On The Political Economy of Zoning

Stephen Calabrese                      Dennis Epple                             Richard Romano
University of South Florida    Carnegie Mellon and NBER         University of Florida

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Abstract. Households choose a community in a metropolitan area and collectively set a minimum housing quality and a property tax to finance a local public good. The collective imposition of a lower bound on housing consumption induces an income-stratified equilibrium in a specification where meaningful community differentiation would not arise without zoning. We show computationally that zoning restrictions are likely to be stringent, with a majority facing a binding constraint in communities that permit it. By inducing a stratified equilibrium, zoning causes Tiebout-welfare gains in aggregate but with large welfare transfers. Relative to stratified equilibrium without zoning, the zoning equilibrium is significantly more efficient as it reduces housing-market distortions.

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1. Introduction

Few policies of local governments are more ubiquitous or more controversial than zoning. Critics argue that much zoning is fiscally motivated, a tool enabling residents of wealthy communities to restrict entry of poorer households who would contribute less to tax revenues than the cost of the public services they consume. The terms “fiscal zoning” and “exclusionary zoning” have been coined to describe such exercise of community zoning powers.¹ Bitter, repeated court battles attest to the intense hostility to zoning and the equally intense resistance by communities to restrictions on their use of zoning.²

While economic analysis has illuminated the incentive issues associated with use of zoning³, the quest for a model characterizing zoning in a multi-community setting with heterogeneous households has proven elusive. Such an effort encounters two key difficulties. One is that there appears to be a chicken-and-egg problem: a community’s residents set zoning, but one cannot determine who those residents are without knowing the community’s zoning policy. The other difficulty is that efforts to model zoning and property taxation as the outcomes of a collective choice process confront the well-know (Plott, 1967) problems of existence of voting equilibrium in multi-dimensional settings.

¹ Ladd (1998), Chapter 1, provides a comprehensive overview of the issues and a history of legal developments related to use of zoning.
² The Mount Laurel New Jersey case is perhaps the most prominent example. In 1975, the New Jersey State Supreme Court declared unconstitutional Mount Laurel’s zoning restrictions, and the NJ Court mandated that every community accept its “fair share” of low income housing. In several subsequent decisions, the NJ Court has attempted to force adherence to its fair-share mandate (Ladd, 1998; Schuck, 2002).
³ White (1975a, 1975b) and Ohls, Weisberg, and White (1974, 1976) are leading papers introducing the formal analysis of zoning and characterizing the incentives for use of zoning. See also Epple, Filimon, and Romer (1988).
In this paper, we develop a model of zoning in multi-community equilibrium. We now briefly outline key features of the model, both to highlight its important elements and to indicate how the model resolves the two problems just discussed. We assume collective choice of property tax to finance a local public good and choose a specification of preferences for which, absent zoning, there is no mechanism for households to stratify across communities.\(^4\) This permits us to provide a precise characterization of the role of zoning in inducing any stratification that emerges in equilibrium. We resolve the chicken-and-egg problem with the following timing of events. Households first buy land in a community. They then vote simultaneously on the property tax rate and on the minimum amount of housing that a dwelling must provide.\(^5\) Households may then relocate to another community or adjust their land holdings within the community. They then acquire housing, respecting the zoning restriction imposed in their chosen community, the public good levels are determined, and consumption occurs. At the voting stage, the potential non-existence of equilibrium is resolved by invoking the representative-democracy model of Besley and Coate (1997).

To anticipate our results, we find that communities adopt very stringent zoning ordinances. In light of this, it is important to note that, if anything, our model understates the incentives for restrictive zoning. It is often argued, quite plausibly, that incentives for restrictive zoning are particularly strong when a group of early community arrivals can impose zoning requirements, exempting themselves, while binding future residents. This

\(^4\) While, empirical evidence indicates preferences are such that stratification would occur in the absence of zoning (e.g., Epple and Sieg, 1999), we believe that the incentives for zoning are most effectively illuminated in a model in which stratification would not otherwise occur. Also, as becomes clear in our computational analysis, it is highly likely that similar incentives will be present in a model in which preferences would induce stratification.

\(^5\) We assume that adequate tools are available (e.g., building codes and restrictions on minimum lot size and floor space) to permit a community, effectively, to specify the minimum number of units of housing services that a dwelling must provide.
is not permitted in our model. All residents must adhere to the community zoning ordinance. Thus, when voting on a zoning ordinance, voters know that they will be required to adhere to the ordinance if they stay in the community. Our finding that communities adopt restrictive zoning thus emerges because the pivotal voter finds it in his interest to bind himself in order to bind others.

Zoning induces income stratification, and aggregate welfare gains arise from better Tiebout matching of preferences to levels of public-good provision. Gains are far from evenly distributed, however, with significant welfare losses to poorer households and a majority worse off. Setting aside the distributional effects, the aggregate gain is achieved relatively efficiently: The housing market distortions under zoning are much lower than those in a perturbed equilibrium that is stratified without zoning, stratification in the latter supported by (the usual) ascending property taxes and consumer housing prices. The zoning equilibrium is also shown to be very similar to equilibrium with head- and property taxation as argued by Hamilton (1975).

The state-of-the-art in modeling zoning in a multi-community equilibrium, and the closest antecedent to our work, is that of Fernandez and Rogerson (1997). They consider an environment with two communities, one of which may zone. They first consider the case in which zoning is exogenously determined, investigating the implications of the zoning constraint for stratification and other properties of equilibrium. In the presence of an exogenous zoning constraint, they demonstrate that equilibrium exists in the vote over property taxes and that the richer community becomes more exclusive due to the zoning constraint. They next endogenize zoning by assuming that households first commit to choice of a community. Policy is then chosen sequentially,
first with a vote on zoning followed by a vote on the tax rate. Households then purchase housing in their community and consume. They present an illuminating illustration of the potential non-existence of majority voting equilibrium. Our model differs in the following respects: Voting is conducted simultaneously over zoning and the community tax rate. Because community residents can then move, voters recognize that zoning is an instrument that may be used to restrict access to the community. Voters are owner-occupants who take account of the effects of zoning on the value of the housing they own. There is no incentive in our main model, absent zoning, for stratification of households across communities. Also, our computational model has several communities.

