Chapter 4: Understanding interest rate

Math Joke:
A mathematician organizes a raffle in which the prize is an infinite amount of money paid over an infinite amount of time. Of course, with the promise of such a prize, his tickets sell like hot cakes. When the winning ticket is drawn, and the jubilant winner comes to claim his prize, the mathematician explains the mode of payments: “1 dollar now, 1/2 dollar next week, 1/3 dollar the week after that ... ”

1.1 Present Value (Present Discounted Value)

- The two rationales of discounting
  * Time preference (people are impatient)
  * Opportunity cost of forgone investment

- Present value: \( PV = \frac{CF}{(1+i)^n} \),
  - \( CF \) = cash flow
  - \( n \) = number of periods
  - \( i \) = interest rate

1.2 Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today.

- Measuring yield to maturity of four basic credit market instruments
  - Simple loan
    * Loans which must be repaid at the maturity date along with an additional payment for the interest
    * \( PV = \frac{CF}{(1+i)^n} \)
  - Fixed payment loan
    * Loans which must be repaid by making the same payment every periods
    * Loan value: \( LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \ldots + \frac{FP}{(1+i)^n} = FP \frac{1-\frac{1}{(1+i)^n}}{i} \)
  - Coupon bond
* Pays a fixed interest payment (coupon payment) every period until the maturity date, when a specified final amount (face value) is repaid

\[ P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n} \]

**Special case**: consol or perpetuity

- No maturity date and no repayment of principal \((n = \infty)\)
- Fixed coupon payments of $C

\[ P_c = \frac{C}{i} \]

- Discount bond (zero-coupon bond)

* is bought at a price below its face value (at a discount) and the face value is repaid at the maturity date.

\[ P = \frac{F}{1+i} \text{ or } i = \frac{F-P}{P} \]

* Note

- Current bond prices and interest rates are negatively related
- Style of questions that can be tested could be either:
  * How to calculate the yield to maturity of each credit market instruments?, or
  * How to calculate yearly payment?

### 1.3 Interest rates vs Returns

- **Rate of return** = current yield + rate of capital gain

\[ R = i_c + g = \frac{C}{P_0} + \frac{P_{t+1}-P_t}{P_t}, \text{ where } i_c = \frac{C}{P_0} \text{ is the current yield (the coupon payment over the purchase price), and } g = \frac{P_{t+1}-P_t}{P_t} \text{ is the rate of capital gain (the change in the bond’s price relative to the initial price).} \]

- Interest-rate Risk:
  * When interest rates change, a bond with a longer term to maturity has a larger change in its price and hence more interest-rate risk than a bond with a shorter term to maturity.
  * Duration, also called the effective maturity measures the average lifetime of a debt security’s stream of payments. It is a measure of interest rate risk.
  * No interest-rate risk for any discount (zero-coupon) bond whose time to maturity matches the holding period.

### 1.4 Real vs Nominal Interest Rates

- Real Interest Rate: the interest rate that is adjusted by subtracting expected changes in the price level
  - Why real? It more accurately reflects the true cost of borrowing
  - Fisher Equation: \( i = r + \pi_e \)

### 2 Chapter 5 Review: Behavior of Interest Rates

Change in Quantity Demanded vs. Change in Demand

Two useful frameworks to understand the behavior of interest rate
2.1 Bond Market\(^1\) (P/i - B or supply and demand framework of the bond market):

2.1.1 Demand for bonds increases (shifts to the right) when

- Wealth (↑) - the total resources owned by the individual, including all assets
- Expected rate of return (↑), (the return expected over the next period) on one asset relative to alternative assets
  - Expected inflation ↓
  - Expected interest rate ↓
- Risk (↓), (the degree of uncertainty associated with the return) on one asset relative to alternative assets
- Liquidity (↑), (the ease and speed with which an asset can be turned into cash) relative to alternative assets

2.1.2 Supply of bonds increases (shifts to the right) when

- i. Expected profit of investment opportunities ↑
- ii. Expected inflation ↑
- iii. Government budget deficit ↑

2.1.3 Note:

- Bond prices are negatively correlated with interest rates
- Fisher effect: When expected inflation rises, the demand for bonds decreases, and the supply for bonds increases. Bond prices will fall and interest rates will rise.

