure of Cobb-Douglas.

[27] LP suggest a similar approach in an appendix.

ductivity, but uncorrelated with just the innovation to productivity.

4.3 Dynamic Panel

An alternative approach employed in the empirical literature is to use dynamic panel methods (Arellano and Bond (1991) and Blundell and Bond (1998, 2000)). Under a linear parametric restriction on the evolution of ω_{jt}, these methods take advantage of the conditional moment restrictions (equation (1)) implied by Assumptions 2 and 3, which allow for the use of appropriately lagged inputs as instruments.

If one were willing to step outside the model described in Section 2.1 and replace Assumption 3 with an assumption that ALL inputs are quasi-fixed (i.e., rule out the existence of flexible inputs), then it would be possible to use dynamic panel methods to estimate the production function and productivity. However, the bulk of empirical work based on production function estimation has focused on environments in which some inputs are quasi-fixed (namely capital and labor) *and* some inputs are flexible (namely intermediate inputs). It is this setting that motivates our problem and distinguishes our approach from the dynamic panel literature.

4.4 Proxy Variable Methods

It is also instructive to compare our empirical strategy with the literature on the structural estimation of production functions à la OP/LP/ACF. In particular, the ACF approach identifies the ex-post shock ε_{jt} from a first stage regression of output on all the inputs and a proxy variable. The main insight of LP was that intermediate inputs M_{jt} can act as the proxy variable, and ACF largely focus their attention on this case. In a second stage, having separated ε_{jt} from the production function, moment conditions with the innovation η_{jt} are used to identify the production function parameters.

It is important to emphasize that the identification problem we reveal in Section 2.2 is distinct from the one that is the focus of ACF. ACF point out that under the same basic Assumptions (1, 2, and 4) that underlie the OP/LP methodology, the adjustment frictions introduced in Assumption 3 are necessary for identification.[28] Our key contribution in Theorem 1 is that although Assumption 3 is necessary, it is still not sufficient for identification. However, in contrast to the dynamic panel literature discussed above, it is not possible here to apply the ACF solution for labor (i.e., introducing adjustment frictions) to intermediate inputs. The reason is that then intermediate inputs would no longer serve the proxy variable role it plays in Assumption 4.[29]

In contrast to the proxy variable approach, we identify the ex-post shock ε_{jt} in a first stage using the nonparametric share regression (8) rather than a proxy equation. Our nonparametric first stage also allows us to recover the output elasticity of the intermediate inputs. In this sense, the nonparametric share regression contains more information than the nonparametric proxy regression. It is this additional information that allows us to solve the identification problem caused by flexible inputs in the production function. Our identification strategy otherwise makes the same assumptions as these structural methods.

5 Value Added and the Identification Problem

A common empirical approach that seemingly avoids the identification problem caused by intermediate inputs is to exclude them from the model and redefine the object of interest to be a value-added function. The goal is then to estimate firm-level productivity $e^{\omega_{jt}+\varepsilon_{jt}}$ and certain features of the production function $F\left(L_{jt}, K_{jt}, M_{jt}\right)$ (e.g., output elasticities of inputs) with respect to the "primary inputs", capital and labor, from this value-added function. The use of value added is typically justified in one of two ways: via the restricted profit function

[28] This means in particular that the labor coefficient in a Cobb-Douglas specification can no longer be identified in the first stage of OP/LP.

[29] An alternative to using intermediate inputs as the proxy is to use investment. However, as pointed out by LP, the use of investment as a proxy variable may be problematic as many firms (about half in our data) have zero investment in any given year. This violates the strict monotonicity assumption required to implement the proxy variable approach to begin with. One would also need to show that the source of the adjustment frictions on intermediate inputs does not violate the invertibility assumption for the investment proxy equation.

or using structural production functions. The two approaches differ both in their underlying assumptions and in how the value-added function is constructed from the underlying gross output production function. We now show that although intermediate inputs do not appear in the value added function, the use of value added does not solve the fundamental identification problem described in Section 2.2.

5.1 Restricted Profit Value-Added

The first approach to justifying the use of value added is based on the duality results in Bruno (1978) and Diewert (1978). We first briefly discuss their original results, which were derived under the assumption that intermediate inputs are flexibly chosen, but excluding the ex-post shocks. In this case, they show that by replacing intermediate inputs with their optimized value in the profit function, the empirical measure of value added, $VA_{jt}^{E} = Y_{jt} - M_{jt}$, can be expressed as:[30]

$$\begin{aligned} VA_{jt}^{E} &= F_t\left(L_{jt}, K_{jt}, \mathbb{M}_t\left(L_{jt}, K_{jt}, e^{\omega_{jt}}\right)\right) e^{\omega_{jt}} - \mathbb{M}_t\left(L_{jt}, K_{jt}, e^{\omega_{jt}}\right) \\ &\equiv \mathcal{V}_t\left(L_{jt}, K_{jt}, e^{\omega_{jt}}\right), \end{aligned} \tag{15}$$

where we use $\mathcal{V}_t\left(\cdot\right)$ to denote value-added in this setup. This formulation is sometimes referred to as the restricted profit function (see Lau, 1976; Bruno, 1978; McFadden, 1978). It is important to emphasize here that the results of this earlier work attempt to provide conditions under which estimates of value-added objects can be used to recover their gross output counterparts. They do not show that the two are equivalent.

In an index number framework, Bruno (1978) shows that elasticities of gross output with respect to capital, labor, and productivity can be locally approximated by multiplying estimates of the value-added counterparts by the firm-level ratio of value added to gross output,

[30] For simplicity, throughout this section we use the double deflated version of empirical value added. Our results do not depend on the method of deflating value added. See Bruno (1978) for discussion of the differences between the alternative ways of deflating value added.

$\frac{VA^E_{jt}}{Q_{jt}} = (1 - S_{jt})$.[31] For productivity, the result is as follows:

$$\left(elas^{GO_{jt}}_{e^{\omega_{jt}}}\right) = \left(elas^{VA^E_{jt}}_{e^{\omega_{jt}}}\right)\left(\frac{VA^E_{jt}}{GO_{jt}}\right) = \left(elas^{VA^E_{jt}}_{e^{\omega_{jt}}}\right)(1 - S_{jt})\,. \tag{16}$$

See Appendix C for the details of this derivation. Analogous results hold for the elasticities with respect to capital and labor by replacing $e^{\omega_{jt}}$ with K_{jt} or L_{jt}.

This derivation suggests that estimates of (log) productivity from the restricted-profit value-added function can simply be multiplied by $(1 - S_{jt})$ to recover estimates of the underlying gross output (log) productivity, and similarly for the output elasticities of capital and labor. However, there are several important problems with the relationship in equation (16), and the implied re-scaling.

First, this approach relies on being able to obtain consistent estimates of the value-added elasticities. Recent methods for estimating production functions rely on the structural form of the production function, e.g., the Hicks-neutrality of productivity. However, in general, the structural form of the value-added function in equation (15) will not correspond to that of the underlying gross output production function (equation 4).[32] As a result, it is not clear that the recent structural methods can be used to estimate $\mathcal{V}_t(\cdot)$ to begin with.

Second, this approach is based on a local approximation. While this may work well for small changes in productivity, for example looking at productivity growth rates (the original context under which these results were derived), it may not work well for large differences in productivity, such as analyzing cross-sectional productivity differences.

Third, this approximation does not account for ex-post shocks to output. As we show in Appendix C, when ex-post shocks are accounted for, the relationship in equation (16) be-

[31] These results were derived under a general form of technical change. We have augmented the results here to correspond to the standard setup with Hicks-neutral technical change as discussed in Section 2.1.

[32] For example, even if productivity $e^{\omega_{jt}+\varepsilon_{jt}}$ enters the gross output production function in a Hicks-neutral fashion, it will, in general, not enter $\mathcal{V}_t(\cdot)$ in the same fashion outside of Cobb-Douglas. For example, even a CES production function will not generate the required separability of productivity in $\mathcal{V}_t(\cdot)$.

comes:

$$\underbrace{\left(\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}_{jt}}{GO_{jt}}\right)}_{elas^{GO_{jt}}_{e^{\omega_{jt}}}} = \underbrace{\left(\frac{\partial VA^{E}_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}_{jt}}{VA^{E}_{jt}}\right)}_{elas^{VA^{E}_{jt}}_{e^{\omega_{jt}}}}(1-S_{jt}) + \left[\frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}_{jt}}{GO_{jt}}\left(\frac{e^{\varepsilon_{jt}}}{\mathcal{E}}-1\right)\right] \tag{17}$$

The term in brackets is the bias introduced due to the ex-post shock. Ex-post shocks drive a wedge between the local equivalence of value added and gross output objects. Analogous results for the output elasticities of capital and labor can be similarly derived.

As a result of the points discussed above, estimates from the restricted profit value-added function cannot simply be "transformed" by re-scaling with the firm-specific share of intermediate inputs to obtain estimates of the underlying production function and productivity. How much of a difference this makes is ultimately an empirical question, which we address in the next section. Previewing our results, we find that rescaling using the shares, as suggested by equation (16), performs poorly.

5.2 "Structural" Value-Added

The second approach to justifying the use of value added is based on specific parametric assumptions on the production function, such that a value-added production function of only capital, labor, and productivity can be both isolated and measured (see Sims, 1969 and Arrow, 1972). We refer to this version of value added as the "structural value-added production function".

The empirical literature on value-added production functions often appeals to the extreme case of perfect complements (i.e., Leontief).[33] A standard representation is

$$Y_{jt} = \min\left[\mathcal{H}\left(L_{jt}, K_{jt}\right)e^{\omega_{jt}+\varepsilon_{jt}}, \mathcal{C}\left(M_{jt}\right)\right],$$

[33] See Appendix D for a discussion of other potential assumptions.

where $\mathcal{C}(\cdot)$ is a monotone increasing and concave function. The main idea underlying the Leontief justification is that under the assumptions that the firm can set

$$\mathcal{H}(L_{jt}, K_{jt}) e^{\omega_{jt}+\varepsilon_{jt}} = \mathcal{C}(M_{jt}) \tag{18}$$

and that $\mathcal{C}(\cdot)$ is linear (i.e., $\mathcal{C}(M_{jt}) = cM_{jt}$), a structural value-added production function can be obtained: $VA^E_{jt} = \left(1 - \frac{1}{c}\right) \mathcal{H}(L_{jt}, K_{jt}) e^{\omega_{jt}+\varepsilon_{jt}}$.

Besides the strong parametric assumptions about how M enters the production function, the key problem with this approach is that, given the assumptions of the model, the relation in equation (18) will not generally hold. The first reason is that ε is realized after input decisions are made. Second, under Assumption 3, firms either cannot adjust capital and labor in period t or can only do so with some positive adjustment cost. The key consequence is that firms may optimally choose to *not* equate $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$ and cM_{jt}. Therefore, even under this specific version of Leontief, the value-added production function cannot be identified because the critical relationship (18) will not hold.[34]

6 Data and Application

Based on the results presented in the previous section, a natural concern is that the use of value added may lead to misleading inferences about the production technology and productivity. Nevertheless, it has become common practice to relate the empirical measure of value added (what we call VA^E) to capital and labor as a means of recovering productivity. We now ask the question: what happens when the relationship between the empirical measure of value added and the primary inputs (i.e., excluding intermediate inputs) is estimated when the model of production follows the general gross output setup outlined in Section 2.1?

The first thing to note is that, as in the classic omitted variable bias problem, if we only

[34] In Appendix D we provide a more detailed description of these problems. We also show that allowing ω_{jt} and/or ε_{jt} to be outside of the min function presents a similar set of issues.

control for the variation in *some* inputs (say capital and labor), part of the heterogeneity in output among firms will be due to variation in the excluded inputs (intermediate inputs) in addition to productivity. Since intermediate input usage is positively correlated with productivity (see equation (7)), the observed variation in output will tend to overstate the true degree of productivity heterogeneity, and similarly for the capital and labor elasticity estimates.

We apply our empirical strategy and measure the magnitude of the effect of using a value-added rather than gross output specification using two commonly employed plant-level manufacturing datasets. The first dataset comes from the Colombian manufacturing census covering all manufacturing plants with more than 10 employees from 1981-1991. This dataset has been used in several studies, including Roberts and Tybout (1997), Clerides, Lach, and Tybout (1998), and Das, Roberts, and Tybout (2007). The second dataset comes from the census of Chilean manufacturing plants conducted by Chile's Instituto Nacional de Estadística (INE). It covers all firms from 1979-1996 with more than 10 employees. This dataset has also been used extensively in previous studies, both in the production function estimation literature (LP) and in the international trade literature (Pavcnik, 2002 and Alvarez and López, 2005).[35]

We estimate separate production functions for the five largest 3-digit manufacturing industries in both Colombia and Chile, which are Food Products (311), Textiles (321), Apparel (322), Wood Products (331), and Fabricated Metal Products (381). We also estimate an aggregate specification grouping all manufacturing together. We estimate the production function in two ways.[36] First, using our approach from Section 3 we estimate a gross output production function using a complete polynomial series of degree 2 for both the elasticity and the

[35] We construct the variables adopting the convention used by Greenstreet (2007) with the Chilean dataset, and employ the same approach with the Colombian dataset. In particular, real gross output is measured as deflated revenues. Intermediate inputs are formed as the sum of expenditures on raw materials, energy (fuels plus electricity), and services. Real value added is the difference between real gross output and real intermediate inputs, i.e., double deflated value added. Labor input is measured as a weighted sum of blue collar and white collar workers, where blue collar workers are weighted by the ratio of the average blue collar wage to the average white collar wage. Capital is constructed using the perpetual inventory method where investment in new capital is combined with deflated capital from period $t-1$ to form capital in period t. Deflators for Colombia are obtained from Pombo (1999) and deflators for Chile are obtained from Bergoeing, Hernando, and Repetto (2003).