Henderson and Thisse (1999, 2001) consider an environment in which a new community in an established metropolitan area is developed by either a single developer or a small group of developers. Their interest is in the challenging problem of characterizing the array of housing types offered within a community when differences in housing types may potentially be used to price discriminate among prospective buyers. We consider simultaneous formation of multiple communities. The “original” owners of the communities’ land are price takers, a single zoning constraint applies to all properties in a community, and resident-voters determine zoning.

Our model also helps to address the efficiency properties of zoning. In an influential paper, Hamilton (1975) argued that zoning may effectively convert the property tax to a head tax, thereby limiting or eliminating the deadweight loss that would

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6 With the choice sequence adopted by Fernandez and Rogerson, a community’s population is fixed before the vote on zoning occurs. Thus, zoning is not a tool that residents can use to restrict access to the community, although initial community choices are affected by the zoning that is anticipated.

7 Our approach can also be applied when stratification would occur in the absence of zoning.
otherwise accompany the use of property taxation. As noted, we investigate this issue, comparing our equilibrium with zoning to equilibrium when communities are permitted to use head taxes.  

Section 2 lays out the theoretical model. Some general properties of equilibrium are developed in Section 3. A computational analysis comprises Section 4. Section 5 concludes. An appendix contains some of the technical analysis.

2. The Model

(a) Communities and Households. The model is of a metropolitan area (MA). The MA consists of J communities that may differ in their land areas, $L_j$, $j = 1,2,\ldots,J$. The default range of the index $j$ (and $i$) is from 1 to $J$. Housing is produced by price-taking firms in each community from land and a non-land factor with a constant returns neoclassical production function. The price of the non-land factor is assumed fixed and uniform throughout the MA. Housing supply in community $j$ is then given by:

$$H_j(p_h) = L_j h_s(p_h),$$

where $p_h$ is the supplier price of housing and $h_s(\cdot)$ is an increasing function.

It is useful to list immediately equilibrium characteristics of community $j$, while describing below the timing of their determination. Equilibrium characteristics of community $j$ are:

(a) property tax rate $t_j$;

(b) supplier housing price $p_h$ and consumer or gross price of housing;

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8 Zoning may also potentially be used to prevent adjacent location of incompatible (e.g., industrial and residential) activities. This has prompted research on whether such zoning accomplishes outcomes different from those that would emerge, a la Coase, by land use that “follows the market.” See Wallace (1988) for a discussion and empirical analysis. This issue does not arise in our model, where household location does not give rise to non-pecuniary externalities.
\[ p_j = (1 + t_j)p_h^1; \]  
\[ (2) \]

(c) competitively determined land price \( p_j' \) satisfying\(^9\):

\[ p_j' = \hat{p}_j'(p_h^1; L_j) \]  
\[ \frac{\partial \hat{p}_j'}{\partial p_h^1} > 0 \]  
\[ \frac{\partial \hat{p}_j'}{\partial L_j} < 0; \]  
\[ (3) \]

(d) measure of households with income \( y \) denoted by:

\[ f_j(y) \]  
\[ (4) \]

where \( S \) is the support of the income distribution in the MA;

(e) congested local public good (e.g., per student public schooling expenditure), \( g_j \), satisfying community budget balance:

\[ g_j \int_{y_{\text{min}}}^{y_{\text{max}}} f_j(y) dy = t_j p_h^1 H_j^1; \]  
\[ (5) \]

and, finally,

(f) minimum required housing consumption \( h_{mj} \) if enacted.

The minimum housing requirement captures the zoning restriction in community \( j \), and its determination and consequences are the focus of this paper.

Turning to households, they are assumed to have the same utility function over \( g, h, \) and a numeraire denoted \( b \), of the form:

\[ U(g, h, b) = v(g)u(h, b). \]  
\[ (6) \]

The function \( v(g) \) is differentiable and increasing. The function \( u(\cdot) \) is differentiable, increasing, quasi-concave, and linearly homogeneous. While our model of zoning applies under more general preference configurations, these assumptions on the utility function

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\(^9\) The \( \hat{p}_j'(\cdot) \) function satisfies land-market clearance given competitive equilibrium in the housing market.
are valuable in clarifying the role of zoning. Under these assumptions, zoning will be seen to be necessary to induce consequential differences in communities.

Households are differentiated ex ante by their exogenous income, \( y \). A continuum of households make up the MA, with continuous p.d.f. \( f(y) \), everywhere positive on the interior of its support \( S \subset [0, \infty) \). Normalize the MA’s population to unity.

Some simple results on household choice are usefully examined before defining equilibrium. Given income \( Y \) (since effective income will depart from \( y \) at one point below) and residence in community \( j \), ordinary housing demand, \( h_d \), solves:

\[
\max_h v(g_j)u(h, Y - p_j h).
\] (7)

Using linear homogeneity of \( u(\cdot) \), this has solution:

\[
h_d(Y, p_j) = Yq(p_j),
\] (8)

for decreasing function \( q(p) \). Given a zoning constraint in community \( j \), housing consumption is given by: \( \max [h_{mj}, Yq(p_j)] \). Noting that ordinary demand for housing is increasing in \( Y \), let \( Y_{mj} \) denote the maximum-income household that would be constrained in community \( j \):

\[
h_d(Y_{mj}, p_j) \equiv h_{mj}.
\] (9)

One can then find indirect utility:

\[
V_j \equiv V(g_j, p_j, h_{mj}; Y) = \begin{cases} 
  v(g_j)u(h_{mj}, Y - p_j h_{mj}) & \text{if } Y \leq Y_{mj} \\
  v(g_j)Yw(p_j) & \text{if } Y \geq Y_{mj},
\end{cases}
\] (10)

for a decreasing function \( w(p) \).

b. Equilibrium. Household choices are made in three stages. First households choose an initial community of residence, this committed by purchase of any amount of land in their

\[10\] Of course, such a household may not exist in community \( j \) or in the entire MA.
chosen community \( j \). Let \( \ell_j(y) \) denote household-type \( y \)’s land purchase in their chosen community \( j \). Since everyone has rational expectations, the price of land in community \( j \) in the first stage equals the final price of land there. Owning land in community \( j \) gives households the right to vote on the policy pair \((t_j, h_{mj})\) in community \( j \). We assume for simplicity that households can purchase land in only one community, but the same results would hold allowing multiple land purchases if households can vote only in one community, perhaps where they hold the most land.

