Midterm Exam 1—Answers

Human Resources and Economic Growth

Answer all questions in your bluebook. Please enter your name and number all bluebooks that you use.
The value of each question is proportional to the number of minutes listed in parentheses. (75 total points)

1. (5 minutes) In the models we’ve studied in class all income is paid to households. Households spend money on consumer goods, where does the market for capital goods come from?

   Answer: Households do not spend all their income. Saved income is deposited in banks who make loans to firms. And we obtain the national income accounting identity, $S = I$.

2. (20 minutes) Recall that the Harrod–Domar Model is:

   $$\frac{s}{\theta} \approx n + g + \delta$$

   where $s$ is the savings rate, $\theta$ is the capital/output ratio, $n$ is the rate of population growth, $g$ is the growth rate and $\delta$ is the rate of depreciation. Using the same notation, the Solow model with exogenous population growth is

   $$(1 + n)k(t + 1) = (1 - \delta)k(t) + sy$$

   and $y$ is income (output) per capita.

   (a) If $s = 12\%$, $\theta = 4$, $n = 1\%$ $\delta = 2\%$ what is the growth rate as predicted by the Harrod–Domar model?

      Answer: $0.12/4 = 0.01 + g + 0.02$ or $g = 0$.

   (b) What is the steady–state ratio of capital output predicted by the Solow model?

      Answer: Steady–state is $k(t + 1) = k(t) = k$. Plug into Solow equation and solve for $k/y = \theta = 4$.

   (c) What is the growth rate $g$ predicted by the Solow model? Answer: $g = 0$

   (d) If the savings rate doubled, what is the growth rate predicted by Harrod–Domar? Answer: $0.24/4 = 0.01 + g + 0.02 = 0.03$ or $g = 3\%$. By Solow? Answer: The Solow model predicts that $g = 0$, as a higher savings rate increases the steady–state income, but does not produce long–run growth.

   (e) Briefly explain the economic rationale why in the Harrod–Domar model an increase in the savings rate increases the long–run growth rate of income per capita.

      Answer: Harrod–Domar assumes that capital–output ratio ($\theta$) is constant and not subject to diminishing returns. The assumption is tantamount to assuming that technology exhibits constant returns to scale.

3. (10 minutes) Consider the following data, for countries X and Z in the years 1960 and 2011. In both countries, the production function is $Y = AK^\alpha(hP)^{1-\alpha}$, where $\alpha = 1/3$. Capital letters denote aggregate variables, while lower case letters denote per capita quantities. As usual, $Y$ is output, $K$ is capital stock, $P$ is working population, and $h$ is the level of human capital, and $A$ is the total factor productivity.

   Question: Which country had higher productivity growth between 1970 and 2011? [Hint: Think before calculating.]

   Answer: This is virtually the same question as included on the study questions. I changed the output numbers and added the superfluous information on the size of the working population.
Table 1: Countries X and Z

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>$y$</th>
<th>$k$</th>
<th>$h$</th>
<th>P (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1960</td>
<td>100</td>
<td>1</td>
<td>1</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td>2011</td>
<td>1,350</td>
<td>27</td>
<td>8</td>
<td>2,000</td>
</tr>
<tr>
<td>Z</td>
<td>1960</td>
<td>20</td>
<td>2</td>
<td>2</td>
<td>4,000</td>
</tr>
<tr>
<td></td>
<td>2011</td>
<td>1,350</td>
<td>54</td>
<td>16</td>
<td>8,000</td>
</tr>
</tbody>
</table>

You do not need to use a calculator to answer this question. First, notice that the accumulation of factors from 1960 to 2011 is the same for countries $X$ and $Z$: $27^{1/3}8^{2/3}$. The country with the higher productivity growth is the one then that has the largest increase in output, which is $Z$.

4. (10 minutes) Recall in the model with human capital that the long–run growth rate of per capita income is

\[
\frac{y(t+1) - y(t)}{y(t)} = g = s^\alpha q^{1-\alpha}
\]

where $y(t)$ is per capita income, $g$ is the growth rate, $s$ is the savings rate, $q$ is the fraction of output that is spent on human capital, and $\alpha$ is the physical capital’s share of income.

**Question:** There is no technological progress ($\pi$) yet this model exhibits long–run growth at rate $g$. Briefly, explain the economic rationale for long–run growth in this model.

**Answer:** The accumulation of human capital augments the capability of labor. Thus human capital accumulation serves to offset diminishing returns of the physical capital. As Ray notes the result is sensitive to the addition of a third factor (e.g., unskilled labor) which would induce diminishing returns in the combination of physical and human capital.

5. (10 minutes) The following table provides data on the annual growth rate of output, physical capital, and human capital for two countries. Assume the income share of physical capital equals 1/3.