[36] For all of the estimates we present, we obtain standard errors by using the nonparametric bootstrap with 200 replications.

integration constant in the production function.[37] That is, we use

$$G_2(L_{jt}, K_{jt}, M_{jt}) = \gamma_0 + \gamma_l l_{jt} + \gamma_k k_{jt} + \gamma_m m_{jt} + \gamma_{ll} l_{jt}^2 + \gamma_{kk} k_{jt}^2 + \gamma_{mm} m_{jt}^2 + \gamma_{lk} l_{jt} k_{jt} + \gamma_{lm} l_{jt} m_{jt} + \gamma_{km} k_{jt} m_{jt}$$

to estimate the intermediate input elasticity and

$$\mathscr{C}_2(L_{jt}, K_{jt}) = \alpha_l l_{jt} + \alpha_k k_{jt} + \alpha_{ll} l_{jt}^2 + \alpha_{lk} k_{jt}^2 + \alpha_{lk} l_{jt} k_{jt}$$

for the constant of integration. Putting all the elements together, the gross output production function we estimate is given by:

$$y_{jt} = \begin{pmatrix} \gamma_0 + \gamma_k k_{jt} + \gamma_l l_{jt} + \frac{\gamma_m}{2} m_{jt} + \gamma_{ll} l_{jt}^2 + \gamma_{kk} k_{jt}^2 \\ + \frac{\gamma_{mm}}{3} m_{jt}^2 + \gamma_{lk} l_{jt} k_{jt} + \frac{\gamma_{lm}}{2} l_{jt} m_{jt} + \frac{\gamma_{km}}{2} k_{jt} m_{jt} \end{pmatrix} m_{jt} - \alpha_l l_{jt} - \alpha_k k_{jt} - \alpha_{ll} l_{jt}^2 - \alpha_{lk} k_{jt}^2 - \alpha_{lk} l_{jt} k_{jt} + \omega_{jt} + \varepsilon_{jt}, \quad (19)$$

since, from equation (10), it is immediate that $y_{jt} = \int G(L_{jt}, K_{jt}, M_{jt}) \frac{dM_{jt}}{M_{jt}} - \mathscr{C}(L_{jt}, K_{jt}) + \omega_{jt} + \varepsilon_{jt}$.

Second, we estimate a value-added specification using the commonly-applied method developed by ACF, also using a complete polynomial series of degree 2:

$$va_{jt} = \beta_l l_{jt} + \beta_k k_{jt} + \beta_{ll} l_{jt}^2 + \beta_{kk} k_{jt}^2 + \beta_{lk} l_{jt} k_{jt} + \upsilon_{jt} + \epsilon_{jt}, \quad (20)$$

where $\upsilon_{jt} + \epsilon_{jt}$ represents productivity in the value-added model.

In Table 1 we report estimates of the average output elasticities for each input, as well as the sum, for both the value-added and gross output models. In every case but one, the value-added model overestimates the sum of elasticities relative to gross output, with an average

[37] In order to improve performance of the estimators, we restrict both the gross output and value added production functions to not vary over time.

difference of 2% in Colombia and 6% in Chile.

We also report the ratio of the mean capital and labor elasticities, which measures the capital intensity (relative to labor) of the production technology in each industry. In general, the value-added estimates overstate the capital intensity of the technology relative to gross output, although the differences are small. According to both measures, the Food Products (311) and Textiles (321) industries are the most capital intensive in Colombia, and in Chile the most capital intensive are Food Products (311), Textiles (321), and Fabricated Metals (381). In both countries, Apparel (322) and Wood Products (331) are the least capital intensive industries, even compared to the aggregate specification denoted "All" in the tables.

Value added also recovers dramatically different patterns of productivity as compared to gross output. Following OP, we define productivity (in levels) as the sum of the persistent and unanticipated components: $e^{\omega_{jt}+\varepsilon_{jt}}$.[38,39] In Table 2 we report estimates of several frequently analyzed statistics of the resulting productivity distributions. In the first three rows of each panel we report ratios of percentiles of the productivity distribution, a commonly used measure of productivity dispersion. There are two important implications of these results. First, value added suggests a much larger amount of heterogeneity in productivity across plants *within* an industry, as the various percentile ratios are much smaller under gross output. For Colombia, the average 75/25, 90/10, and 95/5 ratios are 1.88, 3.69, and 6.41 under value added, and 1.33, 1.78, and 2.23 under gross output. For Chile, the average 75/25, 90/10, and 95/5 ratios are 2.76, 8.02, and 17.93 under value added, and 1.48, 2.20, and 2.95 under gross output. The value-added estimates imply that, with the same amount of inputs, the 95th percentile plant would produce more than 6 times more output in Colombia, and almost 18 times more output in Chile, than the 5th percentile plant. In stark contrast, we find that under gross output, the

[38] Since our interest is in analyzing productivity heterogeneity we conduct our analysis using productivity in levels. An alternative would be to measure productivity in logs. However, the log transformation is only a good approximation for measuring percentage differences in productivity across groups when these differences are small, which they are not in our data. We have also computed results based on log productivity. As expected, the magnitude of our results changes, however, our qualitative results comparing gross output and value added still hold.

[39] We have also computed results using just the persistent component of productivity, $e^{\omega_{jt}}$. The results are qualitatively similar.

95th percentile plant would produce only 2 times more output in Colombia, and 3 times more output in Chile, than the 5th percentile plant with the same inputs.

In addition, the ranking of industries according to the degree of productivity dispersion is not preserved moving from the value added to gross output estimates. For example, in Chile, the Fabricated Metals industry (381) is found to have the smallest amount of productivity dispersion under value added, but the largest amount of dispersion under gross output, for all three dispersion measures.

The second important result is that value added also implies much more heterogeneity *across* industries, which is captured by the finding that the range of the percentile ratios across industries are much tighter using the gross output measure of productivity. For example, for the 95/5 ratio, the value-added estimates indicate a range from 4.36 to 11.01 in Colombia and from 12.52 to 25.08 in Chile, whereas the gross output estimates indicate a range from 2.02 to 2.38 and from 2.48 to 3.31. The surprising aspect of these results is that the dispersion in productivity appears far more stable both across industries and across countries when measured via gross output as opposed to value added. In the conclusion we sketch some important policy implications of this finding for empirical work on the misallocation of resources.

In addition to showing much larger overall productivity dispersion, results based on value added also suggest a substantially different relationship between productivity and other dimensions of plant-level heterogeneity. We examine several commonly-studied relationships between productivity and other plant characteristics. In the last four rows of each panel in Table 2 we report percentage differences in productivity based on whether plants export some of their output, import intermediate inputs, have positive advertising expenditures, and pay above the median (industry) level of wages.

Using the value-added estimates, for most industries exporters are found to be more productive than non-exporters, with exporters appearing to be 83% more productive in Colombia and 14% more productive in Chile across all industries. Once we account for intermediate inputs using the gross output specification, these estimates of productivity differences fall to 9%

in Colombia and 3% in Chile, and actually turn negative (although not statistically different from zero) in some cases.

A similar pattern exists when looking at importers of intermediate inputs. The average productivity difference is 14% in Colombia and 41% in Chile using value added. However, under gross output, these numbers fall to 8% and 13% respectively. The same story holds for differences in productivity based on advertising expenditures. Moving from value added to gross output, the estimated difference in productivity drops for most industries in Colombia, and for all industries in Chile. In several cases it becomes statistically indistinguishable from zero.

Another striking contrast arises when we compare productivity between plants that pay wages above versus below the industry median. Using the productivity estimates from a value-added specification, firms that pay wages above the median industry wage are found to be substantially more productive, with the estimated differences ranging from 34%-63% in Colombia and from 47%-123% in Chile. In every case the estimates are statistically significant. Using the gross output specification, these estimates fall to 9%-22% in Colombia and 19%-30% in Chile, representing a fall by a factor of 3, on average, in both countries.

As explained above, using empirical value added as the measure of output when it is not appropriate to do so will result in biased estimates, since intermediate input usage is not properly controlled for. As a consequence, we would expect to see the largest discrepancies between the value-added and gross output productivity heterogeneity estimates in industries which are intensive in intermediate input usage. By looking at Tables 1 and 2 we can confirm that, for the most part, this is the case. When comparing the value added and gross output productivity estimates, the largest deviations tend to occur in the most intermediate input intensive industries, which are Food Products (311) in Colombia and Food Products (311) and Wood Products (331) in Chile. However, consistent with the fact that the exclusion of intermediate inputs is not the only driver of the differences, this is not always the case. For example, in Chile, the difference between the gross output and value added estimates of the average productivity comparing advertisers and non-advertisers is actually the smallest in the Wood Products (331) industry.

In order to isolate the importance of the value-added/gross output distinction separately from the effect of transmission bias, in Table 3 we repeat the above analysis without correcting for the endogeneity of inputs. We examine the raw effects in the data by estimating productivity using simple OLS to estimate both gross output and value-added specifications, using a complete polynomial of degree 2. As can be seen from Table 3, the general pattern of results, that value added overstates productivity differences across many dimensions, is similar to our previous results both qualitatively and quantitatively.

While the results in Table 3 may suggest that transmission bias is not empirically important, in Table 4 we show evidence to the contrary. In particular, we report the average input elasticities based on estimates for the gross output model using OLS and using our method to correct for transmission bias. A well-known result is that failing to control for transmission bias leads to overestimates of the coefficients on more flexible inputs. The intuition is that the more flexible the input is, the more it responds to productivity shocks and the higher the degree of correlation between that input and unobserved productivity. The estimates in Table 4 show that the OLS results substantially overestimate the output elasticity of intermediate inputs in every case. The average difference is 34%, which illustrates the importance of controlling for the endogeneity generated by the correlation between input decisions and productivity.

An important implication of our results is that, while controlling for transmission bias certainly has an effect, the use of value added has a much larger effect on the productivity estimates than the transmission bias in the gross output production function. This suggests that the use of gross output versus value added may be more important from a policy perspective than controlling for the transmission bias that has been the primary focus in the production function literature. Our approach avoids the mismeasurement caused by value added by allowing for the use of gross output production functions while simultaneously correcting for the transmission bias.

6.1 Robustness Checks

Fixed Effects An advantage of our identification and estimation strategy is that we can easily incorporate fixed effects in the production function. As described in Section 3.2, the production function allowing for fixed effects, a_j, is given by $Y_{jt} = F(L_{jt}, K_{jt}, M_{jt}) e^{a_j + \omega_{jt} + \varepsilon_{jt}}$, where now productivity is defined as $e^{a_j + \omega_{jt} + \varepsilon_{jt}}$. A common drawback of models with fixed effects is that the differencing of the data needed to subtract out the fixed effects can remove a large portion of the identifying information in the data. In the context of production functions, this often leads to estimates of the capital coefficient and returns to scale that are unrealistically low, as well as large standard errors (see GM).

In Online Appendix O2, we report estimates corresponding to those in Tables 1 and 2, using our method to estimate the gross output production function allowing for fixed effects. The elasticity estimates for intermediate inputs are exactly the same as in the specification without fixed effects, as the first stage of our approach does not depend on the presence of fixed effects. We do find some mild evidence in Colombia of the problems mentioned above as the sample sizes are smaller than those for Chile. Despite this, the estimates are very similar to those from the main specification for both countries, and the larger differences are associated with larger standard errors.

Alternative Flexible Inputs Our new identification and estimation strategy takes advantage of the first-order condition with respect to a flexible input. As intermediate inputs have been commonly assumed to be flexible in the literature, we have used intermediate inputs (the sum of raw materials, energy, and services) as the flexible input. In some applications, researchers may not want to assume that all intermediate inputs are flexible, or they may want to test the sensitivity of their estimates to this assumption. As a robustness check we also estimate two different specifications in which we allow some of the components of intermediate inputs to be quasi-fixed. In particular, the production function we estimate is of the form $F(L_{jt}, K_{jt}, RM_{jt}, NS_{jt}) e^{\omega_{jt} + \varepsilon_{jt}}$, where RM denotes raw materials and NS denotes energy plus services. In one specification we assume RM to be quasi-fixed and NS to be flexible, and in the other specification we assume the opposite. See online Appendix O3 for these results. Overall the results are sensible and qualitatively similar to our main results. In addition, they maintain the relationship in the main set of results between productivity estimates based on gross output and value added.

Adjusting the Value Added Estimates As discussed in Section 5.1, in the absence of ex-post shocks, the derivation provided in equation (16) suggests that the differences between gross output and value added can be eliminated by rescaling the value-added estimates by a factor equal to the plant-level ratio of value added to gross output, i.e., one minus the share of intermediate inputs in total output. While this idea has been known in the literature for a while, this correction is very rarely applied in practice.[40] As shown in Section 5.1, there are several reasons why this correction may not work. In particular, as we demonstrate in equation (17), this correction is no longer valid once ex-post shocks are accounted for. In order to investigate how well the re-scaling of value added estimates performs, we apply the correction implied by equation (16) and re-scale the value-added estimates of output elasticities of inputs and productivity by the ratio of value added to gross output $\left(\frac{VA^E_{jt}}{GO_{jt}}\right)$, a quantity readily available in the data. We find that this correction performs quite poorly in recovering the underlying gross output estimates of the production function and productivity, leading to biased estimates that are in some cases even worse than the value-added estimates themselves.

