A Note on Two Models of Sibling Resemblance

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This note compares two structural equation models of educational and occupational resemblance among sibling pairs. Each model decomposes the regression of occupational status on schooling into a between-family regression of common factors and a pair of within-family regressions. In model 1, the within-family regressions are written in unique, within-family factors, and in model 2, the within-family regressions are written in the total educational and occupational variables. The two models are equivalent when within-family regressions are the same for each member of the sibling pair. Otherwise, they are not equivalent, and the second model has an undesirable logical implication. Under certain conditions, either model may exhibit symptoms of near-underidentification, or it may be underidentified; this problem may be more likely to occur in model 2 than in model 1.

Sociologists and economists have long worked to measure the effects of schooling. Its influence on such measures of success as occupational status and earnings serves, on the one hand, as an indicator of the role of educational institutions in fostering (or hampering) social mobility and, on the other, as an indicator of the productivity of personal and public

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1 This research was carried out with support from the Spencer Foundation, the National Science Foundation (SES-80-10640), the National Institute of Mental Health (MH-6275), the Kenneth D. Brody Foundation, and the Graduate School of the University of Wisconsin—Madison. Computations were performed using facilities of the Center for Demography and Ecology of the University of Wisconsin—Madison, which are supported by the Center for Population Research of the National Institute of Child Health and Human Development (HD-5876). I thank Richard T. Campbell, Michael W. Gillespie, Arthur S. Goldberger, David Grusky, Christopher S. Jencks, Robert D. Mare, Peter A. Mossel, Michael R. Olineck, Willem Saris, Arthur L. Stinchcombe, Elizabeth Thomson, Richard A. Williams, Raymond Sin-Kwok Wong, and Yu Xie for helpful advice, and I thank Taissa S. Hauser, Brian Clarridge, and Nancy Bode for assistance in research. The opinions expressed herein are those of the author. Requests for reprints should be sent to Robert M. Hauser, Department of Sociology, University of Wisconsin, 1180 Observatory Drive, Madison, Wisconsin 53706.

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0002-9602/88/9306-0004$01.50
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investments in schooling. At the same time, it is well known that social and economic success may depend directly on conditions of upbringing and personal characteristics that may also affect the length and quality of schooling. For these reasons, it is by no means obvious that an association of schooling with social or economic success can be interpreted in causal terms, and many studies have attempted to determine the degree to which causal inferences are warranted.

The effects of social background, broadly conceived, on achievement can be taken into account by modeling the similarity of siblings. That is, a research design based on sibling pairs (or $n$-tuples) permits a decomposition of the cross-sibling variance-covariance matrix into "between-family" and "within-family" components. If fraternal differences in schooling lead to differences in adult success, we can be more confident that the association of schooling with success is not merely an artifact of school success running in families that are also economically successful. Statistical controls for common family influences are by no means sufficient to eliminate problems of omitted-variable bias in the measured effects of schooling; other, sibling-specific factors that jointly determine schooling and economic success must also be controlled. Still, the prospect of controlling common family influences has helped to instigate a number of studies of the stratification process that are based on samples of siblings, rather than samples of the general population. Such studies begin with that of Blau and Duncan (1967, pp. 316-28), with the most notable of them being the two major studies by Jencks and his associates (Jencks et al. 1972, 1979). Moreover, the use of sibling data has called attention to the larger issue of the role of families in the stratification process.

In this paper, I use an analysis of educational attainment and occupational status among Wisconsin men (Hauser and Mossel 1985, 1988) to compare and contrast two structural-equation models of sibling resemblance. Figure 1 displays path diagrams of these two models. Each model permits a full specification of an analysis of covariance with random group (family) effects and heterogeneous (between- and within-family) slopes when there are two observations per group. With some changes of notation, model 1 is the same as that in figure 2 of Hauser and Mossel (1985; also see Hauser, Sewell, and Clarridge 1982; Hauser 1984; Hauser and Mossel 1988), while model 2 is similar to the specifications of Jencks et al. (1972), Hauser and Dickinson (1974), Olneck (1976, 1977), and Corcoran and Datcher (1981).

I shall describe and compare the two

However, between-family slopes have sometimes not been treated explicitly under the latter model. For example, in Jencks et al. (1972, app. B), the differences of within-family and between-family regressions appear only implicitly in correlations among family factors that are treated as exogenous in the model.

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Fig. 1.—Two models of sibling resemblance in educational attainment and occupational status with errors in variables and latent family factors.
models, show where they yield the same and different results, and explain and illustrate their differences. Despite their apparent simplicity, these models give rise to methodological problems that occur in larger and more realistic models, and studying them may alert analysts who use structural-equation models to some subtle problems in model specification and estimation.

The methodological issues addressed here also occur in other areas of social scientific research, for example, in analyses of neighborhood effects (Bielby 1981), husband-wife interaction (Thomson and Williams 1982), fertility (Clarridge 1983), political identification (Jennings and Niemi 1981, chap. 4), and, more generally, in the analysis of change over time (Jöreskog and Sörbom 1977; Kenny 1979; Kessler and Greenberg 1981).

The upper half of each model in figure 1 pertains to the educational attainment and occupational status of a primary respondent (1957 high school graduate) in the Wisconsin Longitudinal Study (Sewell and Hauser 1980; Hauser 1983; Sewell, Hauser, and Hauser 1983), and the lower half pertains to the educational attainment and occupational status of a randomly selected brother of the primary respondent (Hauser, Sewell, and Clarridge 1982). The sampling plan of the Wisconsin study allows parameters of the “within-family” models to differ between primary respondents and their brothers. For example, primary respondents must all have completed high school, but about 7% of their brothers did not, and, unlike respondents, brothers were drawn from many birth cohorts. The central part of each model pertains to the between-family regression of occupational status on schooling.