Land owners in each community collectively determine the policy pair \((t_j, h_{mj})\) in the second stage according to a variant of the Besley-Coate (1997) representative democracy model.\(^\text{11}\) A land owner with a majority preferred policy pair among land owners’ ideal points is elected mayor, who then enacts his preferred policy pair. Below we detail the beliefs about the future that voters and candidates have during the voting stage. The basic idea of the Besley-Coate model is that a candidate for mayor cannot commit to a policy pair so it can be anticipated that, once elected, the candidate will choose his ideal point. This restricts the set of feasible policy pairs to be a subset of ideal points of community members. Because the voting problem is multi-dimensional, a Condorcet winner will not generally exist (Plott, 1967), but the restriction on feasible policies implied by the representative democracy model leads to an equilibrium in our computational model. Specifically, we find the equilibrium assuming 0 entry costs to stand for election. If a policy pair in the set of all land owners’ ideal points is majority preferred in that set, then this is an equilibrium.\(^\text{12,13}\) Moreover, only such points can be

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\(^{11}\) See also Osborne and Slivinski (1996).

\(^{12}\) Besley and Coate also allow voters to have preferences for particular candidates independent of the candidate’s policy preference, e.g., for good looking candidates. We assume no such preferences, that every voter’s incentive is to maximize utility as it has been defined.
equilibrium points. We confirm that a unique such ideal point arises in each community in our computational model.

In the third and final stage, households can frictionlessly re-optimize with respect to their choice of residence and land, consumption levels are chosen, and the local public-good levels are determined. In determining the equilibrium conditions for the third stage, we assume that households take as given the equilibrium characteristics of the J communities.\textsuperscript{14} The first-stage land purchases are per se irrelevant to equilibrium in the third stage because the first-stage land prices equal those established in the third stage, implying no capital gains or losses from possible sale of land. (Such potential capital gains or losses will play a role in the second stage as clarified below.)

The third-stage equilibrium conditions are (2), (5), (9), and:

\begin{equation}
\int_{\eta_{mn}}^{\eta_{ms}} h_{mj} f_j(y) dy + \int_{\eta_{mj}}^{\eta_{mt}} y q(p_j) f_j(y) dy = L_j h_s(p_j/[1 + t_j]);
\end{equation}

\begin{equation}
\begin{cases}
0 & \text{if } V(g_{j},p_{j},h_{mj};y) < \max_{i \neq j} V(g_{i},p_{i},h_{mi};y) \\
f(y) & \text{if } V(g_{j},p_{j},h_{mj};y) > \max_{i \neq j} V(g_{i},p_{i},h_{mi};y) \\
[0, f(y)] & \text{if } V(g_{j},p_{j},h_{mj};y) = \max_{i \neq j} V(g_{i},p_{i},h_{mi};y),
\end{cases}
\end{equation}

and

\begin{equation}
\sum_{i=1}^{J} f_i(y) = f(y).
\end{equation}

Conditions (2) and (9) are definitional. Condition (5) determines the $g_i$'s. Condition (11) is housing-market clearance, with the left-hand side housing demand, this condition

\textsuperscript{13} This and the next statement are essentially restatements of Corollary 1 in Besley and Coate (1997, p. 92). This case corresponds to their single-candidate equilibrium. Besley and Coate do not, however, examine the case of 0 entry cost. With 0 entry cost, everyone except a resident with majority-preferred ideal policy pair is indifferent to entry in equilibrium. A resident with majority-preferred ideal policy pair has a strict preference for entering and is then elected.

\textsuperscript{14} At this stage, the values of $(t_j, h_{mj})$ have already been determined. Taking as given the remaining community characteristics is implied by the rationality and atomism of households.
determining $p_j$. Condition (12) determines the $f_j(y)$’s, with each household choosing an optimal residence along with land and housing. Condition (13) ensures a feasible split of households in cases where income-$y$ households are indifferent among communities. As clarified in the next section, we will focus on income-stratified equilibria in which the measure of the latter types is zero.

Consider now how voters and candidates in the second stage determine the consequences of policy pairs. (Since every community member is both a candidate and voter, we refer to them as just “voters.”) Let $x_j$ denote the vector of $x_i$’s, $i = 1,2,\ldots,J$, $i \neq j$. We assume voters in community $j$ take as given the third-stage equilibrium vector $(p_{j},g_{j},h_{m,j})$ when voting, while otherwise anticipating the effects on the equilibrium allocation of their community’s policy pair. Hence, when voting, residents of community $j$ take as given utilities of all types (including themselves) should another community be chosen. Let $p_j(t_j,h_{m_j})$, $g_j(t_j,h_{m_j})$, $h_{m,j}$, and $f_j(y;t_j,h_{m_j})$ denote, respectively, a voter in community $j$’s anticipated values of his community’s gross housing price, land price, public good provision, and the community income distribution. In equilibrium, these anticipated values will be correct. Mathematically, they satisfy (2), (3),

$$

\begin{align*}
\max_{y_{\max}} & \min_{y_{\min}} g_j \int f_j dy = \frac{t_j p_j^*}{1 + t_j} H_s(p_j^*/[1 + t_j]); \\
\int_{y_{\min}}^{y_{\max}} h_{m_j} f_j dy + \int_{y_{\min}}^{y_{\max}} [y + (p_{j}^* - p_j) \ell_j(y)] q(p_j^*) f_j dy = H_s(p_j^*/[1 + t_j]);
\end{align*}

(14)

(15)

and

\begin{align*}
\text{In (2), here } p_j = p_j^*, \text{ implying an anticipated net price of housing } p_j^*. \text{ The latter enters (3), determining } p_j^*. \text{ Related to this, note that } p_j^* \text{ in (15) and (16) equals the equilibrium price of land that actually results. It solves (3) at the equilibrium net price of housing.}
\end{align*}

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with $Y_j \equiv y + (p_j - p_j')\ell_j(y)$,

$$
\begin{cases}
0 & \text{if } V(g_j^a, p_j^a, h_{mj}; Y_j) < \max_{i \in j} V(g_i, p_i, h_{mi}; Y_i) \\
= f(y) & \text{if } V(g_j^a, p_j^a, h_{mj}; Y_j) > \max_{i \in j} V(g_i, p_i, h_{mi}; Y_i) \\
\in [0, f(y)] & \text{if } V(g_j^a, p_j^a, h_{mj}; Y_j) = \max_{i \in j} V(g_i, p_i, h_{mi}; Y_i).
\end{cases}
\tag{16}
$$