In Tables 5 and 6, we report the re-scaled estimates, as well as the value-added estimates using ACF and the gross output estimates using our method, for comparison. At first glance, the naive correction appears to be working as the re-scaled value-added estimates move towards the gross output estimates. However, in some cases, the re-scaled estimates of dispersion and the relationship between productivity and other dimensions of firm heterogeneity move only slightly towards the gross output estimates, and remain very close to the original value-added estimates. Moreover, in many cases, however, the re-scaled estimates overshoot

[40] See Petrin and Sivadasan (2013) for an example in which a version of this correction is implemented.

the gross output estimates. Even worse, in many cases the correction moves in the opposite direction and leads to estimates that are even further from the gross output estimates than the original value-added estimates. Finally, in several cases, the re-scaled estimates actually lead to a sign-reversal compared to both the value-added and gross output estimates. Overall, while in some cases the correction applied to the value-added estimates moves them closer to the gross output estimates, it does a poor job of replicating the gross output estimates, and in many cases generates estimates that are even more misleading.

7 Conclusion

In this paper we show that the nonparametric identification of production functions in the presence of quasi-fixed and flexible inputs has remained an unresolved issue. We offer a new identification strategy that closes this loop. The key to our approach is exploiting the nonparametric information contained in the first order condition for the flexible inputs.

Our empirical analysis demonstrates that value added can generate substantially different patterns of productivity heterogeneity as compared to gross output, which suggests that empirical studies of productivity based on value added may lead to fundamentally misleading policy implications. To illustrate this possibility, consider the recent literature that uses productivity dispersion to explain cross-country differences in output per worker through resource misallocation. As an example, the recent influential paper by Hsieh and Klenow (2009) finds substantial heterogeneity in productivity dispersion (defined as the variance of log productivity) across countries as measured using value added. In particular, when they compare the United States with China and India, the variance of log productivity ranges from 0.40-0.55 for China and 0.45-0.48 for India, but only from 0.17-0.24 for the United States. They then use this estimated dispersion to measure the degree of misallocation of resources in the respective economies. In their main counterfactual they find that by reducing the degree of misallocation in China and India to that of the United States, aggregate TFP would increase by 30%-50%

in China and 40%-60% in India. In our datasets for Colombia and Chile, the corresponding estimates of the variance in log productivity using a value-added specification are 0.43 and 0.94, respectively. Thus their analysis applied to our data would suggest that there is similar room for improvement in aggregate TFP in Colombia, and much more in Chile.

However, when productivity is measured using our gross output framework, our empirical findings suggest a much different result. The variance of log productivity using gross output is 0.08 in Colombia and 0.15 in Chile. These significantly smaller dispersion measures could imply that there is much less room for improvement in aggregate productivity for Colombia and Chile. Since the 90/10 ratios we obtain for Colombia and Chile using gross output are quantitatively very similar to the estimates obtained by Syverson (2004) for the United States (who also employed gross output but in an index number framework), this also suggests that the degree of differences in misallocation of resources between developed and developing countries may not be as large as the analysis of Hsieh and Klenow (2009) implies.[41]

Exploring the role of gross output production functions for policy problems such as the one above could be a fruitful direction for future research. A key message of this paper is that insights derived under value added could significantly bias policy conclusions, and the use of gross output production functions is thus possibly critical for policy analysis. Our identification strategy provides researchers with a stronger foundation for using gross output production functions in practice.

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[41] Hsieh and Klenow note that their estimate of log productivity dispersion for the United States is larger than previous estimates by Foster, Haltiwanger, and Syverson (2008) by a factor of almost 4. They attribute this to the fact that Foster, Haltiwanger, and Syverson use a selected set of homogeneous industries. However, another important difference is that Foster, Haltiwanger, and Syverson use gross output measures of productivity rather than value-added measures. Given our results in Section 6, it is likely that a large part of this difference is due to Hsieh and Klenow's use of value added, rather than their selection of industries.

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Appendix A: Non-Identification Proof

Recall that the model of production can be written as

$$y_{jt} = f_t\left(L_{jt}, K_{jt}, M_{jt}\right) + h\left(\underbrace{\mathbb{M}_{t-1}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)}_{\omega_{jt-1}}\right) + \eta_{jt} + \varepsilon_{jt},$$

and that $\Gamma_{jt} = (L_{jt}, K_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}, ..., L_1, K_1, M_1)$. The reduced form of the model is therefore given by

$$\begin{aligned} E[y_{jt} \mid \Gamma_{jt}] &= E[f_t(L_{jt}, K_{jt}, M_{jt}) \mid \Gamma_{jt}] + E[\omega_{jt} \mid \Gamma_{jt}] \\ &= E[f_t(L_{jt}, K_{jt}, M_{jt}) \mid \Gamma_{jt}] + h\left(\mathbb{M}_{t-1}^{-1}(L_{jt-1}, K_{jt-1}, M_{jt-1})\right) \\ &= E[f_t(L_{jt}, K_{jt}, M_{jt}) \mid \Gamma_{jt}] + h_t(L_{jt-1}, K_{jt-1}, M_{jt-1}) \\ &= E[r_t(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}) \mid \Gamma_{jt}] \qquad (21) \end{aligned}$$

where $h_t(L_{jt-1}, K_{jt-1}, M_{jt-1}) \equiv h\left(\mathbb{M}_{t-1}^{-1}(L_{jt-1}, K_{jt-1}, M_{jt-1})\right)$ and $r_t(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}) \equiv f_t(L_{jt}, K_{jt}, M_{jt}) + h_t(L_{jt-1}, K_{jt-1}, M_{jt-1})$.

In order to prove Theorem 1 in Section 2.2, we begin by showing in Lemma 1 that identification of the production function is equivalent to identification of r_t. The function r_t is identified if for any $\bar{r}_t$ such that

$$\begin{aligned} &E[r_t(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}) \mid \Gamma_{jt}] \\ &= E[\bar{r}_t(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}) \mid \Gamma_{jt}] \end{aligned} \qquad a.s.\ [\Gamma_{jt}]$$

implies that $r_t = \bar{r}_t$ almost surely in $(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1})$. Proving Lemma 1 involves the following two technical assumptions (see Newey, Powell, and Vella, 1999).

Assumption L1. The boundary of the support of $(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1})$ in $\mathbb{R}^6$ has a probability measure zero.

Assumption L2. f_t and r_t are differentiable.

Lemma 1. *Under Assumptions 1-4, L1, and L2, f_t is identified up to an additive constant if and only if r_t is identified.*

Proof. Suppose that r_t is identified and that there exist (f_t, h_t) and $(\bar{f}_t, \bar{h}_t)$ such that

$$f_t(L_{jt}, K_{jt}, M_{jt}) + h_t(L_{jt-1}, K_{jt-1}, M_{jt-1}) = \bar{f}_t(L_{jt}, K_{jt}, M_{jt}) + \bar{h}_t(L_{jt-1}, K_{jt-1}, M_{jt-1})$$

almost surely in the support of $(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1})$. Let the interior of the support of $(L_{jt}, K_{jt}, M_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1})$ be denoted by S. Because r_t is continuous and the boundary of the support has measure 0, the value of r_t is uniquely identified at each point $x \in S$ and so are its partial derivatives. Since $\frac{\partial}{\partial z} r_t(x) = \frac{\partial}{\partial z} f_t(x) = \frac{\partial}{\partial z} \bar{f}_t(x)$ for each $z \in \{L_{jt}, K_{jt}, M_{jt}\}$ and each $x \in S$, we have that $f_t - \bar{f}_t = c$ for some constant c for all points $x \in S$. Because the set S has probability one, we have shown that f_t and $\bar{f}_t$ with probability one differ by only a constant. Hence identification of r_r implies identification of f_t up to a constant. For the other direction, assume that f_t is identified. Then $h_t(L_{jt-1}, K_{jt-1}, M_{jt-1}) = E[y_{jt} \mid \Gamma_{jt}] - E[f_t(L_{jt}, K_{jt}, M_{jt}) \mid \Gamma_{jt}]$ is identified almost everywhere, and $r_t = f_t + h_t$ is identified. □

Now let $\gamma_{jt} = L_{jt}, K_{jt}, L_{jt-1}, K_{jt-1}, M_{jt-1}$, a subset of Γ_{jt}. We can re-write our model in terms of r_t as

$$y_{jt} = r_t(\gamma_{jt}, M_{jt}) + \eta_{jt} + \varepsilon_{jt},$$

where the interior of the support of $(\gamma_{jt}, M_{jt}) \in \mathbb{R}^6$ has positive measure and $E[\eta_{jt} + \varepsilon_{jt} \mid \gamma_{jt}] = 0$. We say that r_t is not uniquely identified from the reduced form $E[y_{jt} \mid \gamma_{jt}]$, if there exist $r_t(\gamma_{jt}, M_{jt})$ and $\bar{r}_t(\gamma_{jt}, M_{jt})$ such that

$$E[r_t(\gamma_{jt}, M_{jt}) \mid \gamma_{jt}] = E[\bar{r}_t(\gamma_{jt}, M_{jt}) \mid \gamma_{jt}] \quad a.s.\ [\gamma_{jt}],$$

and $r_t(\gamma_{jt}, M_{jt}) \neq \bar{r}_t(\gamma_{jt}, M_{jt})$ with positive probability in (γ_{jt}, M_{jt}).

In order to prove that r_t is not identified, we use the following lemma.

Lemma 2. *Suppose there exists a random variable δ and functions $\mu_t(\gamma, \delta)$ and $\bar{\mu}_t(\gamma, \delta)$ that satisfy*

$$E[\mu_t(\gamma_{jt}, \delta_{jt}) \mid \gamma_{jt}] = E[\bar{\mu}_t(\gamma_{jt}, \delta_{jt}) \mid \gamma_{jt}] = 0.$$

If

$$r_t(\gamma_{jt}, M_{jt}) + \mu_t(\gamma_{jt}, \delta_{jt}) = \bar{r}_t(\gamma_{jt}, M_{jt}) + \bar{\mu}_t(\gamma_{jt}, \delta_{jt}) \quad a.s.\ [\gamma_{jt}, M_{jt}, \delta_{jt}] \tag{22}$$

then

$$E\left[r_t\left(\gamma_{jt}, M_{jt}\right) \mid \gamma_{jt}\right] = E\left[\bar{r}_t\left(\gamma_{jt}, M_{jt}\right) \mid \gamma_{jt}\right] \quad a.s.\ \left[\gamma_{jt}\right].$$

Proof. Take the conditional expectation of both sides of (22) conditional on γ_{jt} to arrive at the conclusion. □

We now construct two observationally equivalent functions $r_t\left(\gamma_{jt}, M_{jt}\right)$ and $\bar{r}_t\left(\gamma_{jt}, M_{jt}\right)$ such that $r_t\left(\gamma_{jt}, M_{jt}\right) \neq \bar{r}_t\left(\gamma_{jt}, M_{jt}\right)$ using Lemma 2. To do so, let us construct

$$M_{jt} = \varphi_t\left(\gamma_{jt}\right) + \delta_{jt} \tag{23}$$

with $E\left[\delta_{jt} \mid \gamma_{jt}\right] = 0$, i.e., φ_t is the regression of M_{jt} on γ_{jt} in period t. Assume that φ_t is differentiable. Then let us construct an

$$\bar{r}_t\left(\gamma_{jt}, M_{jt}\right) = r_t\left(\gamma_{jt}, M_{jt}\right) + \alpha M_{jt} - \alpha\varphi_t\left(\gamma_{jt}\right) \tag{24}$$

for $\alpha \neq 0$. Observe that because $\alpha \neq 0$ we have that $\frac{\partial r_t}{\partial M_{jt}} \neq \frac{\partial \bar{r}_t}{\partial M_{jt}}$ for all $\left(\gamma_{jt}, M_{jt}\right) \in S$ and thus r_t and $\bar{r}_t$ differ with positive probability.

In order to show that our proposed $\bar{r}_t$ (equation 24) is observationally equivalent to r_t, we now construct a pair of functions μ_t and $\bar{\mu}_t$ that satisfy the conditions of Lemma 2. Let $\mu_t\left(\gamma_{jt}, \delta_{jt}\right) \equiv E\left[\eta_{jt} + \varepsilon_{jt} \mid \gamma_{jt}, \delta_{jt}\right]$ and $\bar{\mu}_t\left(\gamma_{jt}, \delta_{jt}\right) \equiv \mu_t\left(\gamma_{jt}, \delta_{jt}\right) - \alpha\delta_{jt}$. Under our assumptions, it follows that $E\left[\mu_t\left(\gamma_{jt}, \delta_{jt}\right) \mid \gamma_{jt}\right] = E\left[\eta_{jt} + \varepsilon_{jt} \mid \gamma_{jt}\right] = 0$. Since $E\left[\delta_{jt} \mid \gamma_{jt}\right] = 0$ by construction, it follows that $E\left[\bar{\mu}_t\left(\gamma_{jt}, \delta_{jt}\right) \mid \gamma_{jt}\right] = 0$ as required by the first part of Lemma 2. Given our proposed $\bar{r}_t$ in equation (24) it follows that

$$\begin{aligned}
\left(r_t + \mu_t\right) - \left(\bar{r}_t + \bar{\mu}_t\right) &= \left(r_t - \bar{r}_t\right) + \left(\mu_t - \bar{\mu}_t\right) \\
&= \left(-\alpha M_{jt} + \alpha\varphi_t\left(\gamma_{jt}\right)\right) + \left(\alpha\delta_{jt}\right) \\
&= -\alpha\left(\varphi_t\left(\gamma_{jt}\right) + \delta_{jt}\right) + \alpha\varphi_t\left(\gamma_{jt}\right) + \alpha\delta_{jt}, \\
&= 0,
\end{aligned}$$

where the third equality follows from equation (23). Therefore, the conditions for Lemma 2 are satisfied, and we have that r_t and $\bar{r}_t$ generate the same reduced form. Since r_t is not identified from the data, by Lemma 1 we have that the production function f_t is not identified either.