THE ANALYSIS OF COVARIANCE MODEL

In both models 1 and 2, the common family influences on educational attainment and occupational status are specified as unobserved or latent common factors. The distinction between model 1 and model 2 has an analog in the more familiar analysis of covariance, in which family influences are specified as (observable) group means. In order to anticipate the contrast between model 1 and model 2, I will briefly review the

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1 The measured variables are EDEQYR = respondent's years of schooling, EDAT64 = respondent's years of schooling, XEDEQYR = brother's years of schooling, SSBED = brother's years of schooling, OCSXCR = respondent's current occupation, OCSX70 = respondent's 1970 occupation, XOCSXCR = brother's current occupation, OCSSIB = brother's 1975 occupation, and XOCSX70 = brother's 1970 occupation (Hauser and Mossel 1985). The path diagrams omit several correlations between errors in indicators that were ascertained on the same occasion; they also fail to show some equality restrictions on error variances. Complete details of the measurement-error model and model 1, including the LISREL control files, are presented by Hauser and Mossel (1988) and are available from the author on request.
analysis of covariance with a single covariate. Let $x$ be the covariate and $y$ be the dependent variable, where $i$ indexes individuals or cases and $j$ indexes groups or treatments. Thus $x_{ij}$ is the value of the covariate for the $i$th case in the $j$th group, $\bar{x}_j$ is the mean of the covariate in the $j$th group, $y_{ij}$ is the value of the dependent variable for the $i$th case in the $j$th group, and $\bar{y}_j$ is the mean of the covariate in the $j$th group. We assume that the $x_{ij}$ and $y_{ij}$ have each been deviated from their respective population means; thus, without loss of generality, we can ignore intercepts in the regressions of $y$ on $x$. The decomposition of the observed variates into between-group and within-group components is

$$x_{ij} = \bar{x}_j + (x_{ij} - \bar{x}_j),$$ (1)

and

$$y_{ij} = \bar{y}_j + (y_{ij} - \bar{y}_j).$$ (2)

Consider two regressions of $y$ on $x$, the regression of group means or between-group regression,

$$\bar{y}_j = \beta_b \bar{x}_j + \epsilon_j, \quad \text{(3)}$$

and the within-group regression (sometimes called the average within-group regression),

$$y_{ij} - \bar{y}_j = \beta_w (x_{ij} - \bar{x}_j) + \epsilon_{ij}, \quad \text{(4)}$$

where $\beta_b$ is the between-family slope, $\beta_w$ is the within-family slope, and $\epsilon_j$ and $\epsilon_{ij}$ are each independently and identically distributed disturbances. If we add equation (3) and equation (4), we obtain

$$y_{ij} = \beta_b \bar{x}_j + (\beta_w - \beta_w) (x_{ij} - \bar{x}_j) + \epsilon_{ij}, \quad \text{(5)}$$

where the error term ($\epsilon_{ij}$) in equation (5) should be understood to combine those of equations (3) and (4). Grouping coefficients instead of variables, we can rewrite equation (5) as

$$y_{ij} = \beta_w x_{ij} + (\beta_b - \beta_w) \bar{x}_j + \epsilon_{ij}. \quad \text{(6)}$$

Equations (5) and (6) are equivalent expressions of unequal slopes in the within- and between-group regressions of $y$ on $x$. Each says exactly the same thing about the joint distributions of $x$ and $y$. In equation (5) there are distinct slopes for the between-family ($\bar{x}_j$) and within-family ($x_{ij} - \bar{x}_j$) components of the fundamental decomposition in equation (1). In equation (6), because the equation has been rewritten in $x_{ij}$ and $\bar{x}_j$, the former term takes the within-family slope, $\beta_w$, and the latter term takes the

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4 For a more complete discussion of the analysis of covariance model, see Hauser (1972, pp. 15–25).
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difference of the between-family and within-family slopes, $\beta_b - \beta_w$. As shown below, the relationship between model 1 and model 2, when family effects are unobservable, is similar to that between equation (5) and equation (6) in the analysis of covariance.

MODEL ONE

Model 1 says that the educational attainments of respondent, $\eta_2$, and brother, $\eta_3$, are composed of a common between-family component, $\xi_1$, and unique within-family components, $\xi_2$ and $\xi_3$, respectively: $^5$

$$\eta_2 = \gamma_{21}\xi_1 + \xi_2$$  (7)

and

$$\eta_3 = \gamma_{31}\xi_1 + \xi_3,$$  (8)

where $E[\xi_i\xi_j] = 0$ for $i \neq j$. That is, $\xi_1$ is analogous to the mean family level of education in the analysis of covariance, and $\xi_2$ and $\xi_3$ are analogous to the deviations of the brothers' educational attainments from the mean family level of education. Similarly, the occupational statuses of respondent, $\eta_4$, and brother, $\eta_5$, are composed of the common between-family component, $\xi_1$, and the unique within-family components, $\xi_6$ and $\eta_7$:

$$\eta_4 = \beta_{41}\eta_1 + \eta_6,$$  (9)

and

$$\eta_5 = \beta_{51}\eta_1 + \eta_7,$$  (10)

where $E[\eta_1\eta_6] = E[\eta_1\eta_7] = E[\eta_6\eta_7] = 0$. The corresponding within- and between-family variables are connected by three structural equations, in which the slopes of occupational status on educational attainment are $\gamma_{62}$ for primary respondents, $\gamma_{73}$ for brothers, and $\gamma_{11}$ for families:

$$\eta_6 = \gamma_{62}\xi_2 + \xi_6,$$  (11)

$$\eta_7 = \gamma_{73}\xi_3 + \xi_7,$$  (12)

and

$$\eta_1 = \gamma_{11}\xi_1 + \xi_1,$$  (13)

where $E[\xi_i\xi_j] = 0$ for $i = 1, \ldots, 3$ and $j = 1, 6, 7$, and $E[\xi_i\xi_6] = E[\xi_1\xi_7] = E[\xi_6\xi_7] = 0$. That is, in equations (7)–(13), the disturbances, $\xi_i$, are not correlated with one another or with variables on the right-hand

$^5$ As in the analysis of covariance model, I assume that variables have been deviated from population means and write the structural equations without intercepts.

