Economics 702
Macroeconomics
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Problem Set 1
Due in class on February 5.

1. In response to the recent change in federal taxes, firms have reacted in two different ways. Some firms have given bonuses to their workers, while others have raised wages. This problem explores the difference in these reactions for labor supply. Start with the benchmark model of labor supply we had in class, where workers earn wage w, have h hours to devote to leisure or work, and have π unearned income.

   (a) Suppose that the firm now offered a bonus, an additional amount b of income to its workers. To qualify for the bonus the worker must have worked a minimum amount of hours $N \geq \underline{N}$. What would be the impact of this bonus on the labor supply of workers in the firm?

   (b) Now suppose that the firm instead gave workers a two-tiered wage increase. That is, for part-time workers with $0 < N < \underline{N}$ wages increased to $w' > w$ but full time workers got a larger increase to $w'' > w' > w$ for $N > \underline{N}$. What would be the impact of this new wage schedule on the workers in the firm? Be sure to consider intensive and extensive margins, and the possible shifts from part-time to full-time or vice versa.

   (c) Between offering a bonus and increasing wages, which would be more likely to lead to higher output?

2. Consider a static economy where labor N is the only input in production. There is a representative firm who produces according to the production function:

$$Y = zN,$$

   where $z > 0$ is the level of productivity. Time per period is normalized to one and a representative worker has preferences over consumption C and leisure $1 - N$ given by:

$$u(C, 1 - N) = \log C + \log(1 - N).$$

   The worker has no unearned income and earns a wage w for each hour worked.

   (a) Find expressions for the equilibrium values of employment N and the wage w.

   (b) Suppose that there is a government program to subsidize wages paid by firms, which is funded by lump sum taxes on the household (and there is no other government spending). That is, for each hour of labor hired the firm pays $(1 - \tau)w$, while the worker receives the wage w per hour and must now also pay lump sum taxes T. How does this policy affect the equilibrium wage?

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(c) If the government balances its budget so $T = \tau w N$, how does this policy affect the equilibrium employment? Interpret your answer.

3. Consider the static general equilibrium model as in class, with the following functional forms, household utility is given by:

$$u(C, \ell) = \sqrt{C}\sqrt{\ell},$$

while the firm production is:

$$Y = z\sqrt{K}\sqrt{N}.$$

Household own the capital in fixed supply $\bar{K}$ which they rent to firms, and there is no government.

(a) Write down the household's problem of choosing consumption and leisure to maximize utility, find the optimality conditions, and derive the household's labor supply curve.

(b) Suppose there would be an increase in the household's capital $\bar{K}$ but all else was fixed (i.e. this is not yet equilibrium, so suppose w and r are unchanged). How would the household's labor supply change? Interpret your answer.

(c) Write down the firm's profit maximization problem, find the optimality conditions, and find the labor demand curve.

(d) If there were an increase in productivity z, how would the firm's labor demand change?

(e) Now impose market clearing (in all relevant markets) and find expressions for the equilibrium levels of K, N, C, Y, w, r.

(f) Suppose that there were an increase in capital supplied $\bar{K}$. How would the equilibrium change? Discuss your answer.

(g) Suppose that there were an increase in productivity z. How would the equilibrium change? Discuss your answer.

4. Consider a static model with two consumers A and B. The consumers value consumption and leisure but are envious of the other consumer's consumption. That is, consumer A's preferences are given by:

$$U^A(c_A, c_B, \ell_A) = u(c_A) - j(c_B) + v(\ell_A)$$

where c_A and ℓ_A are his consumption and leisure, c_B is the other consumer's consumption, and $j(c_B)$ gives the utility cost of jealousy. Consumer B's preferences are exactly symmetric. Each consumer has one unit of time to supply for labor or leisure so $N_A = 1 - \ell_A$ and $N_B = 1 - \ell_B$, and neither has unearned income so $C_i = w_i N_i$ where w_i is the wage of type i. Output is produced according to:

$$Y = F(N_A) + F(N_B).$$

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(a) Find an expression characterizing a competitive equilibrium in the labor market which relates marginal utilities (marginal rates of substitution) and marginal products for consumer A. Note that consumer A cannot control consumer B's decisions.

(b) Now consider the social planner's problem which is to solve:

$$\max_{c_A, c_B, \ell_A, \ell_B} U^A(c_A, c_B, \ell_A) + U^B(c_B, c_A, \ell_B)$$

subject to:

$$c_A + c_B = F(N_A) + F(N_B).$$

Consider only symmetric allocations where $c_A = c_B$ and $\ell_A = \ell_B$, and again find an expression which relates marginal utilities (marginal rates of substitution) and marginal products for consumer A.

(c) Do your answers for the previous parts differ? Why?