Appendix B: A Parametric Example

In order to illustrate our non-identification result, we consider a parametric example. Suppose that the true production function is Cobb-Douglas $F(L_{jt}, K_{jt}, M_{jt}) = L_{jt}^{\alpha} K_{jt}^{\beta} M_{jt}^{\gamma}$, and productivity follows an AR(1) process $\omega_{jt} = \delta_0 + \delta\omega_{jt-1} + \eta_{jt}$.[42] We can then write output (in logs) as

$$y_{jt} = \alpha l_{jt} + \beta k_{jt} + \gamma m_{jt} + \delta\omega_{jt-1} + \eta_{jt} + \varepsilon_{jt}.$$

If we plug in for M_{jt} using the first-order condition, plug in for ω_{jt-1} using the inverted intermediate inputs equation and group terms, we have the following:

$$\begin{aligned} y_{jt} &= constant + \alpha\left(\frac{1}{1-\gamma}\right) l_{jt} + \beta\left(\frac{1}{1-\gamma}\right) k_{jt} \\ &\quad + \frac{1}{1-\gamma}\delta\left(\mathbb{M}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)\right) + \frac{1}{1-\gamma}\eta_{jt} + \varepsilon_{jt}. \end{aligned}$$

The reduced form is given by

$$E\left[y_{jt} \mid \Gamma_{jt}\right] = constant + \alpha\left(\frac{1}{1-\gamma}\right) l_{jt} + \beta\left(\frac{1}{1-\gamma}\right) k_{jt} + \frac{1}{1-\gamma}\delta\left(\mathbb{M}^{-1}\left(L_{jt-1}, K_{jt-1}, M_{jt-1}\right)\right).$$

Since variation in $(L_{jt-1}, K_{jt-1}, M_{jt-1})$ is used to identify $\mathbb{M}^{-1}$, there are only three sources of variation left, $(L_{jt}, K_{jt}, \mathbb{M}^{-1})$, to identify four parameters $(\alpha, \beta, \gamma, \delta)$, and thus the model is not identified.

In the example above, we did not exploit that $\mathbb{M}^{-1}$ has a specific parametric form that

[42] For simplicity we assume that the production function and the prices of output and intermediate inputs are non-time-varying. Our point applies equally to the general time-varying case.

depends on the parameters of F. We now show that even imposing the parametric restrictions on $\mathbb{M}^{-1}$ is not enough to identify the production function, although we can now recover the AR(1) parameter, δ.

If we plug in for the parametric form of $\mathbb{M}^{-1}$ and take the conditional expectation of both sides we have

$$\begin{aligned} E\left[y_{jt} \mid \Gamma_{jt}\right] &= \tilde{constant} + \alpha\left(\frac{1}{1-\gamma}\right)l_{jt} + \beta\left(\frac{1}{1-\gamma}\right)k_{jt} \\ &\quad +\delta m_{jt-1} - \delta\alpha\frac{1}{1-\gamma}l_{jt-1} - \delta\beta\frac{1}{1-\gamma}k_{jt-1}. \end{aligned} \tag{25}$$

Notice that although there are five sources of variation $(l_{jt}, k_{jt}, m_{jt-1}, l_{jt-1}, k_{jt-1})$, the model is still not identified. Variation in m_{jt-1} identifies δ, but the coefficient on l_{jt} is equal to the coefficient on l_{jt-1} multiplied by $-\delta$, and the same is true for k. In other words, variation in l_{jt-1} and k_{jt-1} do not provide any additional information about the parameters of the production function. As a result, all we can identify is $\alpha\left(\frac{1}{1-\gamma}\right)$ and $\beta\left(\frac{1}{1-\gamma}\right)$. To put it another way, the rank condition necessary for identification of this model is not satisfied.

This is why Doraszelski and Jaumandreu (2013) need to rely on both parametric restrictions and observed price variation as an instrument for identification. Even using the parametric form of the first-order condition to replace for ω in the production function is not enough to achieve identification.

This illustrates an important difference with our approach. In addition to not requiring parametric restrictions (or price variation), we are not using the first-order condition to find a replacement function for ω in the production function. Instead, we use it to form the share regression equation, which gives us a second structural equation that we use in identification and estimation. It is the information contained in this additional equation, combined with the production function, that allows us to solve the identification problem. In terms of our

Cobb-Douglas example, the second equation would be given by the following share equation

$$s_{jt} = \gamma + \varepsilon_{jt}.$$

Since this equation identifies γ (given that $E\left[\varepsilon_{jt}\right] = 0$), this is enough to allow for identification of the whole production function and productivity.

Appendix C: Restricted Profit Functions and Value Added

In this appendix we provide the derivation of the results related to restricted profit functions described in Section 5.1. We derive the results in the context of Hicks-neutral technical change for consistency with the rest of the paper. However, the results we derive, and the presence of the bias term are general and not specific to the assumption of Hicks-neutrality.

Consider the empirical definition of real (double-deflated) value added without ex-post shocks, as in Bruno (1978):

$$VA_{jt}^{E} = Q_{jt} - M_{jt} = F_t\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}} - M_{jt} \equiv \mathcal{V}_t\left(L_{jt} K_{jt}, e^{\omega_{jt}}\right).$$

It can be shown that the total derivative of value added with respect to one of its inputs is equal to the partial derivative of gross output with respect to that input. For example, the total derivative of value added with respect to productivity is given by:

$$\begin{aligned} \frac{dVA_{jt}^{E}}{de^{\omega_{jt}}} &= \frac{d\mathcal{V}_t\left(L_{jt} K_{jt}, e^{\omega_{jt}}\right)}{de^{\omega_{jt}}} \\ &= \left[\frac{\partial F_t\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}}}{\partial e^{\omega_{jt}}} - \left(\frac{\partial F_t\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}}}{\partial M_{jt}} - 1\right) \frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\right] \\ &= \frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}. \end{aligned}$$

Due to the first order condition in equation (6), the term inside the parentheses on the second line is equal to zero, where the relative price of output to intermediate inputs has been normalized to one via deflation. This implies that the elasticity of gross output with respect to capital can be computed by multiplying the elasticity of value added with respect to ω by the ratio of value added to gross output, or equivalently one minus the share of intermediate inputs:

$$\underbrace{\left(\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{GO_{jt}}\right)}_{elas^{GO_{jt}}_{e^{\omega_{jt}}}} = \underbrace{\left(\frac{dVA^E_{jt}}{de^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{VA^E_{jt}}\right)}_{elas^{VA_{jt}}_{e^{\omega_{jt}}}}\frac{VA^E_{jt}}{GO^E_{jt}} = \underbrace{\left(\frac{dVA^E_{jt}}{de^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{VA^E_{jt}}\right)}_{elas^{VA_{jt}}_{e^{\omega_{jt}}}}(1 - S_{jt}).$$

The previous result, however, relies crucially on the absence of ex-post shocks to output. When we add back in the ex-post shocks we have the following relationship:

$$\begin{aligned}\frac{dVA^E_{jt}}{de^{\omega_{jt}}} &= \frac{d\mathcal{V}_t\left(L_{jt}K_{jt}, e^{\omega_{jt}}, e^{\varepsilon_{jt}}\right)}{de^{\omega_{jt}}} \\ &= \left[\frac{\partial F_t\left(L_{jt}, K_{jt}, M_{jt}\right)e^{\omega_{jt}+\varepsilon_{jt}}}{\partial e^{\omega_{jt}}} - \left(\frac{\partial F_t\left(L_{jt}, K_{jt}, M_{jt}\right)e^{\omega_{jt}+\varepsilon_{jt}}}{\partial M_{jt}} - 1\right)\frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\right].\end{aligned}$$

Notice now that the term inside the parentheses is no longer equal to zero, due to the presence of the ex-post shock, ε_{jt}. The reason is that the first order condition, which previously made that term equal to zero, is an *ex-ante* object, whereas what is inside the parentheses is *ex-post*. Therefore, the two derivatives are no longer equal, and we cannot simply transform the value added elasticities into their gross output counterparts by rescaling via the ratio of value added to gross output.

The first order condition implies that $\frac{\partial F_t(L_{jt},K_{jt},M_{jt})e^{\omega_{jt}+\varepsilon_{jt}}}{\partial M_{jt}} = \frac{e^{\varepsilon_{jt}}}{\mathcal{E}}$. In turn, this implies that

$$\begin{aligned}\frac{dVA^E_{jt}}{de^{\omega_{jt}}} &= \left[\frac{\partial F_t\left(L_{jt}, K_{jt}, M_{jt}\right)e^{\omega_{jt}+\varepsilon_{jt}}}{\partial e^{\omega_{jt}}} - \left(\frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1\right)\frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\right] \\ \Rightarrow elas^{VA^E_{jt}}_{e^{\omega_{jt}}} &= elas^{GO_{jt}}_{e^{\omega_{jt}}}\frac{GO_{jt}}{VA^E_{jt}} - \frac{\partial M_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}}}{VA^E_{jt}}\left(\frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1\right).\end{aligned}$$

The equation above can then be rearranged to form relationship between the elasticities as:

$$\underbrace{\left(\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}_{jt}}{GO_{jt}}\right)}_{elas^{GO_{jt}}_{e^{\omega_{jt}}}} = \underbrace{\left(\frac{\partial VA^{E}_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}_{jt}}{VA^{E}_{jt}}\right)}_{elas^{VA^{E}_{jt}}_{e^{\omega_{jt}}}}(1-S_{jt}) + \frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}_{jt}}{GO_{jt}}\left(\frac{e^{\varepsilon_{jt}}}{\mathcal{E}}-1\right).$$

A similar result holds when we analyze the elasticities with respect to the entire productivity shock, $e^{\omega_{jt}+\varepsilon_{jt}}$, instead of just the persistent component,$e^{\omega_{jt}}$. In this case we have the following relationship:

$$\underbrace{\left(\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}+\varepsilon_{jt}}_{jt}}{GO_{jt}}\right)}_{elas^{GO_{jt}}_{e^{\omega_{jt}+\varepsilon_{jt}}}} = \underbrace{\left(\frac{\partial VA^{E}_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}+\varepsilon_{jt}}_{jt}}{VA^{E}_{jt}}\right)}_{elas^{VA^{E}_{jt}}_{e^{\omega_{jt}+\varepsilon_{jt}}}}(1-S_{jt}) + \frac{\partial M_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}+\varepsilon_{jt}}_{jt}}{GO_{jt}}\left(\frac{e^{\varepsilon_{jt}}}{\mathcal{E}}-1\right).$$

Appendix D: "Structural" Value Added

Under the assumptions that 1) the underlying gross output production function takes a nested (i.e., weakly separable) form and 2) productivity enters in a *value-added augmenting* way, we can write the production function as:

$$Y_{jt} = F\left[\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}, M_{jt}\right]. \tag{26}$$

The goal is then to measure a subset of features of the gross output production function in equation (26) using the structural value-added production function

$$\mathcal{H}\left(L_{jt}, K_{jt}\right)e^{\omega_{jt}+\varepsilon_{jt}}. \tag{27}$$

This typically includes productivity $e^{\omega_{jt}+\varepsilon_{jt}}$ and the elasticities of $\mathcal{H}$ with respect to capital and labor.

As has been emphasized in the literature (see e.g., Griliches and Ringstad, 1971; Parks, 1971; Berndt and Wood, 1975; and Denny and May, 1977), the only cases in which the output of the structural value-added production function underlying (26) can be observed are when F takes one of two extreme possible forms: 1) perfect substitution between intermediate inputs and value added, and 2) perfect complementarity between intermediate inputs and value added.

In a special case of perfect substitution, (26) becomes

$$Y_{jt} = \mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}} + M_{jt}.$$

Thus the standard empirical measure of real value added, the difference between deflated output and deflated intermediate inputs $VA^E_{jt} \equiv Y_{jt} - M_{jt}$, equals the latent value added in equation (27).[43] However, perfect substitution is an unreasonable description of a production process, as it implies that final output can be produced from intermediate inputs alone.

As discussed in Section 5.2, for the Leontief case we have

$$Y_{jt} = \min\left[\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}, \mathcal{C}\left(M_{jt}\right)\right]. \tag{28}$$

If it were the case that $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}} = \mathcal{C}\left(M_{jt}\right)$, and if we further assumed that $\mathcal{C}\left(M_{jt}\right) = cM_{jt}$, then the empirical measure of value added would be $VA^E = \left(1 - \frac{1}{c}\right)\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$, and the structural value-added production function would correspond to the empirical measure.

The problem is that the key condition, $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}} = cM_{jt}$, will not hold because of the presence of the ex-post shock ε_{jt}. Even if firms chose to equalize $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}}\mathcal{E}$ and cM_{jt} (not necessarily an optimal solution), since ε_{jt} is realized after this decision is made, the key condition will generally not hold unless the realization of the shock is such that $e^{\varepsilon_{jt}} = \mathcal{E}\ \forall j, t$. Thus VA^E_{jt} does not correspond to the structural value-added production function $\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}}$.