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side of the same equation. Furthermore, all variables on the right-hand side of the same equation are uncorrelated. Ordinarily, the specification of uncorrelated disturbances would cover the latter possibility, but model 1 puts $\xi$'s and $\eta$'s where structural disturbances ($\zeta$'s) might otherwise appear in order to permit the specification of the within-family regressions, $\gamma_{62}$ and $\gamma_{73}$. Thus, $\xi_2$ and $\xi_3$ are similar to disturbances in equations (7) and (8), respectively, while $\eta_6$ and $\eta_7$ are similar to disturbances in equations (9) and (10), respectively. Hauser and Mossel (1985, p. 658) refer to these within-family components as “disturbances” because they are specified to behave something like the disturbances in an oblique factor model (Hauser and Goldberger 1971), but this is not an appropriate term: the within-family components are neither errors in variables nor errors in equations; they pertain to true deviations from family levels of status variables.

As presented in figure 1, model 1 is underidentified. First, the metrics of $\xi_1$ and $\eta_1$ are undefined. This problem can be resolved without loss of generality, for example, by the restriction

$$\gamma_{21} = \beta_{41} = 1,$$

(14)

which is equivalent to a typical normalization in a measurement-error model. Second, the structural model remains underidentified by one restriction, which is typically imposed in the form of an equality between corresponding parameters for primary respondents and their brothers. For example, Hauser, Sewell, and Clarridge (1982, pp. 23–26) impose a single identifying restriction in model 1 that is equivalent to $\psi_6 = \psi_7$, where $\psi_i = \text{Var}[\xi_i]$. In contrast, Hauser and Mossel (1985, 1988) identify the structural model by imposing two additional constraints like those used to normalize $\xi_1$ and $\eta_1$, that is,

$$\gamma_{21} = \gamma_{31} = 1$$

(15)

$$\beta_{41} = \beta_{51} = 1.$$

(16)

Taken together, equations (15) and (16) place a single overidentifying restriction on the structural model. That is, together with equation (14), either equation (15) or equation (16) is sufficient to identify the structural model. Hauser and Mossel impose both restrictions because, in combination, they specify tau-equivalent measurement of educational attainment and occupational status between primary respondent and brother. Thus, it is possible to compare the slopes of the three regressions among brothers and families.6

6 Although the model pertains to a single population of families and of persons drawn from them, the same issues of normalization arise here as in true cross-population comparisons of structural coefficients (Bielby 1986; Williams and Thomson 1986; Sobel and Arminger 1986).
MODEL TWO

In model 2, the equations for educational attainment of the primary respondent and his brother are identical with equations (7) and (8) under model 1. Nothing in model 2 requires the specification of within-family education components as $\xi$'s, rather than $\zeta$'s, but this simplifies later comparisons between model 1 and model 2. In model 2, the three occupation equations differ from those in model 1 because the total educational attainment variables enter the equations for respondent and brother:

$$\eta_4 = \beta_{42} \eta_2 + \beta_{41} \eta_1 + \zeta_4,$$

(17)

$$\eta_5 = \beta_{53} \eta_3 + \beta_{51} \eta_1 + \zeta_5,$$

(18)

and

$$\eta_1 = \gamma_{11} \xi_1 + \zeta_1,$$

(19)

where $E[\xi_4 \eta_1] = E[\xi_4 \eta_2] = E[\xi_5 \eta_1] = E[\xi_5 \eta_3] = E[\xi_1 \zeta_4] = E[\xi_4 \zeta_5] = E[\xi_5 \zeta_1] = E[\zeta_4 \eta_4] = E[\zeta_5 \eta_5] = 0.$ That is, in equations (17)–(19), the disturbances, $\zeta_i$, are not correlated with one another or with variables on the right-hand side of the equation in which they appear. Note the use of $\gamma_{11}^*$ in equation (19) to distinguish it from $\gamma_{11}$ in equation (13). Like model 1, model 2 is underidentified without additional restrictions on equations (7), (8), and (17)–(19), and it is convenient to impose the normalizing and identifying restrictions of equations (15) and (16).

A COMPARISON OF THE TWO MODELS

The two models differ in the specification of the within-family regressions of occupational status on educational attainment. In model 1, these regressions are specified on the within-family components or unique, individual factors of the regressions of individual schooling and status on common family factors (Hauser and Mossel 1985, p. 658). In model 2, the within-family regressions are specified on the total schooling and status variates. Patently, both models are incomplete, for neither includes explicit measurements of socioeconomic background or of the several social and psychological variables that may intervene between social origins and schooling or economic success (Hauser, Tsai, and Sewell 1983); Hauser and Sewell (1986) have elaborated model 1 to include direct measures of socioeconomic background and adult earnings. At the same time, the two models are large enough to be of substantive interest in their own

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7 Note that the disturbances in equations (17) and (18), $\zeta_4$ and $\zeta_5$, correspond exactly to $\zeta_6$ and $\zeta_7$ in equations (11) and (12); there is similarly a one-to-one relationship between their variances, $\psi_4 = \psi_6$ and $\psi_5 = \psi_7$. 

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right (Hauser 1984, pp. 161–63) and to raise methodological problems that also occur in more elaborate models.

One might think that the two models in figure 1 are equivalent. Under certain conditions, they are formally equivalent, but, even so, empirical inconsistencies may arise when each of them is estimated with the same data using LISREL (Jöreskog and Sörbom 1978, 1985). Furthermore, from the path diagrams it may appear that the between-family regressions are the same in the two models, but this is not the case even when the two models are otherwise equivalent. Because of the change in the within-family regressions, there is an implicit reparameterization of the between-family regressions.