The conditions describing the anticipated equilibrium values as functions of the policy vector in community $j$ are essentially the same as the conditions describing the equilibrium that actually arises in the third stage. Because the just listed conditions must allow out-of-equilibrium values of the policy pair $(t_j, h_{mj})$, the effective income determining anticipated housing demand and indirect utilities includes a potential capital gain or loss on initially purchased land: $(p_j^a - p_j')\ell_j(y)$. Since the price of land in the first stage, $p_j'$, is rationally assumed to equal the eventual equilibrium price of land (see the previous footnote), no capital gain or loss results in equilibrium. However, the potential for such gain or loss impacts voter preferences in the second stage and thus equilibrium. When a resident of community $j$ votes over policy alternatives, the household maximizes: $\max [V(g_j^a(t_j, h_{mj}), p_j^a(t_j, h_{mj}), h_{mj}; y + (p_j^a - p_j')\ell_j(y), V(\cdot)]$, with each element $i$ of $V_{-j}$ evaluated at $(g_i, p_i, h_{mi}; y + (p_j^a - p_j')\ell_j(y))$. Note, importantly, that while voters in community $j$ take as given utility of every household if another community is chosen, they anticipate movement of households into and out of their own community as their community’s policy, and thus utility in their community, changes. Hence, the effects of zoning changes on final community choices are anticipated.

Last, consider the first stage choice of initial residence and land. In an equilibrium, households must be indifferent to these choices by the following argument. Assume for the moment that a household cannot impact the political equilibrium in its
initially chosen community. Capital gains or losses on initial land purchases fail to arise in any community because the initial equilibrium land price equals the final equilibrium land price. Households can frictionlessly re-optimize with respect to community choice and consumption in the third stage. Hence the initial community and land choice has no impact on the household’s utility given it has no effect on political equilibrium. We next argue that having such an effect is inconsistent with equilibrium.

Because a household is atomistic, its vote in a community and land holdings have no impact. However, if a household’s preferred policy pair is the majority choice in a community and it differs from the majority choice absent that household’s choice of the community, then the household could have a strict preference for the community. A necessary condition for the latter is that the household has different income than other households in the community, since, otherwise, the household’s preferred and majority chosen policy pair would already be the majority choice. Suppose, then, that a household could choose a strictly preferred community where its preferred policy differs and would be majority chosen. Then, since there is a continuum of households, there must also exist households with the same income who do not live in the community and thus do not strictly prefer the community. But this contradicts such an initial allocation of households across communities being an equilibrium. Thus it is impossible for any household in equilibrium to have a strict preference for their initial community (or land) choice.

Because of the indifference over initial choices that must characterize equilibrium, we believe that there is a multiplicity of equilibria. We then restrict our analysis to no-adjustment equilibria, where households choose initially the community
and land corresponding to their optimal third-stage choices.\textsuperscript{16} We find this an intuitive restriction with small adjustment costs in mind, but we have not formally introduced such costs. We do need to make sure in these allocations that in fact no one could initially choose another community and be better off because they offer a distinct and majority preferred policy pair. We verify this in the equilibria we investigate computationally.

3. Theoretical Results

In this section, we develop general properties of the no-adjustment equilibria that we study. The next section pursues a computational analysis where we quantify findings, including of welfare effects, as well as verify existence and uniqueness. We begin with a definition of homogeneous equilibrium: In a homogeneous equilibrium, there exists a positive measure of household types indifferent in their choice of residence among two or more communities. The extreme form of homogeneous equilibrium has all households indifferent among all communities. Zoning provides a means to induce non-homogenous equilibrium (henceforth, NH-equilibrium). Absent zoning, equilibrium must be homogeneous with the assumptions on preferences (6) that we have adopted:

\textit{Proposition 1:} NH-equilibrium can arise only if \( h_{mi} > 0 \) and would be binding on some household if it selects community \( i \).

\textit{Proof of Proposition 1:} Absent potentially binding zoning constraint, utility is given by the lower line of (10) for every household in every community. Given \( V_i > V_j \) for some household and communities \( i \neq j \) as characterizes NH equilibrium, then no household selects \( j \), a contradiction. \( \Box \)

In fact, zoning must characterize equilibrium.

\textsuperscript{16} Households are not committed to the initial community that they choose and the prospect of moving plays a key role on the equilibrium allocation.
Proposition 2: In equilibrium, at least J-1 communities will impose a zoning constraint that would bind some household if it selects the community.\textsuperscript{17}

The proof is long and in the appendix. It is by contradiction and formalizes the following logic. Taking as the point of departure a hypothesized equilibrium in which a pair of communities does not zone, one community introduces zoning. In the zoned allocation, the community retains the same government spending and gross-of-tax housing price as in the equilibrium without zoning. The zoning level is set so as to induce income stratification, with the aggregate housing demand in the zoned allocation equal to that in the equilibrium without zoning. Poorer initial residents of the community that introduces zoning move out, while richer households from the other community move in. This is achieved with a zoning constraint not strictly binding on the marginal resident after zoning, but strictly binding on all individuals poorer than the marginal resident. The per capita tax base is then higher in the zoned allocation. This permits a reduction in the tax rate and, thereby, an increase in the net-of-tax price of housing. All initial residents of the community realize a capital gain on their initial land purchase, making them strictly better off.\textsuperscript{18} The zoning ordinance thus receives unanimous support.

We focus on NH-equilibrium.\textsuperscript{19} The major equity concern raised by zoning is that it is “exclusionary,” limiting access of poor households to suburban communities that offer high levels of locally provided public goods. In our model, zoning is exclusionary if it induces income-stratified NH-equilibria. Hence, we next examine when NH-equilibria

\textsuperscript{17} As is clarified in the proof, to show Proposition 2 we use a very weak condition that communities are not “too small” and/or that housing supplies are not “too elastic.”

\textsuperscript{18} Those that move out of the community that introduces zoning anticipate the same (p,g) pair in the other community as in the initial allocation (using our equilibrium definition), while better off due to the capital gain.

\textsuperscript{19} We thus ignore as uninteresting equilibria, if there are any, in which two or more communities are homogeneous; for example, communities which are replicas of each other and impose the same zoning constraint that binds some of their residents.
exhibit *income stratification*. In an income-stratified equilibrium, the communities can be numbered such that, if household with income $y$ chooses community $i$ and household with different income $y'$ chooses community $j > i$, then $y' > y$. Proposition 3 provides conditions such that equilibrium will be *stratified*, by which we mean income-stratified.