(a) For each country, calculate the growth rate of productivity and factor accumulation.

**Answer:** The growth accounting equation is $\dot{y} = \dot{y} - s_k \dot{k} - s_h \dot{h}$ (using the dot above a variable to denote percent change). So, for Austria the growth rate of productivity is $\dot{y} = 1.90 - (1/3) \times (2.82) - (2/3) \times 0.26 = 0.79\%$. The growth rate of factor accumulation is $1.90 - 0.79 = 1.11\%$. Calculations for Chile yield, the growth rate of productivity equals .91\% and a factor accumulation growth rate of 0.63\%.

(b) In which country does factor accumulation contribute the most to growth?

**Answer:** The share of factor accumulation for Austria is $1.11/1.90 = 58.4\%$ while the share for Chile is $0.63/1.54 = 40.9\%$. So factor accumulation contributes the most to growth for Austria.

6. (20 Minutes) True, False, Uncertain. Please label the following statements as True, False or Uncertain (i.e., truth or falsehood depends on missing information). You must support your answer; no credit for an unsupported label. Be brief.

(a) According to the Harrod–Domar model a higher savings rate increases the rate of growth of per capita income.

**Answer:** True. The first question presents the Harrod–Domar model. An increase in the savings rate increases the lhs, so $g$ must increase on the rhs to keep the equation in balance.
Table 2: Comparison of Austria and Chile

<table>
<thead>
<tr>
<th>Country</th>
<th>Output (%)</th>
<th>Physical Capital (%)</th>
<th>Human Capital (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>1.90</td>
<td>2.82</td>
<td>0.26</td>
</tr>
<tr>
<td>Chile</td>
<td>1.54</td>
<td>0.63</td>
<td>0.63</td>
</tr>
</tbody>
</table>

(b) In the Solow model with exogenous population growth, per capita income grows at the rate of population growth.

**Answer:** False. With only population growth, there is no long–run growth of income per capita.

(c) The market exchange rate depends only on traded goods and therefore is likely to be biased in assessing the standard of living in many developing countries.

**Answer:** True. This is why we use Purchasing Power Parity exchange rate to make cross–country comparisons. The PPP exchange takes account of non–traded goods (e.g., food and housing) which comprise a large share of household expenditures, especially among the poor.

(d) Assuming similar exogenous parameters governing the evolution of the economy, unconditional convergence predicts that history in the sense of different initial conditions does not matter; all countries converge to the same level of per capita income.

**Answer:** True. This is a restatement of unconditional convergence.

(e) If we control for differences in the rate of population growth and savings then conditional convergence predicts the long–run growth rates of all countries will be the same.

**Answer:** True. This is a restatement of conditional convergence; control for factors that determine initial conditions then as technical progress is assumed to be costlessly transmitted all countries should grow at the same rate.

(f) A production function written as $Y = AK^{\alpha}P^{\beta}$ exhibits constant returns to scale.

**Answer:** Uncertain. If $\alpha + \beta = 1$ then the Cobb Douglas production function exhibits constant returns to scale. If $\alpha + \beta < 1$ then there are decreasing returns and if the exponents sum to more than one there are increasing returns.

(g) In models with directed technical progress a firm that develops new technology obtains profits and some market power.

**Answer:** True. Firms invest in research and development to earn an economic profit (as opposed to an accounting profit). It is the prospect of a stream of profits (for the life of a patent) that gives firms an incentive to innovate. We know that (free–entry) in competitive markets eliminate economic profits. Hence, for economic profits to exist implies the innovation also generates some market power (that enable the firm to act monopolistically).

(h) In a model with deliberate (and hence endogenous) technical progress it is possible that multiple equilibrium exist.

**Answer:** Uncertain. It depends on the model of endogenous technical progress. If there are positive externalities among firms such as when the production function for each firm depends on the average level of capital in the economy then a firm’s willingness to invest depends on the anticipated behavior of other firms. As discussed in class it is possible that firms have pessimistic beliefs and invest little or not at low generating a low–growth equilibrium or conversely, they can be optimistic about the future, invest heavily and the high–growth equilibrium can emerge. It is important to realize multiple equilibrium arise due to externalities not simply directed investment decisions.
(i) The Harrod–Domar predicts the “neutrality” of growth rates with respect to per capita income.

*Answer:* True. Notice from question one that per capita income does not enter into the equation. Thus, the prediction on the growth rate is independent of per capita income.

(j) In a model with endogenous technical change, if technical change diffuses rapidly through the economy then convergence to the equilibrium will be rapid.

*Answer:* False. We’ve studied only comparative statics — comparing one equilibrium versus another. We know nothing about the speed of moving from one equilibrium to another. Conditional and unconditional notions of convergence are long-run, after all the transitory changes have occurred.