Conditional on the identifying and normalizing restrictions that $\gamma_{21} = \gamma_{31} = \beta_{41} = \beta_{51} = 1$, it is straightforward to compare the specifications of within- and between-family regressions in model 1 and model 2. Consider first the within-respondent and between-family regressions in model 1. The respondent's occupational status is

$$\eta_4 = \eta_1 + \eta_6, \quad (20)$$

where

$$\eta_1 = \gamma_{11} \xi_1 + \xi_1 \quad (21)$$

and

$$\eta_6 = \gamma_{62} \xi_2 + \xi_6. \quad (22)$$

Substituting equations (21) and (22) into equation (20), the reduced form equation is

$$\eta_4 = \gamma_{11} \xi_1 + \gamma_{62} \xi_2 + \xi_1 + \xi_6. \quad (23)$$

Given the fact that the respondent's educational attainment is

$$\eta_2 = \xi_1 + \xi_2, \quad (24)$$

where $E[\xi_1 \xi_2] = 0$, it is clear that $\gamma_{11}$ and $\gamma_{62}$ are the respective between-family and within-respondent regressions of occupational status on schooling. Similarly, the reduced form equation for brothers under model 1 is

$$\eta_5 = \gamma_{11} \xi_1 + \gamma_{73} \xi_3 + \xi_1 + \xi_7, \quad (25)$$

where $\gamma_{73}$ is the within-brother regression of status on schooling. Note that the between-family slope, $\gamma_{11}$, is the same in equations 23 and 25.

Under model 2, the occupational status of the respondent is

$$\eta_4 = \eta_1 + \beta_{42} \eta_2 + \xi_4, \quad (26)$$
where
\[ \eta_1 = \gamma^*_1 \xi_1 + \zeta_1 \]  
(27)
and, again,
\[ \eta_2 = \xi_1 + \xi_2. \]  
(28)
Thus, under model 2, the reduced form equation for respondents is
\[ \eta_4 = (\gamma^*_1 + \beta_{42})\xi_1 + \beta_{42}\xi_2 + \xi_1 + \xi_4. \]  
(29)
The two reduced-form equations, (23) and (29), are the same, but their expression of within-family and between-family slopes in terms of structural parameters is different. From equations (28) and (29), we see that \( \beta_{42} = \gamma_{62} \) is the within-respondent slope under model 2, whereas the between-family slope is \( \gamma^*_{11} + \beta_{42} = \gamma^*_{11} + \gamma_{62} = \gamma_{11} \). Thus, in model 2, \( \gamma^*_{11} = \gamma_{11} - \gamma_{62} \) is the difference between the within-respondent and between-family slopes, while in model 1, \( \gamma_{11} \) is the between-family slope. In model 2, the hypothesis of "no family bias" or homogeneity of regressions across respondents and families is tested by specifying that \( \gamma^*_{11} = 0 \), while the same hypothesis is tested by specifying \( \gamma_{11} = \gamma_{62} \) in model 1.

While models 1 and 2 have the same numbers of parameters, there is a nontrivial difference between them. In model 2, brother’s occupational status is
\[ \eta_5 = \eta_1 + \beta_{53}\xi_3 + \zeta_5, \]  
(30)
where
\[ \eta_3 = \xi_1 + \xi_3. \]  
(31)
Substituting equations (27) and (31) into equation (30), we obtain the reduced form equation for brothers,
\[ \eta_5 = (\gamma^*_1 + \beta_{53})\xi_1 + \beta_{53}\xi_3 + \xi_1 + \xi_5. \]  
(32)
Again, equation (32) shows that, under model 2, \( \gamma^*_{11} \) is the difference of the within-brother and between-family regressions.

Comparing equations (29) and (32), we see that model 2 implies there are two distinct “between-family” regressions, one for respondents, \( \gamma^*_{11} + \beta_{42} \), and one for brothers, \( \gamma^*_{11} + \beta_{53} \), whose difference is just the difference between the two within-family regressions, \( \beta_{42} - \beta_{53} \). That is, strictly as a function of differences between siblings in within-family regressions, model 2 implies corresponding differences in the total effects of families. In contrast, as shown by equations (23) and (25), under model

\[ I \text{ thank Arthur S. Goldberger for demonstrating this to me. For a related example, see Goldberger (1978, pp. 965–66).} \]
1 the specification of distinct within-family regressions for respondent and
brother does not imply that there are two "between-family" regressions. It
is difficult to imagine circumstances in which it would be logical to specify
two between-family regressions, and this implication of model 2 leads me
to reject it when there are distinct within-respondent and within-brother
regressions.

In the special case where the two within-family regressions are homo-
genous, so that

\[ \gamma_{62} = \gamma_{73} \text{ in model 1} \]  \hspace{1cm} (33)

and

\[ \beta_{42} = \beta_{53} \text{ in model 2,} \]  \hspace{1cm} (34)

each model implies only one between-family regression, and the two
models are equivalent. This restriction might hold by construction, as in
the case of Olneck's (1976, 1977) Kalamazoo brothers sample. There,
brothers were drawn from a sample of families, and there was no natural
order to the members of the sibling pairs. The restriction may also be met
empirically, rather than by design. The Wisconsin data of Hauser and
Mossel (1985, pp. 661, 667) are consistent with this restriction. However,
there is no necessity that this symmetry restriction will hold; for example,
Hauser (1984) found that it did not hold for the effects of schooling on
occupational status in brother-sister pairs.

AN EMPIRICAL ILLUSTRATION OF THE COMPARISON
To illustrate the differences between model 1 and model 2, I turn to
analyses of educational and occupational resemblance among 518 Wis-
consin brother pairs, initially reported by Hauser and Mossel (1985,
pp. 662–66). These analyses are based on the path models in figure 1. In
Hauser and Mossel's final measurement model, \( L^2 = 24.39 \), with 24 \( df \),
and the fit and \( df \) of more restrictive models should be compared with that
baseline.\(^9\) For example, when model 1 is restricted by equations (15) and
(16), \( L^2 = 26.07 \), with 25 \( df \), so the single overidentifying restriction in
the structural model accounts for \( L^2 = 1.78 \), with 1 \( df \).