**Proposition 3:** a. If communities can be numbered such that in equilibrium

$$\left( \frac{u_b}{u} \right)_i \leq \left( \frac{u_b}{u} \right)_{i+1}$$

for all $y \in S$, $i=1,2,...,J-1$, with strict inequality for $V_i = V_{i+1}$, then NH-equilibrium is stratified.

b. If utility is Cobb-Douglas, $U = g^\beta h^\alpha b^{1-\alpha}$, then NH-equilibrium is stratified.

**Proof of Proposition 3:** a. Differentiating gives

$$\frac{\partial [V_{i+1} - V_i]}{\partial y} = \left( \frac{V_i u_b}{u} \right)_{i+1} - \left( \frac{V_i u_b}{u} \right)_i.$$

Hence, if $V_i = V_{i+1}$ for household-type $y$, then all households with $y' > y$ strictly prefer community $i+1$ over community $i$.

b. See the appendix. □

**Interpretation of Proposition 3:** The economic interpretation and role of zoning in Proposition 3a can be seen as follows. Inverting the ratio in the condition of Proposition 3a, we have:

$$\frac{u}{u} = \frac{u_b}{u} b + \frac{u_b}{u} h = y - ph + MV^h h,$$

where $MV^h \equiv \frac{u_b}{u}$ is the household’s marginal valuation of housing and the first equality follows from linear homogeneity of $u$. The condition of Proposition 3a can then be restated as:

$$(MV^h_i - p_i) h_i \geq (MV^h_{i+1} - p_{i+1}) h_{i+1},$$

with strict inequality for the indifferent household type with $V_i = V_{i+1}$. Observe that the indifferent household and by continuity the lowest-income households in the richer community $i+1$ must face a binding zoning constraint
there: \( h_{i+1} = h_{m,i+1} > h_d \) and \( MV_{i+1}^h < p_{i+1}. \) If these households were unconstrained in community \( i+1, \) then \( h_{i+1} = h_d, \) implying \( (MV_{i+1}^h - p_{i+1})h_{i+1} = 0, \) and the strict inequality condition cannot be satisfied.\(^{21}\) A relatively high zoning constraint in community \( i+1 \) induces income sorting because it has a stronger deterrent effect on poorer households, confirming the criticism that zoning is exclusionary. Since housing demand rises with income, the utility reduction from a binding housing constraint declines as income rises. Proposition 3b provides an example, used in computational model below, where NH-equilibrium is always stratified.

Some benefit of zoning must arise, of course, for it to be an element of equilibrium. Zoning that induces stratification increases the tax base in two or three ways. Housing consumption rises directly due to binding at least relatively poorer households in a community and indirectly by increasing incomes in the community and thus housing demand. The third potential gain might arise from reducing the number of users of the (congested) public good. We have:

**Proposition 4:** Assuming not all residents of community \( i \) are constrained, in NH-stratified equilibrium, \( g_{i+1} \geq g_i \) and/or \( p_{i+1} \leq p_i \) at least one with strict inequality.

**Proof of Proposition 4:** Given the zoning constraint is binding on some households in community \( i+1, \) if the conditions of Proposition 4 are not satisfied, then \( V_{i+1} < V_i \) for some residents of community \( i+1, \) a contradiction. If no residents of community \( i+1 \) are

\(^{20}\) That \( MV_{i+1}^h < p_{i+1} \) for a subset of consumers in communities that impose zoning will imply elevated demand for inputs that produce housing and thus higher land rents holding fixed taxes and public provision in a community. The observed price of land will exceed the marginal value of land. Such an observation serves as the basis for one of the tests in Glaeser and Gyourko (2002) of the impact of zoning on housing affordability. They find evidence that, where housing prices significantly exceed construction costs, zoning constraints “play the dominant role in making housing expensive (from the abstract)” relative to land scarcity.

\(^{21}\) It must be that \( (MV - p_i)h_i \leq 0. \)
constrained, then those with income just below the minimum-income type in community i+1 would be unconstrained in both communities and would share the preference for community i+1 with those residing there; also a contradiction.  

To characterize further stratified equilibria, consider the following lemma:

**Lemma 1:** Let $J_y$ denote the set of communities over which income-type $y$ would be unconstrained, i.e., $h_d(y,p_i) \geq h_{mi}$ for $i \in J_y$. In NH-stratified equilibrium, households $y$ and $y'$ who are unconstrained in equilibrium and both reside in $J_y \cap J_{y'}$ must reside in the same community.

**Proof of Lemma 1:** Utility of households $y$ and $y'$ is given by the lower line of (10) in $J_y \cap J_{y'}$. If either type strictly prefers his equilibrium choice, then the other type cannot make a different choice in this set of communities. If one type is indifferent between his equilibrium choice and another community in this set, then the other type would share this indifference, which contradicts non-homogeneity.

Using Lemma 1, we then can show the following property of stratified equilibrium:

**Proposition 5:** In NH-stratified equilibrium, households that reside in community $i$ and are unconstrained would be constrained in community $j > i$.

**Proof of Proposition 5:** Given income stratification, if everyone residing in community $j$ is constrained, then everyone residing in community $i$ would be constrained in $j$ too. Suppose, then, that some residents of $j$ are unconstrained. Since they have higher income then those residing in community $i$ and some are unconstrained in community $i$, then the

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22 It is shown below (Proposition 6) that in fact some residents of community $i+1$ must be constrained.
residents of j would be unconstrained in community i. The Proposition then follows from Lemma 1.

Given equilibrium is stratified, it is easy to confirm that a binding zoning constraint must arise without imposing any parameter restrictions.23

*Proposition 6: In NH-stratified equilibrium, every community but possibly the poorest community will have a subset of residents constrained.*

*Proof of Proposition 6:* Suppose that no one is constrained in community 2. Then $h_d(y_{m2},p_2) > h_{m2}$, where $y_{m2}$ is the minimum income household residing in community 2. By continuity, income types in community 1 exist who would be unconstrained in community 2. If they are unconstrained in community 1, then Proposition 5 is contradicted. If these households are constrained in community 1, then they would prefer to live in community 2, a contradiction. The latter holds since, if lower $h_{m1}$ rendered them unconstrained in community 1 then they would share the same preference (for community 2) as those unconstrained there (by Lemma 1), and their equilibrium utility is lower yet in community 1. The same argument establishes the result for communities $i > 2$.