[43] For simplicity, throughout this section we use the double-deflated version of empirical value added. Our results do not depend on the method of deflating value added. See Bruno (1978) for a discussion of the differences between the alternative ways of deflating value added.

In addition, even ignoring the fact that epsilon is realized after input decisions are made, there is also the problem associated with L and K being quasi-fixed. Under Assumption 3, firms either cannot adjust capital and labor in period t or can only do so with some positive adjustment cost. The key consequence is that firms may optimally choose to *not* equate $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$ and $\mathcal{C}(M_{jt})$, i.e., it may be optimal for the firm to hold onto a larger stock of L and K than can be combined with M, if L and K are both costly (or impossible) to downwardly adjust.[44]

It is also the case that moving ε_{jt} outside of the min function does not help. Suppose that instead of equation (28), one wrote the production function as: $Y_{jt} = \min[\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}}, \mathcal{C}(M_{jt})]e^{\varepsilon_{jt}}$. Since ε_{jt} appears outside of the min function, it is now possible for firms to set the two terms inside the min equal to each other. However, this does not imply that VA^E can be used to measure the structural value-added production function in this case. While it will be true that $Q_{jt} - M_{jt}$ will be proportional to $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}}$ when $\mathcal{C}(M_{jt}) = cM_{jt}$, Q_{jt} is unobserved, where recall that $Q_{jt} = Y_{jt}e^{-\varepsilon_{jt}}$. Furthermore, if we multiply by $e^{\varepsilon_{jt}}$ then we will have that $Y_{jt} - M_{jt}e^{\varepsilon_{jt}}$ is proportional to the value added production function $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$. However, we cannot form $Y_{jt} - M_{jt}e^{\varepsilon_{jt}}$ since ε_{jt} is unobserved. In addition, the problem associated with L and K being quasi-fixed still applies. Neither of these problems are resolved by moving both ω_{jt} and ε_{jt} outside of the min function.

[44] For example, suppose $\mathcal{C}(M_{jt}) = M_{jt}^{0.5}$. For simplicity, also suppose that capital and labor are fixed one period ahead, and therefore cannot be adjusted in the short-run. When $M_{jt}^{0.5} \leq \mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$, marginal revenue with respect to intermediate inputs equals $\frac{\partial \mathcal{C}(M_{jt})}{\partial Mjt}aP_t$. When $M_{jt}^{0.5} > \mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$, increasing M_{jt} does not increase output due to the Leontief structure, so marginal revenue is zero. Marginal cost in both cases equals the price of intermediate inputs ρ_t. The firm's optimal choice of M is therefore given by $M_{jt} = \left(\frac{P_t}{\rho_t}0.5a\right)^2$, if $\left(\frac{P_t}{\rho_t}0.5a\right) < \mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$. But when $\left(\frac{P_t}{\rho_t}0.5a\right) > \mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$, the firm no longer finds it optimal to set $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}} = \mathcal{C}(M_{jt})$, and prefers to hold onto excess capital and labor.

Table 1: Average Input Elasticities of Output

(Structural Estimates: Value Added vs. Gross Ouput)

Industry (ISIC Code)

	Food Products (311)		**Textiles (321)**		**Apparel (322)**		**Wood Products (331)**		**Fabricated Metals (381)**		**All**	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
Colombia												
Labor	0.70	0.22	0.65	0.32	0.83	0.42	0.86	0.44	0.89	0.43	0.78	0.35
	(0.04)	(0.02)	(0.06)	(0.03)	(0.03)	(0.02)	(0.06)	(0.05)	(0.04)	(0.02)	(0.01)	(0.01)
Capital	0.33	0.12	0.36	0.16	0.16	0.05	0.12	0.04	0.25	0.10	0.31	0.14
	(0.02)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.04)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)
Intermediates	--	0.67	--	0.54	--	0.52	--	0.51	--	0.53	--	0.54
		(0.01)		(0.01)		(0.01)		(0.01)		(0.01)		(0.00)
Sum	1.03	1.01	1.01	1.01	0.99	0.99	0.98	0.99	1.14	1.06	1.09	1.04
	(0.03)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.07)	(0.04)	(0.02)	(0.01)	(0.01)	(0.00)
Mean(Capital) / Mean(Labor)	0.47	0.55	0.55	0.49	0.19	0.12	0.14	0.08	0.28	0.23	0.39	0.40
	(0.06)	(0.08)	(0.10)	(0.09)	(0.03)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)	(0.02)	(0.03)
Chile												
Labor	0.77	0.28	0.93	0.45	0.95	0.45	0.92	0.40	0.96	0.52	0.77	0.38
	(0.02)	(0.01)	(0.04)	(0.03)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.03)	(0.01)	(0.01)
Capital	0.33	0.11	0.24	0.11	0.20	0.06	0.19	0.07	0.25	0.13	0.37	0.16
	(0.01)	(0.01)	(0.02)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.00)
Intermediates	--	0.67	--	0.54	--	0.56	--	0.59	--	0.50	--	0.55
		(0.00)		(0.01)		(0.01)		(0.01)		(0.01)		(0.00)
Sum	1.10	1.05	1.17	1.10	1.14	1.08	1.11	1.06	1.22	1.15	1.13	1.09
	(0.02)	(0.01)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.01)	(0.03)	(0.02)	(0.01)	(0.01)
Mean(Capital) / Mean(Labor)	0.43	0.39	0.26	0.24	0.21	0.14	0.21	0.18	0.26	0.25	0.48	0.43
	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathcal{C}$) of our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table 2: Heterogeneity in Productivity

(Structural Estimates)

Colombia

Industry (ISIC Code)

	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
75/25 ratio	2.20	1.33	1.97	1.35	1.66	1.29	1.73	1.30	1.78	1.31	1.95	1.37
	(0.07)	(0.02)	(0.09)	(0.03)	(0.03)	(0.01)	(0.08)	(0.04)	(0.04)	(0.02)	(0.17)	(0.01)
90/10 ratio	5.17	1.77	3.71	1.83	2.87	1.66	3.08	1.80	3.33	1.74	4.01	1.86
	(0.27)	(0.05)	(0.30)	(0.07)	(0.09)	(0.03)	(0.38)	(0.12)	(0.13)	(0.03)	(0.07)	(0.02)
95/5 ratio	11.01	2.24	6.36	2.38	4.36	2.02	4.58	2.24	5.31	2.16	6.86	2.36
	(1.11)	(0.08)	(0.76)	(0.14)	(0.22)	(0.05)	(1.01)	(0.22)	(0.34)	(0.06)	(0.02)	(0.03)
Exporter	3.62	0.14	0.20	0.02	0.16	0.05	0.26	0.15	0.20	0.08	0.51	0.06
	(0.99)	(0.05)	(0.10)	(0.03)	(0.07)	(0.03)	(0.63)	(0.14)	(0.05)	(0.03)	(0.12)	(0.01)
Importer	-0.25	0.04	0.27	0.05	0.29	0.12	0.06	0.05	0.26	0.10	0.20	0.11
	(0.08)	(0.02)	(0.10)	(0.04)	(0.08)	(0.03)	(0.53)	(0.08)	(0.06)	(0.02)	(0.05)	(0.01)
Advertiser	-0.46	-0.03	0.20	0.08	0.13	0.05	0.02	0.04	0.15	0.05	-0.13	0.03
	(0.10)	(0.02)	(0.07)	(0.03)	(0.04)	(0.02)	(0.09)	(0.04)	(0.04)	(0.02)	(0.06)	(0.01)
Wages > Median	0.59	0.09	0.60	0.18	0.41	0.18	0.34	0.15	0.55	0.22	0.63	0.20
	(0.19)	(0.02)	(0.09)	(0.03)	(0.03)	(0.02)	(0.17)	(0.04)	(0.06)	(0.02)	(0.05)	(0.01)

Chile

	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
75/25 ratio	2.92	1.37	2.56	1.48	2.58	1.43	3.06	1.50	2.45	1.53	3.00	1.55
	(0.05)	(0.01)	(0.07)	(0.02)	(0.07)	(0.02)	(0.08)	(0.02)	(0.06)	(0.02)	(0.03)	(0.01)
90/10 ratio	9.02	1.90	6.77	2.16	6.76	2.11	10.12	2.32	6.27	2.33	9.19	2.39
	(0.30)	(0.02)	(0.30)	(0.05)	(0.33)	(0.05)	(0.60)	(0.05)	(0.27)	(0.05)	(0.15)	(0.02)
95/5 ratio	21.29	2.48	13.56	2.91	14.21	2.77	25.08	3.11	12.52	3.13	20.90	3.31
	(0.99)	(0.05)	(0.84)	(0.09)	(0.77)	(0.09)	(2.05)	(0.11)	(0.78)	(0.10)	(0.47)	(0.04)
Exporter	0.27	0.02	0.07	0.02	0.18	0.09	0.12	0.00	0.03	-0.01	0.20	0.03
	(0.10)	(0.02)	(0.07)	(0.03)	(0.08)	(0.03)	(0.12)	(0.03)	(0.06)	(0.03)	(0.04)	(0.01)
Importer	0.71	0.14	0.22	0.10	0.31	0.14	0.44	0.15	0.30	0.11	0.46	0.15
	(0.11)	(0.02)	(0.05)	(0.02)	(0.05)	(0.02)	(0.10)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)
Advertiser	0.18	0.04	0.09	0.04	0.15	0.06	0.04	0.03	0.07	0.01	0.14	0.06
	(0.05)	(0.01)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)
Wages > Median	1.23	0.21	0.47	0.19	0.62	0.22	0.68	0.21	0.56	0.22	0.99	0.30
	(0.09)	(0.01)	(0.06)	(0.02)	(0.06)	(0.02)	(0.08)	(0.02)	(0.06)	(0.02)	(0.04)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the nonparametric functions (G and $\mathcal{C}$) of our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile value added implies that a firm that advertises is, on average, 18% more productive than a firm that does not advertise.

Table 3: Heterogeneity in Productivity

(Uncorrected OLS Estimates)

Colombia

Industry (ISIC Code)

	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)
75/25 ratio	2.17	1.16	1.86	1.21	1.65	1.17	1.72	1.23	1.78	1.23	1.93	1.24
	(0.06)	(0.01)	(0.06)	(0.01)	(0.03)	(0.01)	(0.06)	(0.02)	(0.04)	(0.01)	(0.02)	(0.00)
90/10 ratio	5.15	1.42	3.50	1.51	2.81	1.44	3.05	1.57	3.30	1.53	3.96	1.58
	(0.27)	(0.02)	(0.18)	(0.04)	(0.08)	(0.02)	(0.22)	(0.06)	(0.12)	(0.02)	(0.06)	(0.01)
95/5 ratio	10.86	1.74	5.77	1.82	4.23	1.74	4.67	2.01	5.22	1.82	6.81	1.94
	(0.94)	(0.05)	(0.55)	(0.08)	(0.20)	(0.04)	(0.72)	(0.15)	(0.31)	(0.04)	(0.15)	(0.02)
Exporter	3.42	0.09	-0.03	-0.01	0.10	0.00	0.21	0.10	0.12	0.03	0.45	0.01
	(0.99)	(0.04)	(0.04)	(0.01)	(0.05)	(0.01)	(0.19)	(0.09)	(0.04)	(0.02)	(0.12)	(0.01)
Importer	-0.23	-0.02	0.09	0.00	0.21	0.02	0.02	-0.03	0.20	0.05	0.14	0.04
	(0.07)	(0.01)	(0.06)	(0.01)	(0.06)	(0.01)	(0.06)	(0.02)	(0.05)	(0.01)	(0.04)	(0.01)
Advertiser	-0.46	-0.07	0.11	-0.04	0.10	-0.03	0.01	-0.02	0.08	0.00	-0.16	-0.02
	(0.11)	(0.02)	(0.05)	(0.02)	(0.03)	(0.01)	(0.07)	(0.03)	(0.04)	(0.01)	(0.06)	(0.01)
Wages > Median	0.51	0.06	0.49	0.10	0.39	0.13	0.33	0.11	0.50	0.13	0.56	0.13
	(0.15)	(0.02)	(0.07)	(0.02)	(0.03)	(0.01)	(0.08)	(0.03)	(0.04)	(0.01)	(0.05)	(0.01)
Chile												
75/25 ratio	2.91	1.30	2.57	1.40	2.56	1.36	3.07	1.39	2.47	1.46	3.01	1.45
	(0.05)	(0.00)	(0.07)	(0.01)	(0.07)	(0.01)	(0.08)	(0.01)	(0.06)	(0.01)	(0.03)	(0.00)
90/10 ratio	9.00	1.72	6.63	1.97	6.64	1.91	10.21	2.03	6.27	2.14	9.13	2.14
	(0.29)	(0.01)	(0.31)	(0.04)	(0.29)	(0.03)	(0.57)	(0.04)	(0.26)	(0.04)	(0.15)	(0.01)
95/5 ratio	20.93	2.15	13.49	2.57	14.20	2.45	25.26	2.77	12.18	2.80	20.64	2.86
	(0.96)	(0.02)	(0.83)	(0.07)	(0.80)	(0.05)	(2.05)	(0.07)	(0.77)	(0.06)	(0.47)	(0.03)
Exporter	0.17	-0.01	0.04	-0.02	0.12	0.01	0.12	-0.02	0.00	0.00	0.15	-0.01
	(0.09)	(0.02)	(0.06)	(0.02)	(0.08)	(0.02)	(0.09)	(0.02)	(0.06)	(0.02)	(0.04)	(0.01)
Importer	0.57	0.03	0.20	0.04	0.26	0.06	0.41	0.07	0.27	0.06	0.41	0.09
	(0.09)	(0.01)	(0.04)	(0.02)	(0.05)	(0.01)	(0.09)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)
Advertiser	0.12	0.00	0.07	0.01	0.11	0.02	0.02	0.01	0.05	0.01	0.10	0.04
	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)
Wages > Median	1.11	0.12	0.45	0.15	0.58	0.16	0.66	0.13	0.53	0.16	0.94	0.24
	(0.07)	(0.01)	(0.05)	(0.02)	(0.06)	(0.02)	(0.07)	(0.02)	(0.06)	(0.02)	(0.03)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with OLS. The numbers in the second column are based on a gross output specification estimated using a complete polynomial series of degree 2 with OLS.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile value added implies that a firm that advertises is, on average, 12% more productive than a firm that does not advertise.