2.2 Money Market (i-M or Liquidity Preference framework)

2.2.1 Demand for money increases (shifts to the right) when

- Income ↑
- Price-level ↑

2.2.2 Supply for money increases (shifts to the right) when

- Central bank increases money supply

2.3 The linkage of two frameworks

2.3.1 The market for money is in equilibrium ⇐⇒ The bond market is in equilibrium.

\[ M^d + B^d = M^s + B^s \Rightarrow B^d - B^s = -(M^d - M^s) \]

2.3.2 Money has a zero rate of return but bonds have an expected return equal to the interest rate.

2.3.3 Money supply growth rate and interest rates

The liquidity preference framework predicts that an increase in money supply lowers interest rates.

A criticism by Milton Friedman:

\(^1\)Note that savers/lenders demand bonds and borrowers/spenders supply bonds
• Liquidity effect (↑ MS ⇒ ↓ i immediately)
• Income effect (↑ MS ⇒ ↑ income and wealth ⇒ ↑ i slowly)
• Price-level effect (↑ MS ⇒ ↑ price level ⇒ ↑ i slowly)
• Expected-inflation effect (↑ MS ⇒ ↑ πe ⇒ ↑ i slowly or quickly)

3 Practice questions:

Practice questions:
The best way to solve yield to maturity questions involve the following steps
1. Read the question and identify the type of credit market instrument being tested.
2. Identify the variables that are given
3. Identify the unknowns that need to be solved
4. Recall the appropriate formula
5. Substitute in the given values and obtain your answer

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Q1] Everything else held constant, if income tax rates were lowered, then _______.
A) the interest rate on municipal bonds would fall.
B) the interest rate on municipal bonds would rise.
C) the price of Treasury bonds would fall.
D) the interest rate on Treasury bonds would rise.

Q2] When the price of a bond is above the equilibrium price, there is an excess _______ bonds
and price will _______.
A) demand for; fall
B) supply of; rise
C) supply of; fall
D) demand for; rise

Q3] A situation in which the quantity of bonds supplied exceeds the quantity of bonds demanded
is called a condition of excess supply; because people want to sell _______ bonds than others want
to buy, the price of bonds will _______.
A) more; fall
B) fewer; fall
C) more; rise
D) fewer; rise

Q4] A factor that could cause the supply of bonds to increase (shift to the right) is:
A) a decrease in expected inflation.
B) a decrease in government budget deficits.
C) expectations of more profitable investment opportunities.
D) a business cycle recession.

Q5] If a security pays $55 in one year and $133 in three years, its present value is $150 if the
interest rate is _______.
A. 5 percent.
B. 10 percent.
C. 12.5 percent.
D. 15 percent.
[Q6] If you own a $1,000 face value bond with one year remaining to maturity and a five percent coupon rate and new bonds are paying 12 percent, what is the most you can get for your old bond?
A. $1,200
B. $1,100
C. $937.50
D. $1157.60
E. $867.99

[Q7] Which of the following bonds would you prefer to be buying?
A) A $10,000 face-value security with a 10 percent coupon selling for $10,000
B) A $10,000 face-value security with a 9 percent coupon selling for $10,000
C) A $10,000 face-value security with a 10 percent coupon selling for $9,000
D) A $10,000 face-value security with a 7 percent coupon selling for $10,000

[Q8] Which of the following are true for a coupon bond?
A) The yield to maturity is greater than the coupon rate when the bond price is above the par value.
B) The yield is less than the coupon rate when the bond price is below the par value.
C) The price of a coupon bond and the yield to maturity are positively related.
D) When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.

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[Q9] Other things being equal, an increase in the default risk of corporate bonds shifts the demand curve for corporate bonds to the ______ and the demand curve for Treasury bonds to the ______.
A) left; left
B) left; right
C) right; right
D) right; left

[Q10] If the price of bonds is set ______ the equilibrium price, the quantity of bonds demanded exceeds the quantity of bonds supplied, a condition called excess ______.
A) below; supply
B) above; supply
C) above; demand
D) below; demand

[Q11] Everything else held constant, if the expected return on U.S. Treasury bonds falls from 8 to 7 percent and the expected return on corporate bonds falls from 10 to 8 percent, then the expected return of corporate bonds ______ relative to U.S. Treasury bonds and the demand for corporate bonds ______.
A) rises; falls
B) falls; falls
C) rises; rises
D) falls; rises

[Q12] During a recession, the supply of bonds ______ and the supply curve shifts to the ______, everything else held constant.
A) increases; left
B) increases; right
C) decreases; right
D) decreases; left