4. **Computational Model and Results**

a. **Model Specification and Positive Results.** To provide further insight into the properties of the model, we turn to a computational model. We adopt the calibration of Calabrese and Epple (2004). Preferences are Cobb-Douglas: $U=g^\beta h^{\alpha} b^{1-\alpha}$ with $\alpha = .37$ and $\beta = .111$. If all goods were chosen privately, these parameters would yield a gross-of-property-tax housing share of 33% and a local-government spending share of 10%. The

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23 Proposition 2 does not require stratification but then employs the (weak!) parameter restrictions noted in footnote 17.
The housing supply function has a constant price elasticity set equal to three.\textsuperscript{24} As in Calabrese and Epple, we consider five communities. One large “central city” has 40\% of the metropolitan land area. Four “suburbs” have an equal share of the remaining land, each then having 15\% of the metropolitan land area. The MA distribution of income is lognormal: \( \ln(y) \sim N(10.52, 0.89) \).

It is useful to begin with characterization of the equilibrium that would emerge if there were a single metropolitan government and zoning is prohibited. This equilibrium is depicted in Figure 1. The upper left panel displays the response of gross- and net-of-tax housing prices to parametric variation of the property tax rate. The upper right panel displays the variation in government spending per capita with variation in the community tax rate. An owner-occupant’s utility depends on \((p,p_h,g)\), since \(p_h\) maps into the equilibrium land price (and we consider out-of-equilibrium values here). As the top panels of Figure 1 illustrate, a choice of the community tax rate determines these three variables. Thus, substituting expressions for \(p\), \(p_h\), and \(g\) into the median voter’s utility function and differentiating, we can find the optimum tax rate.\textsuperscript{25} The plot of utility in the lower panel of Figure 1 reveals that the maximum occurs at a tax rate of .319. The associated equilibrium values are: \( p = 14.68 \), \( p_h = 11.13 \) and \( g = 4,899 \).

The preceding \((t,p,p_h,g)\) tuple is also an equilibrium in our multi-community model when there is no zoning. It is the homogeneous equilibrium with a prohibition on zoning having the same \((t,p,p_h,g)\) values in each community.\textsuperscript{26} Thus, this equilibrium

\textsuperscript{24} This calibration of the supply equation is from Eppe and Romer (1991), who derive the elasticity by assuming a Cobb-Douglas production function for housing with land- and non-land shares of one fourth and three fourths respectively.
\textsuperscript{25} Since the indirect utility function is linear in \(Y\), every voter has the same preference for \(t\).
\textsuperscript{26} In this equilibrium, the income distribution is the same in each community with number of community residents proportional to the community land endowment.
serves as a natural benchmark for comparison to the equilibrium when zoning is permitted.

Equilibrium when the four suburban communities are permitted to zone is presented in Table 1. We assume that the central city cannot impose a zoning constraint so that no homeless population exists. To verify that the median-income household’s ideal point in each community is the collective choice, we compute the ideal points for each voter in the community. Then the vote favoring the median-income household’s ideal point relative to each of these alternative points is calculated. Figure 2 presents the results for community two. The upper left panel presents voters’ most-preferred zoning constraints and anticipated public provision as a function of incomes of community residents. The upper right panel presents the most-preferred tax rates and gross- and net-of-tax housing prices. As income rises, an increasingly stringent zoning constraint and rising government spending are preferred, with little change in the property tax rate or housing prices. The lower panel presents the vote favoring the median against each of the ideal points in the upper panels. As is evident from the graph, the median ideal point defeats the ideal points of other community residents. The same holds in each of the remaining communities. Thus, the conditions for single-candidate equilibrium in the Besley-Coate voting framework are satisfied in each community (see footnote 13); the ideal policy of the median-income voter is a voting equilibrium in each community.

A technical appendix, available on request from the authors, examines existence and uniqueness of equilibrium (given the no-adjustment restriction). We can show existence generally in a simplified model where community residents ignore potential capital gains or losses on land when they vote. Conditions for uniqueness are also developed, though they are difficult to apply theoretically. In the analysis presented in the paper, we have verified existence computationally and examined uniqueness locally (i.e., with local perturbations).
Voters in each of communities two through five choose binding zoning constraints. The fraction constrained by zoning ranges from 65% (community 5) to 84% (community 2); see Table 1. Thus, the pivotal (median-income) voter in each community prefers a zoning constraint that is binding on a majority of the community’s residents. This in turn implies that, in each community, the pivotal voter chooses a policy that constrains his own housing consumption. The restrictiveness of this zoning constraint is evident from the last four rows of Table 1. The amount by which the zoning constraint exceeds the preferred housing consumption of the poorest community resident varies from 38% (community 3) to 49% (community 5).

Our finding that the pivotal voter constrains himself, that zoning is highly restrictive, warrants an intuitive explanation. The appendix analyzes this issue while here we briefly summarize the key points. At his ideal tax rate, consider the incentive of the pivotal voter to increase his over the range where he would not bind his own choice of housing. Any anticipated effects on gross housing price and land price on the voter are largely offsetting since the household has purchased the efficient amount of land and land prices absorb changes in the net price of housing. More precisely, the latter effects are of order $t/(1+t)$. This is weighed against the effect of increasing the zoning constraint on $g$. Increasing zoning drives out poor households, reducing the number of community members, thus increasing $g$. The remaining effect on $g$ works through the effect on the tax base of changes in housing consumption and net housing price. We find that effects on the net housing price and on housing consumption are small. Hence, $g$ rises and dominates the pricing effects on the pivotal voter. Consequently, the incentive is to increase the zoning constraint into the range where the voter constrains himself.
Equilibrium tax rates range from 30% to 33%, and thus differ little from the rate chosen in the benchmark homogeneous equilibrium (32%). However, in the stratified equilibrium with zoning, government expenditures per capita exhibit dramatic variation across communities, ranging from $2,181 to $24,710. These may be compared to the value of $4,899 in the benchmark homogeneous equilibrium without zoning. The proportionate effect on housing prices is comparatively modest. Housing price gross-of-tax in the poorest community is approximately 10% lower than the value with no zoning allowed while the price in the wealthiest is approximately 10% higher than in the no-zoning case. Net-of-tax prices differ from the no-zoning price by comparable proportions.

b. Normative Effects of Zoning. Figure 3 presents the distributional effects of equilibrium with zoning compared to the no-zoning equilibrium. The upper panel presents the welfare change, measured by the equivalent variation of introducing zoning, as a function of income; the lower panel plots these welfare changes as a proportion of household income. The poorest 77% of households are made worse off by zoning. Households with incomes below approximately $45,000 experience a loss equivalent to approximately 5% of their incomes. These households all reside in the central city in the presence of zoning. As income rises, the welfare loss as a percentage of income declines. Higher income households experience gains relative to income that rise as income rises. While the majority of households lose, the overall welfare gain is positive. The per capita welfare gain (Equivalent Variation) is $1,617 and the per capita increase in economic rent to (absentee) land owners is $441.