Table 4: Average Input Elasticities of Output

(Gross Output: Structural vs. Uncorrected OLS Estimates)

Colombia

	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)
Labor	0.15	0.22	0.21	0.32	0.32	0.42	0.32	0.44	0.29	0.43	0.26	0.35
	(0.01)	(0.02)	(0.02)	(0.03)	(0.01)	(0.02)	(0.03)	(0.05)	(0.02)	(0.02)	(0.01)	(0.01)
Capital	0.04	0.12	0.06	0.16	0.01	0.05	0.03	0.04	0.03	0.10	0.06	0.14
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.00)	(0.01)
Intermediates	0.82	0.67	0.76	0.54	0.68	0.52	0.65	0.51	0.73	0.53	0.72	0.54
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Sum	1.01	1.01	1.03	1.01	1.01	0.99	1.00	0.99	1.05	1.06	1.04	1.04
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.04)	(0.01)	(0.01)	(0.00)	(0.00)
Mean(Capital) / Mean(Labor)	0.27	0.55	0.27	0.49	0.04	0.12	0.08	0.08	0.11	0.23	0.23	0.40
	(0.07)	(0.08)	(0.06)	(0.09)	(0.02)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)	(0.01)	(0.03)

Industry (ISIC Code)

Chile

	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)
Labor	0.17	0.28	0.26	0.45	0.29	0.45	0.20	0.40	0.32	0.52	0.20	0.38
	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.03)	(0.01)	(0.01)
Capital	0.05	0.11	0.06	0.11	0.03	0.06	0.02	0.07	0.07	0.13	0.09	0.16
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Intermediates	0.83	0.67	0.75	0.54	0.74	0.56	0.81	0.59	0.71	0.50	0.77	0.55
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Sum	1.05	1.05	1.06	1.10	1.06	1.08	1.04	1.06	1.10	1.15	1.06	1.09
	(0.00)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.00)	(0.01)
Mean(Capital) / Mean(Labor)	0.28	0.39	0.22	0.24	0.12	0.14	0.12	0.18	0.21	0.25	0.42	0.43
	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.05)	(0.03)	(0.04)	(0.03)	(0.02)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 with OLS. The numbers in the second column are also based on a gross output specification using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table 5: Average Input Elasticities of Output--Rescaled Value Added

(Structural Estimates: Rescaled Value Added vs. Gross Ouput)

Colombia

	Food Products (311)			Textiles (321)			Apparel (322)			Wood Products (331)			Fabricated Metals (381)			All		
	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)
Labor	0.70	0.20	0.22	0.65	0.28	0.32	0.83	0.38	0.42	0.86	0.40	0.44	0.89	0.40	0.43	0.78	0.33	0.35
	(0.04)	(0.01)	(0.02)	(0.06)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.06)	(0.03)	(0.05)	(0.04)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
Capital	0.33	0.08	0.12	0.36	0.15	0.16	0.16	0.07	0.05	0.12	0.05	0.04	0.25	0.11	0.10	0.31	0.13	0.14
	(0.02)	(0.01)	(0.01)	(0.04)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.04)	(0.02)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)
Intermediates	--	--	0.67	--	--	0.54	--	--	0.52	--	--	0.51	--	--	0.53	--	--	0.54
			(0.01)			(0.01)			(0.01)			(0.01)			(0.01)			(0.00)
Sum	1.03	1.01	1.01	1.01	1.00	1.01	0.99	0.99	0.99	0.98	0.99	0.99	1.14	1.06	1.06	1.09	1.04	1.04
	(0.03)	(0.01)	(0.01)	(0.04)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.07)	(0.03)	(0.04)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Mean(Capital) / Mean(Labor)	0.47	0.42	0.55	0.55	0.55	0.49	0.19	0.19	0.12	0.14	0.14	0.08	0.28	0.27	0.23	0.39	0.38	0.40
	(0.06)	(0.05)	(0.08)	(0.10)	(0.10)	(0.09)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)	(0.04)	(0.02)	(0.02)	(0.03)

Chile

	Food Products (311)			Textiles (321)			Apparel (322)			Wood Products (331)			Fabricated Metals (381)			All		
	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)
Labor	0.77	0.21	0.28	0.93	0.37	0.45	0.95	0.37	0.45	0.92	0.32	0.40	0.96	0.43	0.52	0.77	0.29	0.38
	(0.02)	(0.01)	(0.01)	(0.04)	(0.02)	(0.03)	(0.04)	(0.02)	(0.02)	(0.04)	(0.01)	(0.02)	(0.04)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)
Capital	0.33	0.10	0.11	0.24	0.10	0.11	0.20	0.08	0.06	0.19	0.07	0.07	0.25	0.11	0.13	0.37	0.14	0.16
	(0.01)	(0.00)	(0.01)	(0.02)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Intermediates	--	--	0.67	--	--	0.54	--	--	0.56	--	--	0.59	--	--	0.50	--	--	0.55
			(0.00)			(0.01)			(0.01)			(0.01)			(0.01)			(0.00)
Sum	1.10	1.03	1.05	1.17	1.07	1.10	1.14	1.06	1.08	1.11	1.04	1.06	1.22	1.09	1.15	1.13	1.05	1.09
	(0.02)	(0.00)	(0.01)	(0.03)	(0.01)	(0.02)	(0.03)	(0.01)	(0.02)	(0.03)	(0.01)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.00)	(0.01)
Mean(Capital) / Mean(Labor)	0.43	0.46	0.39	0.26	0.26	0.24	0.21	0.21	0.14	0.21	0.22	0.18	0.26	0.27	0.25	0.48	0.49	0.43
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are obtained by raising the value-added estimates to the power of one minus the firm's share of intermediate inputs in total output. The numbers in the third column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table 6: Heterogeneity in Productivity--Rescaled Value Added

(Structural Estimates: Rescaled Value Added vs. Gross Ouput)

Colombia

Industry (ISIC Code)

	Food Products (311)			Textiles (321)			Apparel (322)			Wood Products (331)			Fabricated Metals (381)			All		
	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)
75/25 ratio	2.20	2.29	1.33	1.97	1.85	1.35	1.66	1.88	1.29	1.73	2.74	1.30	1.78	2.00	1.31	1.95	2.15	1.37
	(0.07)	(0.12)	(0.02)	(0.09)	(0.12)	(0.03)	(0.03)	(0.09)	(0.01)	(0.08)	(0.44)	(0.04)	(0.04)	(0.10)	(0.02)	(0.17)	(0.06)	(0.01)
90/10 ratio	5.17	4.99	1.77	3.71	3.28	1.83	2.87	3.60	1.66	3.08	6.27	1.80	3.33	3.81	1.74	4.01	4.68	1.86
	(0.27)	(0.51)	(0.05)	(0.30)	(0.39)	(0.07)	(0.09)	(0.34)	(0.03)	(0.38)	(1.51)	(0.12)	(0.13)	(0.35)	(0.03)	(0.07)	(0.24)	(0.02)
95/5 ratio	11.01	8.79	2.24	6.36	5.00	2.38	4.36	5.41	2.02	4.58	10.53	2.24	5.31	5.85	2.16	6.86	7.91	2.36
	(1.11)	(1.20)	(0.08)	(0.76)	(0.87)	(0.14)	(0.22)	(0.67)	(0.05)	(1.01)	(3.19)	(0.22)	(0.34)	(0.69)	(0.06)	(0.02)	(0.52)	(0.03)
Exporter	3.62	0.84	0.14	0.20	0.07	0.02	0.16	-0.04	0.05	0.26	0.55	0.15	0.20	0.06	0.08	0.51	0.09	0.06
	(0.99)	(0.30)	(0.05)	(0.10)	(0.08)	(0.03)	(0.07)	(0.07)	(0.03)	(0.63)	(0.60)	(0.14)	(0.05)	(0.07)	(0.03)	(0.12)	(0.06)	(0.01)
Importer	-0.25	-0.40	0.04	0.27	0.08	0.05	0.29	-0.09	0.12	0.06	-0.20	0.05	0.26	0.15	0.10	0.20	0.05	0.11
	(0.08)	(0.08)	(0.02)	(0.10)	(0.08)	(0.04)	(0.08)	(0.05)	(0.03)	(0.53)	(0.20)	(0.08)	(0.06)	(0.04)	(0.02)	(0.05)	(0.04)	(0.01)
Advertiser	-0.46	-0.42	-0.03	0.20	-0.16	0.08	0.13	-0.28	0.05	0.02	-0.24	0.04	0.15	-0.05	0.05	-0.13	-0.18	0.03
	(0.10)	(0.10)	(0.02)	(0.07)	(0.07)	(0.03)	(0.04)	(0.06)	(0.02)	(0.09)	(0.13)	(0.04)	(0.04)	(0.04)	(0.02)	(0.06)	(0.03)	(0.01)
Wages > Median	0.59	0.15	0.09	0.60	0.35	0.18	0.41	0.31	0.18	0.34	0.19	0.15	0.55	0.27	0.22	0.63	0.29	0.20
	(0.19)	(0.18)	(0.02)	(0.09)	(0.08)	(0.03)	(0.03)	(0.05)	(0.02)	(0.17)	(0.18)	(0.04)	(0.06)	(0.04)	(0.02)	(0.05)	(0.04)	(0.01)

Chile

	Food Products (311)			Textiles (321)			Apparel (322)			Wood Products (331)			Fabricated Metals (381)			All		
	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)
75/25 ratio	2.92	2.36	1.37	2.56	2.06	1.48	2.58	2.94	1.43	3.06	3.31	1.50	2.45	2.47	1.53	3.00	2.92	1.55
	(0.05)	(0.11)	(0.01)	(0.07)	(0.17)	(0.02)	(0.07)	(0.27)	(0.02)	(0.08)	(0.36)	(0.02)	(0.06)	(0.19)	(0.02)	(0.03)	(0.10)	(0.01)
90/10 ratio	9.02	5.24	1.90	6.77	3.80	2.16	6.76	8.08	2.11	10.12	10.47	2.32	6.27	5.72	2.33	9.19	7.24	2.39
	(0.30)	(0.44)	(0.02)	(0.30)	(0.57)	(0.05)	(0.33)	(1.44)	(0.05)	(0.60)	(2.23)	(0.05)	(0.27)	(0.85)	(0.05)	(0.15)	(0.44)	(0.02)
95/5 ratio	21.29	8.91	2.48	13.56	5.54	2.91	14.21	13.89	2.77	25.08	19.83	3.11	12.52	9.40	3.13	20.90	12.23	3.31
	(0.99)	(0.96)	(0.05)	(0.84)	(1.05)	(0.09)	(0.77)	(2.94)	(0.09)	(2.05)	(5.41)	(0.11)	(0.78)	(1.84)	(0.10)	(0.47)	(0.91)	(0.04)
Exporter	0.27	0.34	0.02	0.07	-0.03	0.02	0.18	0.14	0.09	0.12	0.09	0.00	0.03	0.01	-0.01	0.20	0.12	0.03
	(0.10)	(0.12)	(0.02)	(0.07)	(0.06)	(0.03)	(0.08)	(0.11)	(0.03)	(0.12)	(0.11)	(0.03)	(0.06)	(0.07)	(0.03)	(0.04)	(0.05)	(0.01)
Importer	0.71	0.44	0.14	0.22	0.10	0.10	0.31	0.18	0.14	0.44	0.21	0.15	0.30	0.19	0.11	0.46	0.37	0.15
	(0.11)	(0.15)	(0.02)	(0.05)	(0.05)	(0.02)	(0.05)	(0.07)	(0.02)	(0.10)	(0.12)	(0.03)	(0.05)	(0.07)	(0.02)	(0.03)	(0.04)	(0.01)
Advertiser	0.18	0.05	0.04	0.09	0.00	0.04	0.15	0.06	0.06	0.04	-0.04	0.03	0.07	0.03	0.01	0.14	0.14	0.06
	(0.05)	(0.05)	(0.01)	(0.04)	(0.04)	(0.02)	(0.04)	(0.06)	(0.02)	(0.04)	(0.06)	(0.01)	(0.04)	(0.06)	(0.02)	(0.02)	(0.02)	(0.01)
Wages > Median	1.23	0.66	0.21	0.47	0.34	0.19	0.62	0.57	0.22	0.68	0.53	0.21	0.56	0.47	0.22	0.99	0.89	0.30
	(0.09)	(0.07)	(0.01)	(0.06)	(0.06)	(0.02)	(0.06)	(0.09)	(0.02)	(0.08)	(0.11)	(0.02)	(0.06)	(0.09)	(0.02)	(0.04)	(0.05)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are obtained by raising the value-added estimates to the power of one minus the firm's share of intermediate inputs in total output. The numbers in the third column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile value added implies that a firm that advertises is, on average, 18% more productive than a firm that does not advertise.

