Macroeconomic Models of Economic Growth

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Human Resources and Economic Growth
Transition: History to Contemporary

Spent the last few lectures considering the Industrial Revolution.

Broad picture, considered more than economics.

- Role of ideas; scientific revolution; experimentation
- Governance structure; strong central govt. versus emerging nation–states
- Fissure in Catholic Church; weakened central authority; interacted with nation–states
- National endowments; population; natural resources (coal)

Convert these factors into relative prices and incentives facing individuals.
Now, want to concentrate on economic factors of economic growth.

Recall that development is the process of establishing societal infrastructure for growth.

Models of economic growth, assume structure in place and concentrate on long run economic growth.

Will concentrate on the role of capital ($K$), labor $L$, technological change.
Aggregate Models

Will shift from detailed analyses of separate components of economy to abstract model of economy.

Will think of economy in the aggregate.

- National Income and Product Accounts. \((I = S)\)
- Conceptualize economy as production function \((Y = F(K, L))\)
Will study Harrod–Domar and Solow models of economic growth. Solow’s model is the center of the universe for economic growth models.

Will see that Solow’s model is simple yet it remains highly relevant for economic growth.

Its simplicity means that it is not realistic. Leaves out a lot.

We will use the Solow model as our trusted guided through the land of growth and development economics.
Methodology

1. Specify a model of the aggregate Economy.
   \[ Y = F(K, L, \ldots) \]
2. Work within an equilibrium framework.
3. Do comparative statics. Consider a change that moves the economy from one equilibrium to another.
4. Yields predictions on relationships we should see in the (aggregate) data.
5. Compare predictions in (4) to aggregate data. Compare a country over time, or several countries over time or at a point in time.
6. If model matches data patterns “well—enough summarize” findings. If model doesn’t fit; re–specify model (make more complicated) and repeat steps.
First Model: Macroeconomic Balance

In its simplest terms, economic growth is the result of abstention from *current* consumption.

Commodity production creates income which creates demand for those very commodities.

Two groups of commodities:
1. *consumption goods* produced to satisfy human wants.
2. *capital goods* produced for the purpose of producing other commodities.

Generally, households buy consumption goods; firms buy capital goods.

Households need not spend all their income. HH savings used to finance purchase of capital goods.

**Macroeconomic balance** Investment demand equals the savings of households, \( I = S \).
Accumulation

Focused on the accumulation of (physical) capital ($K$).

Growth occurs when $\Delta K = K(t + 1) - K(t) > 0,$
Discretize time, $t = 0, 1, 2, 3, \ldots$

Capitals denote aggregate variables.

- $Y$ total output
- $K$ total capital stock
- $C$ total consumption
- $S$ total savings
- $I$ total investment
Impose Macroeconomic Balance

\[ Y(t) = C(t) + S(t) \quad \text{HH} \]
\[ Y(t) = C(t) + I(t) \quad \text{Firms} \]
\[ S(t) = I(t), \quad \text{equilibrium} \]
Let $\delta$ be the fraction of the capital stock that depreciates each period.

$$K(t + 1) = K(t) - \delta K(t) + I(t) = (1 - \delta)K(t) + I(t)$$
Two Critical Concepts:

1. Savings rate: \( s = \frac{S(t)}{Y(t)} \)

2. Capital–output ratio \( \theta = \frac{K(t)}{Y(t)} \)
Harrod–Domar

Combine macroeconomic balance $S = I$, and equation for change in capital stock (and some manipulation) to obtain:

$$\frac{s}{\theta} = g + \delta$$

where $g$ is the overall rate of growth of output.
The growth rate of the economy is linked to two fundamental variables: the ability of the economy to save and the capital–output ratio.

- By increasing savings increase growth.
- Increase the rate at which capital produces output (a lower $\theta$) increases growth.

See textbook to add (exogenous) population growth $n$.

$$\frac{s}{\theta} = (1 + g^*)(1 + n) - (1 - \delta)$$

$$\frac{s}{\theta} \approx g^* + n + \delta$$

$g^*$ is growth rate of output per capita.

The approximation assumes that $g^*$ and $n$ are “small” ($g^* n \approx 0$).
H–D Model

\[ \frac{s}{\theta} = g^* + n + \delta. \]

- What if population growth rate increases?
- What if technological change, more \( Y \) for same \( K \)?
- Compare to societies otherwise equal but one has higher \( s \) than the other. What happens?
Experience dictates: \[ \text{What are the critical assumptions of H–D?} \]

Key assumptions:

1. Savings rate is constant \[ S(t)/Y(t) = s(t) \] but written as \( s \).
2. Capital–output ratio assumed constant \[ K(t)/Y(t) = \theta \].

Second assumption tantamount to assuming constant returns to scale.

Reasonable to believe there is diminishing returns to scale. For fixed labor force, increase capital stock expect output to increase but (eventually) by less smaller and smaller amounts.
Retain the first assumption, constancy of the savings rate
\[ S(t) = sY(t) = I(t). \]

Equation for the capital stock: \[ K(t + 1) = (1 - \delta)K(t) + sY(t) \]

Let Population growth \[ P(t + 1) = (1 + n)P(t). \]

\[ (1 + n)k(t + 1) = (1 - \delta)k(t) + sy(t) \quad (3.9) \]

Note equation is expressed in \textit{per capita} quantities (lower case).
Interpretation

Right hand side (RHS) two parts depreciated per capita capital and current per capita savings.

Together they give us (almost) the new per capita stock. True if \( n = 0 \), but population growth puts downward drag on per capita capital stock.

Hence, adjust for population growth by term \( 1 + n \) on LHS.

Note: the larger the rate of population growth, the lower is the per capita capital stock the next period.
Figure 3.3 p. 65 graph of production function (output per capita) vs capital per capita, and show output–capital ratios (angle theta of ray from origin)
Figure 3.3

Production function $y = f(k)$

Output--Capital Ratios

Capital per Capita ($k$)
Figure 3.4 $K(t + 1)$ vs $K(t)$ and show 45 degree line. $y = f(k)$ and $(1 + n)k$
Steady State

1. if $k(0) < k^*$
2. if $k(0) > k^*$
In Solow model the savings rate has no long–run effect on the rate of growth. (contrary to H–D).

What’s the resolution between H–D and Solow?
Savings rate does not affect long–run growth rate of per capita income, but affects the long–run level of income.

The steady state \( k(t + 1) = k(t) = k^* \) and manipulating eqn 3.9 to yield

\[
\frac{k^*}{y^*} = \frac{s}{n + \delta}
\]

Increase \( \delta \) this lowers the RHS. But this means that the capital–output ratio on the LHS must decline this means that \( k^* \) and \( y^* \) decline.

What’s the economic interpretation?
Higher population growth, lowers the steady–state level of per capita income.

But the total income must grow faster as a result.

Economy converges to a SS level of per capita income, which is impossible unless long–run growth of total income equals the rate of population growth.

Labor is both an input in production and a consumer of final goods. First raise total output and drives higher rate of growth of total income; second lowers savings and investment and brings down the SS level of per capita income.
Summary

1. Solow model that parameters such as savings rate has only level effect.

2. Solow model implies there is a steady–state level of per capita income to which the economy must converge.

3. Convergence (in LR) does not depend on historical starting point.

4. Solow model infers regardless of initial per capita capital stock, two countries with similar savings rates, depreciation rates, and population growth rates will converge to similar standards of living (in the LR).