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28 We use equivalent variation simply because it is easier to calculate the effects of income changes on consumption in the no-zoning homogeneous equilibrium. Clearly, very similar results would hold using compensating variation.
One might surmise that the aggregate welfare gains to households from zoning emerge from the familiar Tiebout mechanism—decentralization leads to closer matching of government provision levels with household preferences. While this is the essential explanation, it bears emphasis that zoning as a mechanism to induce stratification and thus this matching is fairly efficient. To clarify this point, consider an $\varepsilon$-perturbation to the household utility function: $U^\varepsilon \equiv g^{1.11}(h^{3.7}b^{6.3}\varepsilon), \varepsilon \geq 0$; and equilibrium assuming zoning is prohibited. With positive $\varepsilon$, no matter how small, stratified equilibrium arises without zoning. The second column of Table 2 presents properties of the stratified equilibrium that results with prohibition on zoning for $\varepsilon$ vanishingly small, with the first column of Table 2 simply representing the values in Table 1 for comparison. (Ignore the third column for now.) Since $\varepsilon$ is vanishingly small, the equilibrium values in the second column are comparable to those with zoning in the first column. While Tiebout matching arises in the stratified no-zoning equilibrium, the per capita equivalent variation of allowing zoning beginning in the stratified no-zoning equilibrium is $2,107$ (and per capita land rents are $532$ lower in the no-zoning equilibrium). Hence, the aggregate welfare gain from zoning is significantly higher with the no-zoning stratified equilibrium as the benchmark as compared to the (non-stratified) homogeneous equilibrium.

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29 Our focus here on the aggregate welfare gains is with the caveats that the distributional effects are large and also ignore potential negative externalities from low achievement in poorer communities when, for example, $g$ is public educational expenditure.

30 The “single crossing condition” for stratified equilibrium is satisfied. Voting preferences over $(t,g)$, or equivalently $(p,g)$, have the property that willingness to bear a tax (or housing price) increase for increased $g$ rises with income. Hence, higher $t$ and $p$ in richer communities can and does keep out relatively poorer households in stratified equilibrium. For examples of such equilibria, see Epple and Romer (1991) and Fernandez and Rogerson (1998). Note, too, that this single-crossing property guarantees existence of voting equilibrium.

31 This means, of course, that aggregate welfare declines in going from the homogeneous equilibrium to the no-zoning stratified equilibrium. We explore the generality of this in Calabrese, Epple, and Romano (in progress).
The Tiebout-matching gains in the stratified no-zoning equilibrium are offset by large inefficiencies in the housing markets. Here housing prices act as the only screening device, with large deadweight losses. Zoning itself acts as a screening device when it is also chosen, permitting stratification and Tiebout matching with substantially lower distortions in the housing markets. Observe in Table 2 that the property tax rates in the stratified no-zoning equilibrium are markedly higher than those in the zoning equilibrium (with one exception), implying larger distortions. Moreover, the zoning constraint itself works to offset the housing market distortion at least for some households, although the welfare gains from this in the housing market accrue to the (absentee) land owners. Hence, zoning permits the realization of Tiebout matching gains without excessive distortions.

C. Comparison to Head Taxation. Motivated by Hamilton’s (1975) analysis, we next turn to investigation of the extent to which zoning is a substitute for head taxation. In the third column of Table 2, we present the equilibrium without zoning but with head taxes and property taxes. When head taxes are permitted, the suburbs rely almost exclusively on those taxes, setting negligible property tax rates. The city continues to rely on property taxes since head taxes are not permitted there (to allow the poorest population some consumption). The equilibrium allocations of population across jurisdictions, government spending levels, and net-of-tax housing prices are strikingly similar in the zoning and

---

32 Public goods provision is lower in each community in the stratified no-zoning equilibrium in spite of the higher tax rates in part because there is “less” stratification: The populations in the suburbs are higher and the mean incomes are lower in this equilibrium relative to the zoning equilibrium. This further implies that more households are subject to the highly distorted housing prices. The next point in the text provides the other reason that per capita government expenditures are lower in the suburbs than with zoning.

33 This model is the analogue of our zoning model but with head taxes, in particular with simultaneous voting over the two taxes in the second stage. Calabrese and Epple (2004) provide a detailed analysis of this alternative and show that the Plott conditions for voting equilibrium are satisfied. Column (3) of Table 2 is reproduced from their paper.
head-tax equilibria. Gross-of-tax housing prices are quite different, however. Property
taxes drive a wedge between gross- and net-of-tax prices, as is evident in column (1).
With head taxes, there is little difference between gross- and net-of-tax prices in the
suburbs. While the property tax drives a wedge between gross and net housing prices,
zoning restricts ability of households to substitute away from housing as we have
discussed. The bottom rows of Table 2 present housing units consumed by the lowest
income resident of each community in the two equilibria. We see that housing
consumption by those households in the presence of zoning exceeds housing
consumption under head taxation.

The equivalent variation of going from the zoning to head-tax equilibrium reveals
that, on average, households are slightly better off ($85 per capita) in the head-tax
equilibrium. Land rents are slightly lower under the zoning equilibrium ($47 per capita).
Thus, the aggregate welfare difference between the two is quite small. This supports
Hamilton's conjecture that zoning in the presence of property taxation achieves
approximately the same efficiency gains as head taxation.

5. Conclusion

We have seen that zoning must characterize equilibrium and that it may induce
stratification. Perhaps the most striking positive finding of our analysis is the stringency
of zoning restrictions that emerge in such an equilibrium. As we noted in the
Introduction, our framing of zoning policy choice requires voters to abide by the zoning
constraint that is adopted; voters cannot impose a policy on future arrivals while
exempting themselves. Despite this, pivotal voters in all communities choose zoning
levels that constrain their own housing consumption and that of a large majority of
residents of their communities. Thus, our results provide evidence of the powerful incentives for use of fiscal zoning. From the perspective of empirical implications, this finding is also the most distinctive prediction of our model. There should substantial bunching of housing consumption on the lower end within suburban communities and stratification of housing consumption across communities.