Online Appendix O1: Revenue Production Functions

In this appendix we illustrate how our approach can be extended to accommodate imperfect competition. Since firms no longer necessarily charge the same price, when output prices are not observed, deflated revenue no longer properly measures the quantity that the firm produces. As a result, unobserved variation in firm-specific prices needs to be addressed in the production function. One solution to this problem suggested by Klette and Griliches (1996) and recently applied by De Loecker (2011) is to model unobserved prices via an iso-elastic demand system. While this demand system decomposes the problem of unobserved prices in the production function in a convenient way, we now show that exactly the same identification problem involving intermediate inputs arises in the resulting *revenue* production function. Furthermore, as the solution involves modeling unobserved output prices, value added is no longer an appropriate measure of output as consumers have demand for gross output and not for value added.

Suppose we follow Klette and Griliches (1996) and De Loecker (2011) and specify an iso-elastic demand system derived from an underlying representative CES utility function,

$$\frac{P_{jt}}{\Pi_t} = \left(\frac{Q_{jt}}{Q_t}\right)^{\frac{1}{\sigma}} e^{\chi_{jt}}, \tag{O1-1}$$

where P_{jt} is the output price of the firm, Π_t is the industry price index, Q_t is a quantity index that plays the role of an aggregate demand shifter, χ_{jt} is an observable (to the firm) demand shock, and σ is the assumed constant elasticity of demand.

What we observe in the data is the firm's real revenue, which in logs is given by $r_{jt} = (p_{jt} - \pi_t) + y_{jt}$. Recalling equation (4), and replacing (O1-1) into the log revenue equation gives

$$r_{jt} \;=\; \left(1+\frac{1}{\sigma}\right) f(L_{jt}, K_{jt}, M_{jt}) - \frac{1}{\sigma} q_t + \chi_{jt} + \left(1+\frac{1}{\sigma}\right)\omega_{jt} + \varepsilon_{jt}. \tag{O1-2}$$

Thus, the persistent part of the residual is a linear combination of the demand shock and productivity shock, i.e., $\chi_{jt} + \left(1 + \frac{1}{\sigma}\right)\omega_{jt}$. However, it is precisely this same linear combination of the demand and productivity shocks that shifts the intermediate input demand. To see why, observe that short-run profits are given by

$$\begin{aligned} \text{SRProfits}_{jt} &= P_{jt}Q_{jt} - \rho_t M_{jt} \\ &= \Pi_t \left(\frac{1}{Q_t}\right)^{\frac{1}{\sigma}} \left(F\left(L_{jt}, K_{jt}, M_{jt}\right)\right)^{1+\frac{1}{\sigma}} e^{\chi_{jt}+\left(1+\frac{1}{\sigma}\right)\omega_{jt}} - \rho_t M_{jt}. \end{aligned}$$

Notice that the productivity and demand shocks (ω_{jt}, χ_{jt}) enter profits only through the sum, $\chi_{jt} + \left(1 + \frac{1}{\sigma}\right)\omega_{jt}$. It is only this linear combination that matters for short-run profits and hence for any static optimization problems, including the demand for intermediate inputs M_{jt}, i.e., $M_{jt} = \mathbb{M}_t(L_{jt}, K_{jt}, \chi_{jt} + \left(1 + \frac{1}{\sigma}\right)\omega_{jt})$.[45] Thus we are left with precisely the same identification problem that was shown in Section 2. Even though we now have two unobservables (ω_{jt}, χ_{jt}), there still does not exist any exclusion restriction that can vary intermediate inputs M_{jt} from outside of the revenue production function (O1-2).

We now show that our empirical strategy can be extended to the setting with imperfect competition and revenue production functions such that 1) we solve the identification problem with flexible inputs and 2) we can recover time-varying industry markups.[46] In fact, our empirical strategy allows for the identification of pieces of the production function as well as the time pattern (but not the level) of markups without having to specify any particular demand system.

Letting Λ_{jt} denote a firm's marginal cost, the first order condition with respect to M_{jt} for a cost minimizing firm is: $\Lambda_{jt} F_M\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}} = \rho_t$. Following the same strategy as before, we can rewrite this expression in terms of the observed log revenue share, which

[45] As opposed to the flexible inputs, it is not clear how the demand for quasi-fixed inputs (e.g., capital) will depend on ω_{jt} and χ_{jt}, i.e., whether it will depend on the same linear combination or on each component independently (and whether it will be monotone in each shock).

[46] This stands in contrast to the Klette and Griliches (1996) approach that can only allow for a markup that is time-invariant.

becomes

$$s_{jt} = -\psi_{jt} + \ln\left(G\left(L_{jt}, K_{jt}, M_{jt}\right)\mathcal{E}\right) - \varepsilon_{jt}, \tag{O1-3}$$

where $\psi_{jt} = \ln \frac{P_{jt}}{\Lambda_{jt}}$ is the log of the markup, $G(\cdot)$ is the output elasticity of intermediate inputs, and ε_{jt} is the ex-post shock. Equation (O1-3) nests the one obtained for the perfectly competitive case in (8), the only difference being the addition of the log markup ψ_{jt} which is equal to 0 under perfect competition. The two key differences between the perfectly competitive case and this case are that 1) we no longer restrict the firm's price to be constant, and 2) the firm's anticipated revenue share no longer equals the input elasticity directly, but rather it equals the input elasticity divided by the markup charged by the firm.

We now show how to use the share regression (O1-3) to identify production functions among imperfectly competitive firms. As opposed to the Klette and Griliches (1996) setup, in which markups are restricted to be constant, $\psi_{jt} = \psi$, we allow for markups to change over time, $\psi_{jt} = \psi_t$. In this case (O1-3) becomes

$$s_{jt} = -\psi_{t} + \ln\left(G\left(L_{jt}, K_{jt}, M_{jt}\right)\mathcal{E}\right) - \varepsilon_{jt}. \tag{O1-4}$$

The intermediate input elasticity can be rewritten so that we can break it into two parts: a component that varies with inputs and a constant μ, i.e., $\ln \xi_{jt} = \ln G\left(L_{jt}, K_{jt}, M_{jt}\right) = \ln G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) + \mu$. Then, equation (O1-4) becomes

$$\begin{aligned} s_{jt} &= \left(-\psi_t + \mu\right) + \ln \mathcal{E} + \ln G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) - \varepsilon_{jt} \\ &= -\delta_t + \ln \mathcal{E} + \ln G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) - \varepsilon_{jt}. \end{aligned} \tag{O1-5}$$

As equation (O1-5) makes clear, without having to specify a demand system and without observing prices, we can nonparametrically recover the ex-post shock ε_{jt} (and hence $\mathcal{E}$), the output elasticity of intermediate inputs up to a constant $\ln \xi^{\mu}_{jt} = \ln G^{\mu}(L_{jt}, K_{jt}, M_{jt}) = \ln G(L_{jt}, K_{jt}, M_{jt}) - \mu$, and the time-varying markups up to the same constant, $\delta_t = \psi_t - \mu$.

This is, to the best of our knowledge, a new result.[47] Recovering the growth pattern of markups over time is useful as an independent result as it can, for example, be used to check whether market power has increased over time, or to analyze the behavior of market power with respect to the business cycle.

As before, we can correct our estimates for $\mathcal{E}$ and solve the differential equation that arises from equation (O1-5). However, because we can still only identify the elasticity up to the constant μ, we have to be careful about keeping track of it as we can only calculate $\int G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}} = e^{-\mu} \int G\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}}$. It follows that

$$f\left(L_{jt}, K_{jt}, M_{jt}\right) e^{-\mu} + \mathscr{C}\left(L_{jt}, K_{jt}\right) e^{-\mu} = \int G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}}.$$

From this equation it is immediately apparent that, without further information, we will not be able to separate the integration constant $\mathscr{C}\left(L_{jt}, K_{jt}\right)$ from the unknown constant μ.

To see how both the constant μ and the constant of integration can be recovered, we specify a generalized version of the demand system in equation (O1-1)

$$\frac{P_{jt}}{\Pi_t} = \left(\frac{Q_{jt}}{Q_t}\right)^{\frac{1}{\sigma_t}} e^{\chi_{jt}}, \tag{O1-6}$$

where we allow for time-varying markups and hence $\psi_t = -\ln\left(1 + \frac{1}{\sigma_t}\right)$.[48] In this case the observed log-revenue production function (O1-2) becomes

$$r_{jt} = \left(1 + \frac{1}{\sigma_t}\right) f(L_{jt}, K_{jt}, M_{jt}) - \frac{1}{\sigma_t} q_t + \chi_{jt} + \left(1 + \frac{1}{\sigma_t}\right) \omega_{jt} + \varepsilon_{jt}. \tag{O1-7}$$

[47] In contrast to the results in Hall (1988) and Basu and Fernald (1995, 1997), which are based on index number methods that allow them to recover a firm and time invariant markup, we recover the growth pattern of markups but not the level. However, we do not need to impose the restriction that all inputs are flexibly and competitively chosen, impose restrictions on the shape of the production function (e.g., homogeneity), or compute/estimate the rental rate of capital/profit for the entrepreneur. As we show below, we can recover the level of markups with the addition of standard restrictions on product demand.

[48] We can also allow for time-varying firm-specific markups. If we let $\Upsilon_{jt} > 0$ be an independent demand shock that is realized after inputs are chosen, then *expected markups* will be equalized across firms, i.e., $E\left(\Psi_{jt}\right) = \Psi_t$ and χ_{jt} will enter into the firm's period t input decisions. That is, while actual markups $\Psi_{jt} = \frac{P_{jt}}{\Lambda_{jt}}$ will be firm specific due to the Υ_{jt} demand shocks, firms will still have ex-ante symmetric markups.

However, we can write $\left(1+\frac{1}{\sigma_t}\right)=e^{-\gamma_t}e^{-\mu}$. We know γ_t from our analysis above, so only μ is unknown. Replacing back into (O1-7) we get

$$\begin{aligned} r_{jt} &= e^{-\gamma_t}e^{-\mu}q_{jt}-\left(e^{-\gamma_t}e^{-\mu}-1\right)q_t+\chi_{jt}+\varepsilon_{jt} \\ &= e^{-\gamma_t}e^{-\mu}f\left(K_{jt},L_{jt},M_{jt}\right)-\left(e^{-\gamma_t}e^{-\mu}-1\right)q_t \\ &\quad +\left[\left(e^{-\gamma_t}e^{-\mu}\right)\omega_{jt}+\chi_{jt}\right]+\varepsilon_{jt}. \end{aligned} \tag{O1-8}$$

We then follow a similar strategy as before. As in equation (11) we first form an observable variable

$$\mathcal{R}_{jt}\equiv\ln\left(\frac{P_{jt}Y_{jt}}{e^{\varepsilon_{jt}}e^{e^{-\gamma_t}\int G^{\mu}(L_{jt},K_{jt},M_{jt})\frac{dM_{jt}}{M_{jt}}}}\right),$$

where we now use revenues (the measure of output we observe), include $e^{-\gamma_t}$, as well as using G^{μ} instead of the now unobservable G. Replacing into (O1-8) we obtain

$$\mathcal{R}_{jt} = -e^{-\gamma_t-\mu}\mathscr{C}\left(L_{jt},K_{jt}\right)-\left(e^{-\gamma_t}e^{-\mu}-1\right)q_t+\left[\left(e^{-\gamma_t}e^{-\mu}\right)\omega_{jt}+\chi_{jt}\right].$$

From this equation it is clear that the constant μ will be identified from variation in the observed demand shifter q_t. Without having recovered γ_t from the share regression first, it would not be possible to identify time-varying markups. Note that in equation (O1-7) both σ_t and q_t change with time and hence q_t cannot be used to identify σ_t unless we restrict $\sigma_t=\sigma$ as in Klette and Griliches (1996) and De Loecker (2011).

Finally, we can only recover a linear combination of productivity and the demand shock, $\left(1+\frac{1}{\sigma_t}\right)\omega_{jt}+\chi_{jt}$. The reason is clear: since we do not observe prices, we have no way of disentangling whether, after controlling for inputs, a firm has higher revenues because it is more productive (ω_{jt}) or because it can sell at a higher price (χ_{jt}). We can write $\omega^{\mu}_{jt}=$

$\left(1+\frac{1}{\sigma_t}\right)\omega_{jt}+\chi_{jt}$ as a function of the parts that remain to be recovered

$$\omega^{\mu}_{jt}=\mathcal{R}_{jt}+e^{-\gamma_t-\mu}\mathscr{C}\left(L_{jt},K_{jt}\right)+\left(e^{-\gamma_t}e^{-\mu}-1\right)q_t,$$

and impose the Markovian assumption on this combination:[49] $\omega^{\mu}_{jt}=h\left(\omega^{\mu}_{jt-1}\right)+\eta^{\mu}_{jt}$. We can use similar moment restrictions as before, $E\left(\eta^{\mu}_{jt}|k_{jt},l_{jt}\right)=0$, to identify the constant of integration $\mathscr{C}\left(L_{jt},K_{jt}\right)$ as well as μ (and hence the level of the markups).

References Appendix O1

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[49] In this case, the assumption is that the weighted sum of productivity ω_{jt} and the demand shock, χ_{jt} is Markovian. It is not necessarily the case that the two components will be Markovian individually.