Table 1 gives parameter estimates and fit statistics for several versions
of model 1 and model 2. As shown in row A, the expected results are
obtained under the restrictions of equations (15) and (16) combined with
(33) (model 1) or (34) (model 2). The fit is the same under both models, and

\(^9\) The final measurement model of Hauser and Mossel differs slightly from the models
shown in fig. 1 because it includes some correlated errors in variables. These differ-
ences are not consequential.
### TABLE 1

**SELECTED PARAMETER ESTIMATES AND FIT STATISTICS FROM ALTERNATIVE SPECIFICATIONS OF SIBLING RESEMBLANCE: WISCONSIN BROTHERS DATA FROM HAUSER AND MOSEL (N = 518)**

<table>
<thead>
<tr>
<th>Restrictions</th>
<th>Model 1</th>
<th></th>
<th></th>
<th>Model 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_{62}$</td>
<td>$\gamma_{11}$</td>
<td>$\gamma_{13}$</td>
<td>$L^3$</td>
<td>$\beta_{42}$</td>
<td>$\gamma_{11}^*$</td>
</tr>
<tr>
<td>A. Eqq. (15), (16), and (33) or (34)</td>
<td>.728 (.047)</td>
<td>.678 (.062)</td>
<td>.728 (.047)</td>
<td>26.74</td>
<td>.728 (.047)</td>
<td>-.050 (.091)</td>
</tr>
<tr>
<td>B. Eqq. (15), (16), and (33), (35)</td>
<td></td>
<td>.708 (.029)</td>
<td>.708 (.029)</td>
<td>.708 (.029)</td>
<td>27.03</td>
<td>.708 (.029)</td>
</tr>
<tr>
<td>C. Eqq. (15) and (16)</td>
<td>.674 (.081)</td>
<td>.684 (.062)</td>
<td>.756 (.057)</td>
<td>26.07</td>
<td>.724 (.058)</td>
<td>-.049 (.092)</td>
</tr>
<tr>
<td>D. Eqq. (14) and (15)</td>
<td>.621 (.096)</td>
<td>.768 (.090)</td>
<td>.792 (.064)</td>
<td>24.39</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>E. Eqq. (14) and (16)</td>
<td>.633 (.067)</td>
<td>.768 (.090)</td>
<td>.827 (.110)</td>
<td>24.39</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>F. Eqq. (14) and (37)</td>
<td>.628 (.073)</td>
<td>.768 (.090)</td>
<td>.816 (.099)</td>
<td>24.39</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

**Note.**—See text for explanation.
the two models yield equivalent parameter estimates. The within-group slope estimates (.728) are the same under each model, and the family slope estimate in model 2 (-.050) is the difference between the within- (.708) and between-family (.678) slope estimates in model 1. Because the regressions are virtually homogeneous between and within families, the family slope is close to zero in model 2.

Also, the two models yield equivalent findings under the additional restriction of homogeneity between persons and families,

\[ \gamma_{62} = \gamma_{73} = \gamma_{11} \]  \hspace{2cm} (35)

in model 1 and

\[ \gamma_{11}^* = 0 \]  \hspace{2cm} (36)

in model 2. These results are shown in row B of table 1. Equivalent results are obtained, also, under models that make stronger assumptions of symmetry between brothers, that is, by adding constraints on the variances of unobservable variables.

So long as one makes minimal assumptions about symmetry of structure between brothers, one has little to choose between the two models. When families are the sampling units, and the members of brother pairs are not distinct, this will be the case. However, as stated by Hauser and Mossel, asymmetry between siblings is inherent in the sampling plan of the Wisconsin study, and there are many instances in which this could occur, for example, in comparisons of brothers and sisters (Hauser 1984) or of older and younger siblings (Hauser and Wong 1987). Thus, it seems worthwhile to undertake additional comparisons between model 1 and model 2 under conditions where symmetry between siblings cannot be assumed.

Row C of table 1 gives estimates obtained under the identifying and normalizing constraints of equations (15) and (16), but without additional symmetry restrictions. Models 1 and 2 each converge rapidly and yield plausible parameter estimates, yet the fit is slightly better under model 1, and each model implies different within-family and between-family slopes.\(^{10}\) In the case of model 1, the within-respondent slope is \(\hat{\gamma}_{62} = .674\), but in model 2 it is \(\hat{\beta}_{42} = .724\). In the case of model 1, the within-brother slope is \(\hat{\gamma}_{73} = .756\), but in model 2 it is \(\hat{\beta}_{53} = .730\). In model 1, the between-family regression is \(\hat{\gamma}_{11} = .684\), but in model 2 there are two between-family regressions, \(\hat{\gamma}_{11}^* + \hat{\beta}_{42} = -.049 + .724 = .675\) and \(\hat{\gamma}_{11}^* + \hat{\beta}_{53} = -.049 + .730 = .681\). As stated above, I reject model 2 in this situation because of its implication that there are two distinct between-family regressions.

\(^{10}\) Model 1 and model 2 are not nested, so no formal comparison of their fit is possible.
PROBLEMS IN ESTIMATING MODEL TWO

Given the algebraic and empirical evidence of the two preceding sections, I was convinced that model 2 and model 1 were essentially interchangeable, with the exception of the undesirable logical implication of model 2 when the within-family regressions were not homogeneous. Yet, every time I used LISREL to estimate a model that was less restrictive than those of the preceding section, I obtained quite satisfactory results under model 1 and perverse results under model 2.

For example, rows D–F of table 1 show estimates obtained under three alternative, just-identified specifications of the structural portion of model 1. In row D, equations (14) and (15) were used to identify the model; relative to the model of row A, $\beta_{51}$ was made a free parameter, and its estimate was plausible ($\hat{\beta}_{51} = 0.81$). In row E, equations (14) and (16) were used to identify the model; relative to the model of row A, $\gamma_{31}$ was made a free parameter, and its estimate was plausible ($\hat{\gamma}_{31} = 1.23$). In row F, the model was identified using equation (14) and, in addition, the restriction that the variances of the disturbances in occupational status were equal for the primary respondent and his brother:

$$\psi_6 = \psi_7.$$  \hspace{1cm} (37)

The pooled estimate, $\hat{\psi}_6 = \hat{\psi}_7 = 1.78$, was little different from the unrestricted estimates under the model of row A, $\hat{\psi}_6 = 1.86$ and $\hat{\psi}_7 = 1.79$, respectively. Not only does each of these models yield plausible parameter estimates, but each fits exactly as the unrestricted measurement model. Further, convergence of the estimation process occurred very rapidly in these runs, usually in 5–10 steepest descent trials, as it had, also, in the models of rows A and B.

