1) Suppose that two individuals, Jon and David, form a community and would like to construct a communal fort that would protect them from attacks. They consume both good $X$, a private good, and the protection from the fort, $P$. One unit of good $X$ costs 1 unit while one unit of $P$ costs 2 units, so the budget constraint for each is given by: $X + 2P = 100$. Both Jon and David have an income of 100 and a utility function of the form:

$$U = \log(X) + 2\log(P + P_D)$$

a. 10 points): How much protection, $P$, will be privately provided? What is optimal consumption of $X$, the private good?

The problem is to maximize $U = \log(X) + 2\log(P + P_D)$ subject to $X + 2P = 100$. Substitute from the budget constraint into utility and get $\log(100 - 2P) + 2\log(P + P_D)$ and differentiate with respect to $P_1$, finding $\frac{-2}{100 - 2P_1} + \frac{2}{P_1 + P_2} = 0$. Manipulate and you will find the reaction function $3P_1 = 100 - P_2$. The problem is symmetric, so the reaction function for David (Mr. 2) is $3P_2 = 100 - P_1$. Solve these and find $P_1 = P_2 = 25$, and $X_1 = X_2 = 50$. You can also find the reaction functions by calculating the MRS and setting it equal to the price ratio. Specifically, the MRS for Mr. 1 is $\frac{P_1 + P_2}{2X_1}$, and the price ratio is $\frac{1}{2}$.

This will give you the same reaction function as given above.

b. 10 points): What are the socially optimal amounts of protection, $P$, and consumption, $X$, of the private good? This is an example of what phenomenon discussed in class?

A straightforward way to solve for the social optimum is to maximize the sum of utility, subject to the budget constraint, so the problem would read, maximize $U = \log(X_1) + \log(X_2) + 4\log(P_1 + P_2)$, subject to $X_1 + X_2 + 2P = 200$, where $P_1 + P_2 = P$. Given the problem is symmetric, we also know $X_1 + X_2 = X_2$, and $X_1 = X_2 = \frac{1}{2}X$. So we can write the problem for the socially optimal solution, substituting in from the budget constraint, as

$$\max U = 2\log[100 - P] + 4\log(P) \Rightarrow \frac{-2}{100 - P} + \frac{4}{P} = 0.$$  This implies $P_1 = P_2 = X_1 = X_2 = 33\frac{1}{3}$. The phenomenon this problem illustrates is the general idea that public goods will be underprovided by the private market.

For those solving the problem by equating the sum of the marginal rates of substitution equal to the price ratio, your answer would note that $X_1 + X_2 = X$, and $X_1 = X_2 = \frac{1}{2}X$. Set up the problem as maximizing

$$U = 2\log(\frac{1}{2}X) + 4\log(P)$$ subject to $X + 2P = 200$. The sum of the MRS is $\frac{MU_X}{MU_P} = \frac{2P}{4X} = MRT = \frac{1}{2}$.

This, along with the budget constraint yields the previous answer.

2) Suppose that the demand for a chemical is given by $Q = 100 - 2P$, where quantity is
measured in pounds. The market supply is given by $MC = 5$. Assume that the marginal external damage of this product is $3/unit.

a. 3 points): What is the equilibrium price and quantity of this good without government intervention?

Solving $P$ in terms of $Q$ from the demand curve, we get that inverse demand is given by $P = 50 – 0.5Q$. Setting that equal to the supply, we get $5 = 50 – 0.5Q$, which implies that $Q = 90$. Find the price by plugging 90 in for $Q$ in the inverse demand, and we find that $P = 5$.

b. 7 points): What is the socially optimal $P$ and $Q$? What Pigouvian tax or subsidy would generate the socially efficient level of the good?

The social marginal cost curve is $8. Then we find that $8 = 50 – 0.5Q$, which means that $Q = 84$ and $P = 8$ is the socially optimal amount and price. That is, this the amount at which social marginal benefit equals social marginal cost. A Pigouvian tax on the producer of this product equal to $3/pound for every pound it will result in this outcome.

3, 10 points): Suppose that Mary and Beth live on the same street. In the winter, both of them like the snow on their street to be plowed. Beth's demand is given by $Q = 40 – P$ and Mary's demand is given by $Q = 30 – 2P$. Suppose that the marginal cost of plowing the snow is constant at $35$. What is the socially efficient amount of plowing that should be done?

Add the demands vertically. Solving for both inverse demand curves, we get $P = 40 – Q$ and $P = 15-0.5Q$. Adding those where prices are both positive, we find that the segment of the social marginal benefit curve from where $0 < Q < 30$ is given by $P = 55 – 1.5Q$. Mary is no longer willing to pay anything when $Q$ exceeds 30, so there the social marginal benefit curve is $P = 40 – Q$. Find the quantity where the social marginal benefit is equal to social marginal cost, which is $35$. So $35 = 55 – 1.5Q \rightarrow Q = 13.33$. Since this is less than 30, we are indeed using the correct segment of the social marginal benefit curve.