Online Appendix O2: Fixed Effects

Table O2-1: Average Input Elasticities of Output--Fixed Effects
(Structural Estimates: Gross Ouput)

Colombia

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
Labor	0.22	0.24	0.30	0.42	0.37	0.31
	(0.07)	(0.10)	(0.28)	(0.07)	(0.06)	(0.02)
Capital	0.04	0.19	0.15	-0.05	0.05	0.13
	(0.16)	(0.10)	(0.22)	(0.10)	(0.11)	(0.02)
Intermediates	0.67	0.54	0.52	0.51	0.53	0.54
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
Sum	0.93	0.97	0.97	0.87	0.95	0.99
	(0.10)	(0.13)	(0.11)	(0.10)	(0.08)	(0.02)
Mean(Capital) / Mean(Labor)	0.17	0.81	0.50	-0.13	0.13	0.42
	(0.43)	(125.40)	(85.23)	(13.59)	(0.23)	(0.07)

Chile

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
Labor	0.25	0.42	0.45	0.34	0.50	0.33
	(0.01)	(0.04)	(0.03)	(0.03)	(0.03)	(0.01)
Capital	0.11	0.13	0.02	0.08	0.23	0.19
	(0.01)	(0.10)	(0.04)	(0.04)	(0.05)	(0.01)
Intermediates	0.67	0.54	0.56	0.59	0.50	0.55
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
Sum	1.03	1.09	1.03	1.01	1.23	1.07
	(0.02)	(0.10)	(0.04)	(0.03)	(0.05)	(0.01)
Mean(Capital) / Mean(Labor)	0.46	0.30	0.05	0.24	0.47	0.57
	(0.07)	(0.27)	(0.09)	(0.12)	(0.12)	(0.04)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers are based on a gross output specification with fixed effects and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table O2-2: Heterogeneity in Productivity--Fixed Effects
(Structural Estimates: Gross Output)

Colombia

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
75/25 ratio	1.44	1.39	1.34	1.42	1.47	1.41
	(1.32)	(0.38)	(0.55)	(0.17)	(0.45)	(0.02)
90/10 ratio	1.99	1.92	1.80	2.16	2.05	1.97
	(20.18)	(2.25)	(4.58)	(0.92)	(2.34)	(0.06)
95/5 ratio	2.57	2.54	2.24	3.02	2.58	2.50
	(112.81)	(5.26)	(28.82)	(2.24)	(7.29)	(0.09)
Exporter	0.27	0.13	0.01	0.51	0.30	0.16
	(0.69)	(0.61)	(0.24)	(0.58)	(0.23)	(0.05)
Importer	0.14	0.12	0.10	0.27	0.26	0.20
	(0.43)	(0.61)	(0.27)	(0.32)	(0.21)	(0.04)
Advertiser	0.01	0.11	0.00	0.13	0.17	0.08
	(0.09)	(0.27)	(0.17)	(0.11)	(0.14)	(0.02)
Wages > Median	0.23	0.22	0.17	0.28	0.37	0.27
	(0.76)	(0.44)	(0.32)	(0.20)	(0.22)	(0.04)

Chile

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
75/25 ratio	1.37	1.49	1.46	1.50	1.65	1.55
	(0.01)	(0.23)	(0.05)	(0.03)	(0.10)	(0.01)
90/10 ratio	1.90	2.22	2.17	2.32	2.63	2.40
	(0.03)	(1.64)	(0.12)	(0.08)	(0.34)	(0.02)
95/5 ratio	2.48	2.93	2.94	3.16	3.62	3.35
	(0.06)	(6.88)	(0.22)	(0.17)	(0.65)	(0.05)
Exporter	0.03	-0.03	0.25	0.02	-0.17	0.01
	(0.06)	(1.77)	(0.11)	(0.11)	(0.08)	(0.03)
Importer	0.15	0.06	0.23	0.19	-0.06	0.13
	(0.06)	(1.23)	(0.07)	(0.11)	(0.08)	(0.02)
Advertiser	0.05	0.02	0.12	0.05	-0.08	0.05
	(0.02)	(0.56)	(0.05)	(0.03)	(0.05)	(0.01)
Wages > Median	0.21	0.16	0.30	0.23	0.05	0.27
	(0.04)	(0.78)	(0.06)	(0.06)	(0.08)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers are based on a gross output specification with fixed effects and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile a firm that advertises is, on average, 5% more productive than a firm that does not advertise.

Online Appendix O3: Alternative Flexible Inputs

Table O3-1: Average Input Elasticities of Output--Energy+Services Flexible
(Structural Estimates: Gross Ouput)

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
<u>Colombia</u>						
Labor	0.15	0.21	0.37	0.28	0.29	0.22
	(0.02)	(0.03)	(0.03)	(0.06)	(0.03)	(0.01)
Capital	0.06	0.05	0.05	0.01	0.04	0.08
	(0.01)	(0.02)	(0.01)	(0.03)	(0.02)	(0.01)
Raw Materials	0.71	0.69	0.49	0.63	0.58	0.63
	(0.02)	(0.03)	(0.04)	(0.07)	(0.03)	(0.01)
Energy+Services	0.08	0.11	0.09	0.10	0.11	0.11
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sum	1.01	1.05	1.00	1.02	1.02	1.04
	(0.01)	(0.02)	(0.01)	(0.03)	(0.01)	(0.01)
Mean(Capital) / Mean(Labor)	0.43	0.23	0.12	0.04	0.14	0.37
	(0.08)	(0.14)	(0.04)	(0.08)	(0.06)	(0.04)
<u>Chile</u>						
Labor	0.18	0.28	0.31	0.29	0.31	0.22
	(0.02)	(0.03)	(0.03)	(0.04)	(0.02)	(0.01)
Capital	0.06	0.08	0.04	0.06	0.08	0.11
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
Raw Materials	0.77	0.65	0.65	0.59	0.63	0.67
	(0.02)	(0.02)	(0.02)	(0.05)	(0.02)	(0.01)
Energy+Services	0.07	0.07	0.06	0.11	0.07	0.07
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sum	1.08	1.08	1.06	1.05	1.10	1.08
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.00)
Mean(Capital) / Mean(Labor)	0.36	0.28	0.13	0.20	0.26	0.49
	(0.04)	(0.05)	(0.04)	(0.06)	(0.04)	(0.03)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers are based on a gross output specification in which energy+services is flexible and raw materials is quasi-fixed. The results are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, raw materials, and energy+services elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table O3-2: Heterogeneity in Productivity--Energy+Services Flexible

(Structural Estimates: Gross Output)

Colombia

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
75/25 ratio	1.20	1.25	1.24	1.30	1.28	1.33
	(0.02)	(0.03)	(0.03)	(0.06)	(0.02)	(0.01)
90/10 ratio	1.50	1.62	1.60	1.75	1.68	1.83
	(0.05)	(0.10)	(0.07)	(0.16)	(0.06)	(0.03)
95/5 ratio	1.87	2.09	2.00	2.26	2.04	2.43
	(0.09)	(0.22)	(0.11)	(0.24)	(0.11)	(0.08)
Exporter	0.14	-0.04	0.02	0.14	0.08	0.01
	(0.04)	(0.06)	(0.03)	(0.12)	(0.02)	(0.03)
Importer	0.00	-0.03	-0.03	-0.04	0.10	-0.02
	(0.02)	(0.06)	(0.03)	(0.05)	(0.02)	(0.05)
Advertiser	-0.12	-0.13	-0.10	-0.07	0.05	-0.16
	(0.03)	(0.11)	(0.04)	(0.06)	(0.02)	(0.05)
Wages > Median	0.06	0.13	0.14	0.09	0.19	0.10
	(0.02)	(0.05)	(0.02)	(0.05)	(0.02)	(0.04)

Chile

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
75/25 ratio	1.31	1.42	1.41	1.44	1.48	1.49
	(0.01)	(0.02)	(0.02)	(0.04)	(0.02)	(0.01)
90/10 ratio	1.76	2.04	2.01	2.13	2.23	2.26
	(0.03)	(0.05)	(0.04)	(0.12)	(0.05)	(0.02)
95/5 ratio	2.22	2.69	2.65	2.94	2.94	3.08
	(0.05)	(0.12)	(0.07)	(0.21)	(0.08)	(0.04)
Exporter	-0.01	-0.06	0.01	0.01	-0.05	-0.03
	(0.03)	(0.03)	(0.03)	(0.05)	(0.03)	(0.01)
Importer	0.02	0.04	0.07	0.09	0.05	0.07
	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)	(0.01)
Advertiser	-0.01	-0.01	0.01	0.01	0.00	0.02
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)
Wages > Median	0.12	0.15	0.19	0.17	0.16	0.25
	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers are based on a gross output specification in which energy+services is flexible and raw materials is quasi-fixed. The results are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile a firm that advertises is, on average, 1% less productive than a firm that does not advertise.

Table O3-3: Average Input Elasticities of Output--Raw Materials Flexible
(Structural Estimates: Gross Ouput)

Colombia

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
Labor	0.13	0.21	0.33	0.28	0.30	0.24
	(0.02)	(0.04)	(0.03)	(0.06)	(0.02)	(0.01)
Capital	0.05	0.07	0.02	0.01	0.05	0.06
	(0.01)	(0.03)	(0.02)	(0.04)	(0.02)	(0.02)
Raw Materials	0.60	0.44	0.41	0.42	0.42	0.43
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
Energy+Services	0.22	0.30	0.23	0.26	0.28	0.29
	(0.02)	(0.05)	(0.03)	(0.07)	(0.04)	(0.02)
Sum	1.00	1.01	1.00	0.97	1.05	1.02
	(0.01)	(0.04)	(0.03)	(0.05)	(0.04)	(0.01)
Mean(Capital) / Mean(Labor)	0.39	0.33	0.07	0.04	0.16	0.26
	(0.12)	(0.26)	(0.06)	(0.15)	(0.07)	(0.09)

Chile

	Food Products (311)	Textiles (321)	Apparel (322)	Wood Products (331)	Fabricated Metals (381)	All
Labor	0.19	0.34	0.35	0.38	0.42	0.26
	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.02)
Capital	0.06	0.07	0.06	0.06	0.12	0.11
	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.01)
Raw Materials	0.59	0.47	0.49	0.46	0.43	0.47
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
Energy+Services	0.21	0.18	0.15	0.16	0.18	0.21
	(0.02)	(0.03)	(0.04)	(0.03)	(0.06)	(0.02)
Sum	1.05	1.06	1.05	1.07	1.14	1.05
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.01)
Mean(Capital) / Mean(Labor)	0.32	0.20	0.16	0.16	0.28	0.41
	(0.04)	(0.06)	(0.04)	(0.06)	(0.07)	(0.04)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers are based on a gross output specification in which raw materials is flexible and energy+services is quasi-fixed. The results are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, raw materials, and energy+services elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table O3-4: Heterogeneity in Productivity--Raw Materials Flexible

(Structural Estimates: Gross Output)

Colombia

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
75/25 ratio	1.24	1.27	1.25	1.24	1.24	1.31
	(0.02)	(0.07)	(0.03)	(0.07)	(0.07)	(0.02)
90/10 ratio	1.58	1.64	1.59	1.62	1.57	1.72
	(0.03)	(0.16)	(0.08)	(0.19)	(0.19)	(0.05)
95/5 ratio	1.92	2.10	1.90	2.01	1.89	2.13
	(0.06)	(0.24)	(0.13)	(0.41)	(0.30)	(0.07)
Exporter	0.09	-0.01	0.04	0.05	0.01	0.04
	(0.04)	(0.09)	(0.07)	(0.18)	(0.11)	(0.01)
Importer	0.02	0.03	0.08	-0.03	0.05	0.07
	(0.02)	(0.09)	(0.10)	(0.15)	(0.08)	(0.01)
Advertiser	-0.06	0.01	-0.01	-0.02	-0.02	-0.03
	(0.02)	(0.05)	(0.04)	(0.05)	(0.05)	(0.01)
Wages > Median	0.04	0.11	0.14	0.09	0.11	0.13
	(0.02)	(0.07)	(0.03)	(0.06)	(0.07)	(0.01)

Chile

	Food Products (311) Gross Output (GNR)	**Textiles (321)** Gross Output (GNR)	**Apparel (322)** Gross Output (GNR)	**Wood Products (331)** Gross Output (GNR)	**Fabricated Metals (381)** Gross Output (GNR)	**All** Gross Output (GNR)
75/25 ratio	1.34	1.44	1.40	1.50	1.52	1.51
	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.01)
90/10 ratio	1.82	2.13	2.05	2.31	2.26	2.32
	(0.07)	(0.06)	(0.06)	(0.06)	(0.13)	(0.02)
95/5 ratio	2.32	2.80	2.70	3.09	2.98	3.19
	(0.12)	(0.10)	(0.10)	(0.11)	(0.25)	(0.04)
Exporter	-0.02	0.01	0.05	-0.01	-0.03	0.01
	(0.06)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)
Importer	0.06	0.06	0.09	0.12	0.06	0.13
	(0.06)	(0.03)	(0.03)	(0.04)	(0.03)	(0.01)
Advertiser	0.00	0.02	0.02	0.03	-0.02	0.04
	(0.02)	(0.02)	(0.03)	(0.01)	(0.02)	(0.01)
Wages > Median	0.13	0.15	0.18	0.19	0.17	0.25
	(0.04)	(0.03)	(0.03)	(0.02)	(0.04)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers are based on a gross output specification in which raw materials is flexible and energy+services is quasi-fixed. The results are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (G and $\mathscr{C}$) of our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile a firm that advertises is, on average, 0% less productive than a firm that does not advertise.