Under model 2, the same restrictions yielded most unsatisfactory results. For example, in one run of the specification of row D, the SE of $\beta_{51}$ became huge, as did all of the SEs of estimated parameters in $\Psi$, the vector of variances of disturbances in the $\zeta$'s. In one run of the specification of row E, $\gamma_{31}$ appeared to converge toward zero, one of the variances in $\Phi$, the vector of variances in the $\xi$'s, was very large and negative, and convergence was not achieved in 250 iterations. In one run of the specification of row F, convergence was very rapid, as in the corresponding run of model 1, but, again, an element of $\Phi$ was very large and negative. In none of these runs was the likelihood-ratio test statistic as low as it should have been, that is, $L^2 = 24.39$, with 24 df, under the otherwise unrestricted measurement model.

---

11 In the model of row F, under model 2 the specification $\psi_4 = \psi_5$ is equivalent to that of equation (37) under model 1.
AN EXPLANATION

In mid-1982, when I first ran into this problem, I took the easy way out and adopted the specification of model 1. It represents the within- and between-family covariance structure adequately, it follows a traditional specification in covariance analysis (Walker and Lev 1953; Duncan, Cuzzort, and Duncan 1961), and it is computationally tractable. At the same time, the perverse behavior of model 2 continued to bother me. I pestered my colleagues and students about the matter. I fiddled with alternative specifications of model 2, and I rewrote the program instructions several times in order to eliminate the possibility that some clerical or mechanical error was responsible for the problems in estimation. I also ran the model with a variety of starting values and in three versions of LISREL (IV, V, and VI). None of this solved the problem.

After some time, it occurred to me that the problem was perhaps not in the model per se but in the estimation of model 2 with the data used by Hauser and Mossel (1985). All my efforts to estimate model 2 had been based on the same variance-covariance matrix. The distinguishing feature of those data—as surprising to me as to others—is the homogeneity of regressions within and between families. In model 1, this aspect of the data makes its appearance in similar values of $y_{62}$, $y_{73}$, and $y_{11}$, while in model 2, it is displayed in the near-zero estimates of $y_{11}$. The latter account for my problems in estimating model 2.

By way of illustration, consider the version of model 2 in which the structural model is identified by equations (14) and (15), so the 10 parameters of the structural model are $\beta_{42}$, $\beta_{53}$, $\beta_{51}$, $y_{11}^*$, $\text{Var}[\xi_1] = \phi_{11}$, $\text{Var}[\xi_2] = \phi_{22}$, $\text{Var}[\xi_3] = \phi_{33}$, $\text{Var}[\xi_4] = \psi_{11}$, $\text{Var}[\xi_5] = \psi_{44}$, and $\text{Var}[\xi_3] = \psi_{55}$. In the population, the expectations of products of the true education and occupation variates are as follows:12

\begin{align*}
E[\eta_2 \eta_2] &= \sigma_{22} = \phi_{11} + \phi_{22} & (38) \\
E[\eta_2 \eta_3] &= \sigma_{23} = \phi_{11} & (39) \\
E[\eta_2 \eta_4] &= \sigma_{24} = \beta_{42} \phi_{22} + (\beta_{42} + y_{11}^*) \phi_{11} & (40) \\
E[\eta_2 \eta_5] &= \sigma_{25} = (\beta_{53} + \beta_{51} y_{11}^*) \phi_{11} & (41) \\
E[\eta_3 \eta_3] &= \sigma_{33} = \phi_{11} + \phi_{33} & (42) \\
E[\eta_3 \eta_4] &= \sigma_{34} = (\beta_{42} + y_{11}^*) \phi_{11} & (43) \\
E[\eta_3 \eta_5] &= \sigma_{35} = \beta_{53} \phi_{33} + (\beta_{53} + \beta_{51} y_{11}^*) \phi_{11} & (44)
\end{align*}

12 Similar results would hold if the expectations of products pertained to observables, rather than to latent variables, and for the specifications of model 2 in rows E and F of table 1.
\[
E[\eta_4 \eta_4] = \sigma_{44} = \beta_{42}^2(\phi_{11} + \phi_{22}) + 2\beta_{42}^* \gamma_{11}^* \phi_{11} \\
+ \gamma_{11}^2 \phi_{11} + \psi_{11} + \psi_{44}
\] (45)

\[
E[\eta_4 \eta_5] = \sigma_{45} = [\beta_{42} \beta_{53} + \beta_{42} \beta_{51} \gamma_{11}^* + \beta_{53} \gamma_{11}^* \\
+ \beta_{51} \gamma_{11}^2] \phi_{11} + \beta_{51} \psi_{11}
\] (46)

\[
E[\eta_5 \eta_5] = \sigma_{55} = \beta_{53}^2(\phi_{11} + \phi_{33}) + \beta_{53}^2(\gamma_{11}^2 \phi_{11} + \psi_{11}) \\
+ 2\beta_{53} \beta_{51} \gamma_{11}^* \phi_{11} + \psi_{55},
\] (47)

where \(\sigma_{ij}\) denotes \(E[\eta_i \eta_j]\) for convenience in exposition. In this case, knowing the \(\sigma_{ij}\), we can solve equations (38)–(44) for the \(\phi_{ii}\) and for the four slopes.\(^{13}\) Subtracting equation (39) from equation (38),

\[
\phi_{22} = \sigma_{22} - \sigma_{23},
\] (48)

and subtracting equation (43) from equation (40),

\[
\beta_{42} \phi_{22} = \sigma_{24} - \sigma_{34},
\] (49)

so

\[
\beta_{42} = (\sigma_{24} - \sigma_{34})/(\sigma_{22} - \sigma_{23}).
\] (50)

Subtracting equation (39) from equation (42),

\[
\phi_{33} = \sigma_{33} - \sigma_{23},
\] (51)

and subtracting equation (41) from equation (44),

\[
\beta_{53} \phi_{33} = \sigma_{35} - \sigma_{25},
\] (52)

so

\[
\beta_{53} = (\sigma_{35} - \sigma_{25})/(\sigma_{33} - \sigma_{23}).
\] (53)

Then, from equation (43),

\[
\gamma_{11}^* = \sigma_{34}/\phi_{11} - \beta_{42},
\] (54)

and from equation (41),

\[
\beta_{51} = (\sigma_{25}/\sigma_{11} - \beta_{53})/\gamma_{11}^*.
\] (55)

When we note that \(\gamma_{11}^*\) appears in the denominator of equation (55), we are no longer surprised that \(\beta_{51}\) becomes indeterminate as \(\gamma_{11}^*\) approaches zero.