4, 15 points): An advisor to the Mayor of Madison (Mayor Dave) came into his office one day with the following idea. The Advisor had just read an article about subsidies and realized that, since almost everyone loves ice cream, that providing a city subsidy to ice cream vendors would raise consumer surplus and producer surplus. Consequently, Mayor Dave should adopt the subsidy. Mayor Dave knows you are taking Econ 441, and so he would like your advice on the idea.

a. 2 points): Suppose the demand for ice cream in Madison is $Q=100-P$. Supply is $Q=(1/3)P$. Describe the market equilibrium.

Set demand equal to supply and find equilibrium $P=75$ and $Q=25$.

b. 5 points): Mayor Dave’s advisor proposes a $20 per unit subsidy on the production side of the good. Describe the post-subsidy market equilibrium.

The subsidy shifts the supply curve to equal $P=-20+3Q$. Set the post-subsidy supply curve equal to the demand curve and find $P=70$ and $Q=30$. 

2
c. 8 points): Calculate the change in consumer surplus generated by the subsidy, and the change in producer surplus generated by the subsidy. Is the Advisor correct (explain clearly)?

The change in consumer surplus is $5(25) + 0.5(5)(5) = 137.5$. The change in producer surplus is calculated from the original supply curve – subsidies (and taxes) drive a wedge between the price consumers pay and the price producers receive. The producers receive $90$ per unit (the $70$ market price and a subsidy of $20$). So producer surplus increases by $15(25) + 0.5(5)(15) = 412.5$. So in some (limited) sense Mayor Dave’s advisor was right. Consumer and producer surplus goes up as a consequence of the subsidy. The city, however, must finance the subsidy. It is the subsidy multiplied by the post-subsidy quantity, or $20(30) = 600$. This exceeds the sum of the additional consumer and producer surplus (which is $550$). The difference of $50$ is the deadweight loss associated with the subsidy. Mayor Dave should get rid of this advisor (and hire you)!

5, 5 points): Suppose you wish to determine the effect of attending a charter school relative to attending a traditional public school. You assume that learning is measured by SAT scores of high school seniors at both types of schools. Name one potential problem that could lead to biased results if you were to identify the effect of attending a charter school by comparing the scores of the two groups?

The advantage of identifying the effect in this way is that it is simple. However, there are several potential pitfalls to such a simple approach. To the extent that students at charter schools are different from students at public schools in any way other than the type of school they attended, this identification strategy may lead to a biased estimate if the difference also affects achievement. For example, if a higher proportion of students who attend charter schools are from disadvantaged families relative to public school students (or vice versa), the estimate will be biased.

6, 5 points): Suppose that there are 1,000 voters in your city. A total of 400 are willing to pay up to $25 each for the construction of a park while the other 600 are willing to pay only $10. The construction of the park will cost $12,000, and someone proposes a vote of whether to tax each citizen $12 in order to finance the park.

a. What will be the result according to the median voter model? Is this result socially efficient? Explain.

b. How would your answer to part a change if instead of being willing to pay up to $25 each, the 400 residents were willing to pay up to $50 each?

a. The median voter in this case is willing to pay at most $10. Consequently, any proposal for the park that will cost that voter more than $10 will not pass, including the proposal of the $12 tax. However, the total willingness to pay of all the residents is $400 * 25 + 600 * 10 = 16,000$, which exceeds the cost of the park. Consequently, the socially efficient outcome is to construct the park, and so the median voter outcome is socially inefficient.

b. The answer would not change since the median voter did not change.

7, 5 points): If government believes that Tiebout sorting is optimal, which of the following is true?
A) School spending should be equalized across districts.
B) School spending should not be equalized across districts.
C) Local property taxes should be linked to local education spending.
D) Both a and c are correct.
E) Both b and c are correct.

E

8, 5 points): A city is trying to decide whether to build a community pool. The pool costs $50,000 to construct and operate for the first year, and costs $5,000 for annual maintenance every year into the future (assume maintenance costs begin next year). The city estimates that every person who visits the pool benefits by $7 per visit. Because the first summer or operation is expected to be unusually hot, the city estimates that 5,000 visits will be made the first year, but only 1,000 will visit every year after that. The social discount rate is 10 percent. Assume that these are the only costs and benefits that the city decision-makers care about. Should the city build the pool or not? Explain clearly.

The city should build the pool. The PDV of the benefits exceed the PDV of the costs, as the expressions below show.

\[
PDV_{\text{Costs}} = 50,000 + \sum_{t=1}^{\infty} \frac{5,000}{(1 + 0.1)^t} = 50,000 + \frac{5,000}{0.1} = 100,000.
\]

\[
PDV_{\text{Benefits}} = 7 \times (5,000 + \sum_{t=1}^{\infty} \frac{1,000}{(1 + 0.1)^t}) = 7 \times (5,000 + \frac{1,000}{0.1}) = 105,000.
\]