\(^{13}\) Once the other parameters are known, the \(\psi_{ii}\) can be determined by simple substitution in equations (45), (46), and (47).
In addition, suppose that $\gamma^*_{11} = 0$ in the population. Then, equations (38)-(47) become

$$E[\eta_2 \eta_2] = \sigma_{22} = \phi_{11} + \phi_{22}$$  \hfill (56)

$$E[\eta_2 \eta_3] = \sigma_{23} = \phi_{11}$$  \hfill (57)

$$E[\eta_2 \eta_4] = \sigma_{24} = \beta_{42}(\phi_{11} + \phi_{22})$$  \hfill (58)

$$E[\eta_2 \eta_5] = \sigma_{25} = \beta_{53}\phi_{11}$$  \hfill (59)

$$E[\eta_3 \eta_3] = \sigma_{33} = \phi_{11} + \phi_{33}$$  \hfill (60)

$$E[\eta_3 \eta_4] = \sigma_{34} = \beta_{42}\phi_{11}$$  \hfill (61)

$$E[\eta_3 \eta_5] = \sigma_{35} = \beta_{53}(\phi_{33} + \phi_{11})$$  \hfill (62)

$$E[\eta_4 \eta_4] = \sigma_{44} = \beta_{42}^2(\phi_{11} + \phi_{22}) + \psi_{11} + \psi_{44}$$  \hfill (63)

$$E[\eta_4 \eta_5] = \sigma_{45} = \beta_{42}\beta_{53}\phi_{11} + \beta_{51}\psi_{11}$$  \hfill (64)

$$E[\eta_5 \eta_5] = \sigma_{55} = \beta_{53}^2(\phi_{11} + \phi_{33}) + \beta_{51}^2\psi_{11} + \psi_{55}.$$  \hfill (65)

This makes the structural model overidentified, for

$$\beta_{42} = \sigma_{24}/\sigma_{22} = \sigma_{34}/\sigma_{23}$$  \hfill (66)

and

$$\beta_{53} = \sigma_{25}/\sigma_{23} = \sigma_{35}/\sigma_{33},$$  \hfill (67)

but at the same time, the model is underidentified. Given the other parameters determined by equations (56)-(62), only the product $\beta_{51}\psi_{11}$ is determined by equation (64), and equations (63) and (65) each add another unknown ($\psi_{44}$ and $\psi_{55}$).

Thus, in figure 1, without a restriction that identifies each of $\gamma_{21}$, $\gamma_{31}$, $\beta_{31}$, and $\beta_{51}$, model 2 is underidentified when $\gamma^*_{11} = 0$. Similarly, model 1 is underidentified when $\gamma_{11} = 0$. Without $\gamma^*_{11}$ in model 2 and without $\gamma_{11}$ in model 1, there is no third variable in either model that is related to $\eta_2$ and $\eta_3$ through $\xi_1$ or to $\eta_4$ and $\eta_5$ through $\eta_1$ and thus capable of identifying the relative loadings of each pair of variables on the family factor (Werts, Joreskog, and Linn 1973). Without $\gamma^*_{11}$ and $\gamma_{11}$, model 2 and model 1 each contain two single-factor models with two observables per factor, and none of the loadings is identified.

Of course, the empirical circumstances in which $\gamma_{11} = 0$ are quite different from those in which $\gamma^*_{11} = 0$. In the first case, the between-family regression is nonexistent, and in the second case, the within- and between-family regressions are homogeneous. In model 1 in the Hauser-Mossel data, $\gamma_{11}$ never approaches zero, and no problems of estimation
occur. In the several versions of model 2, $\gamma_{11}$ is always very close to zero in the Hauser-Mossel data. In the models where $\gamma_{21}$, $\gamma_{31}$, $\beta_{41}$, and $\beta_{51}$ are not all fixed, the estimation process therefore displays many of the signs of underidentification, even though the model is not formally underidentified. Jöreskog and Sörbom (1985, I, p. 36) note the possibility that a model may be “nearly non-identified,” but I do not know of any other good, or perhaps I should say bad, examples of it that have reached publication. If, in a set of data, the between-family slope were zero and the within-family slope were nonzero, I would expect model 1 to exhibit the same pathological behavior as does model 2 here.

A POSITIVE EXAMPLE OF MODEL TWO

Educational and occupational attainments were ascertained for a randomly selected brother or sister of each respondent in the 1975 follow-up of the Wisconsin sample, but the data used by Hauser and Mossel (1985) pertain to a subset of brother pairs in which both siblings were interviewed. Moments of educational and occupational status for the full set of 1,623 brother pairs are reported in table 2. All the data are based on reports by the primary respondents about themselves and their brothers in 1975, and there is only one indicator per variable. Thus, for the present analysis, I ignore response variability and treat each indicator as an error-free measure of the corresponding construct.

Table 3 shows selected parameter estimates and measures of fit of

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It has been suggested to me that Roosa, Fitzgerald, and Carlson (1982) may deserve this dubious honor.
<table>
<thead>
<tr>
<th>Restrictions</th>
<th>Model 1</th>
<th></th>
<th></th>
<th>Model 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_{62}$</td>
<td>$\gamma_{11}$</td>
<td>$\gamma_{12}$</td>
<td>$L^2$</td>
<td>$df$</td>
<td>$\beta_{42}$</td>
</tr>
<tr>
<td>A. Eqq. (15), (16), and (33) or (34)</td>
<td>.490 (.022)</td>
<td>.595 (.031)</td>
<td>.490 (.022)</td>
<td>8.02</td>
<td>2</td>
<td>.490 (.022)</td>
</tr>
<tr>
<td>B. Eqq. (15), (16), and (33), (35)</td>
<td>.530 (.013)</td>
<td>.530 (.013)</td>
<td>.530 (.013)</td>
<td>13.26</td>
<td>3</td>
<td>.530 (.013)</td>
</tr>
<tr>
<td>or (34), (36)</td>
<td>.425 (.032)</td>
<td>.599 (.031)</td>
<td>.539 (.028)</td>
<td>.18</td>
<td>1</td>
<td>.457 (.026)</td>
</tr>
<tr>
<td>C. Eqq. (15) and (16)</td>
<td>.418 (.036)</td>
<td>.613 (.045)</td>
<td>.544 (.031)</td>
<td>0</td>
<td>0</td>
<td>.418 (.036)</td>
</tr>
<tr>
<td>D. Eqq. (14) and (15)</td>
<td>.424 (.031)</td>
<td>.613 (.045)</td>
<td>.543 (.031)</td>
<td>0</td>
<td>0</td>
<td>.466 (.026)</td>
</tr>
<tr>
<td>E. Eqq. (14) and (16)</td>
<td>.423 (.036)</td>
<td>.613 (.045)</td>
<td>.543 (.031)</td>
<td>0</td>
<td>0</td>
<td>.465 (.27)</td>
</tr>
<tr>
<td>F. Eqq. (14) and (37)</td>
<td>.423 (.036)</td>
<td>.613 (.045)</td>
<td>.543 (.031)</td>
<td>0</td>
<td>0</td>
<td>.465 (.27)</td>
</tr>
</tbody>
</table>

**Note.**—See text for explanation. Wisconsin brothers data from table 2. $N = 1,623$. 
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models based on the data in table 2. I estimated exactly the same set of models here as in table 1, altered only to reflect the changes in the measurement model. In the models of rows A and B, where the within-family regressions are the same for each brother, model 1 and model 2 yield equivalent estimates. It is worth noting that, even with more than three times as many observations as in the sample with interviews of siblings, the evidence of heterogeneity in slopes between and within families is not very strong; the contrast between the models of row A and row B yields a test statistic of $L^2 = 5.24$, with 1 df. As is suggested in the previous analysis, there are discrepancies in fit and in parameter estimates between model 1 and model 2 in row C, where the two within-family regressions are permitted to differ.

The major difference between these analyses and those reported in table 1 is that each of the versions of model 1 and of model 2 in rows D, E, and F of table 3 yields plausible parameter estimates, SEs, and overall measures of fit. Evidently, the between-family regression is steep enough, relative to the within-family regressions, so that estimation of model 2 yields no signs of underidentification. However, there is an ironic twist to this example: Hauser (1984) has shown that in this full sample, as in the subsample used by Hauser and Mossel, the evidence of heterogeneous slopes between and within families disappears when the variances of educational attainments are adjusted for response variability. Thus, if the measurements were corrected for unreliability, we would again be led to prefer model 1 over model 2.

CONCLUSION

I believe that three morals follow from the preceding analyses. First, given the tractable behavior of model 1 throughout these analyses, there is no reason to prefer the set-up of model 2, even though it appears to be more like other models of sibling resemblance that have appeared in recent years. Although there are empirical circumstances in which estimation of model 1 could become problematic, these are evidently less likely to occur than with model 2.\footnote{Willem Saris has pointed out to me that model 2 becomes tractable if additional exogenous variables identify the loadings of the family factors, even when the net within- and between-family regressions are homogenous. For example, see Hauser and Sewell (1986).}

Some readers of this paper have proposed that there is a substantive difference between models 1 and 2. They have suggested (1) that model 2 is a better causal model because it does not rely on the formal decomposi-
tion of within- and between-family components, (2) that there is something needlessly abstract about the deviation of an individual's educational attainment or occupational status from that typical of his or her family, or (3) that there is a substantive difference in the interpretation of reduced-form coefficients in the two models because those of model 2, but not of model 1, include indirect effects of the family (group) factor by way of the individual variate (see equations [29] and [32]). I find little merit in these arguments. To be sure, some decompositions are merely devices for data reduction, but that is not true here; in demography and economics, causal interpretations based on formal decompositions are major theoretical and analytical tools. The deviation of an individual's schooling or occupational standing from that of his or her family is no more an abstraction than the family level of the same variable. Moreover, in model 2, the "indirect effects" of the exogenous family factor reflect the same decomposition that underlies model 1.

Where model 1 and model 2 are formally equivalent, as in the case of symmetry between the two within-family regressions, I see no substantive difference between them, just as there is no substantive difference between equations (5) and (6) in the analysis of covariance. In the case of formal equivalence, the choice between models is one of convenience, and, as I have shown, problems in estimation may dictate that choice. Where model 1 and model 2 are not formally equivalent, they are also substantively different, and the choice between them should rest on theoretical or empirical grounds.

Second, the combination of unfamiliar models and unfamiliar data may be lethal. Had I worked with the data of table 2 at the outset, as well as with those used by Hauser and Mossel (1985), I would have known that some of my difficulties in estimating model 2 were a function of the Hauser-Mossel data as well as of the model. The lesson is that it pays to vary all the factors to which your results may be sensitive, not merely those about which you think you know the least.

Third, the identifiability of a model is not an absolute matter, and identification is sometimes achieved by adding parameters to models and not by eliminating or constraining parameters. A model may only be identified with bounds on parameters; given seemingly equivalent parameterizations of the same model, the researcher should choose one that can be identified without bounds on parameters. Models should be avoided if their parameters are differences between effects that are equal under a maintained hypothesis. Such models should be rewritten to make those effects distinct parameters; the parameters may be compared by imposing an equality constraint on the effects and comparing the fit of the original model with that of the constrained model.
REFERENCES


Siblings


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