The normative result that bears emphasis is the relative efficiency of zoning as a mechanism to realize Tiebout-matching gains. If housing price alone supports socioeconomic sorting among jurisdictions, we find Tiebout-matching gains are offset by large housing-market distortions. Zoning permits aggregate gains and is remarkably similar to head taxation as Hamilton (1975) claimed. We should note in this connection that our framing of the problem, requiring voters to abide by the constraint that is adopted, may yield more favorable efficiency effects that would emerge if voters could choose zoning to bind future residents without binding themselves. An interesting puzzle is why zoning is commonplace while local head taxation is almost always illegal. One should keep in mind that the welfare gains discussed here are anything but evenly distributed. While we believe our model captures many key features of zoning, it is nonetheless a highly simplified representation of the reality of metropolitan housing markets. We know, for example, that the stark prediction of income stratification is at odds with the heterogeneity of populations observed within communities (Epple and Sieg, 1999; Ioannides and Hardman, 1997). Thus, generalizing the model to incorporate heterogeneity of preferences as well as income is an important

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34 When choice of zoning is not disciplined by this constraint, voters may choose more restrictive zoning, leading to the types of adverse effects on housing prices found by Glaeser and Gyuourko (2002).
task. A major challenge posed by such a generalization is extending the characterization of voting equilibrium to a setting with more than one source of household heterogeneity, since single-candidate equilibrium may not exist for such a generalization.

Another avenue for extending the model is to incorporate intergovernmental grant policy. In many states, courts have intervened to mandate some degree of equalization of educational spending across localities (Evans, Murray, and Schwab, 1998). We would expect such equalization policies to reduce the incentives for fiscal zoning. At the same time, however, we would not expect even full equalization to eliminate use of zoning. If peer effects are important in education, incentives to use zoning as a device to limit access to communities will remain even if fiscal incentives are eliminated (see Epple and Romano, 2003 and Calabrese, Epple, and Romano, 2006). Incorporating peer influences is thus another promising avenue for extending the model.

Of course, metropolitan areas and communities develop over time. Capturing the dynamics of such development in a tractable model incorporating zoning is a particularly challenging problem on the research frontier of urban public economics.
Appendix. Proof of Proposition 2: To form a contradiction, suppose there is an equilibrium with two communities, indexed $j = 1, 2$, that do not zone. Let the equilibrium values of the variables in $j$ be $(g_j, p_j, t_j)$. Let the equilibrium population of $j$ be $n_j$, the income density $f_j(y)$, the CDF $F_j(y)$, its complement $\hat{F}_j(y) \equiv 1 - F_j(y)$, and let

$$\hat{G}_j(y) \equiv \int_y^\infty x f_j(x) dx.$$ 

Denote this equilibrium $E_0$. Recall that the housing demand function is given by $h_\delta(p, y) = q(p)y$.

Consider a proposal within one community to adopt zoning. If the lower support of $f_j(y)$ differs between the two communities, let the community adopting zoning have the lower of the lower supports. Otherwise, arbitrarily choose one of the communities. Designate the community adopting zoning to be community $1, C_1$. Let the proposal be a tax rate $t$ and zoning constraint $h_m$ such that, with that tax rate and zoning constraint, the pair $(g_1, p_1)$ is the same as with $E_0$ (which we show exists by construction).

Let $A(t) \equiv q(p_1) \left[ 1 + \left( \frac{P_1}{1 + t} + \frac{P_1}{1 + t_1} \right) q(p_1) \right]$. Note that $A(t_1) = q(p_1)$. Let the minimum incomes who reside in $C_1$ after imposition of the new proposal with zoning, originally residing in communities $1$ and $2$ respectively, be: $y_{b1}(t, h_m) = h_m/A(t)$ and $y_{b2}(h_m) = h_m/A(t_1)$. We assume for now that all initial residents of the two communities who would be unconstrained in $C_1$ after imposition of zoning choose to live in $C_1$, though they are indifferent. A perturbation of the policy would make them strictly prefer $C_1$ as discussed further below (see Remark 2).
Then housing market equilibrium clearing in $C_1$ requires that $t$ and $h_m$ satisfy:

$$
\left[ n_1 \hat{G}_1(y_{b1}(t, h_m)) + \int_{y_{b1}}^\infty xq(p_1)(\frac{P_1}{1+t} - \frac{P}{1+t})f_1(x)dx \right] + n_2 \hat{G}_2(y_{b2}(h_m)) \right]q(p_1) = H^t_1(\frac{P_1}{1+t}) .
$$

(A1)

Assume that, for all $t \in [0, t_1]$, the left-hand-side is greater than the right-hand-side when $h_m = 0$. That is, for all $t \in [0, t_1]$, the housing demands of the combined populations of the two communities would exceed housing supply in $C_1$ at price $p_1$. (The latter is the assumption noted in the footnote to Proposition 2.) For given $t$, the left-hand-side of (A1) declines monotonically in $h_m$, approaching zero as $h_m \to \infty$. Thus, for each $t \in [0, t_1]$, there is a unique $h_m$ satisfying (A1). With a slight abuse of notation, let $h_m(t)$ be the zoning constraint as a function of $t$ that solves (A1).

The population of $C_1$ as a function of $t$ is:

$$
n(t) = [n_1 \hat{F}_1(y_{b1}(t, h_m(t))) + n_2 \hat{F}_2(y_{b2}(h_m(t)))].
$$

(A2)

Three properties of $n(t)$ are used below: $n(0) > 0$, $n(t_1) < n_1$, and $n(t)$ is continuous for $t \in [0, t_1]$. To see that $n(t_1) < n_1$, note that housing supply at $t = t_1$ is the same as in equilibrium $E_0$. In addition, $y_{b1}(t_1, h_m(t_1)) = y_{b2}(h_m(t_1)) = h_m(t_1)/q(p_1)$. Aggregate income in $C_1$ when $t = t_1$ is the same as in $E_0$ by (A1). However, since average income has risen in $C_1$, the population of $C_1$ must be smaller than in $E_0$.

Budget balance in $C_1$ requires:

$$
\frac{t \frac{P_1}{1+t} H^t_1(\frac{P_1}{1+t})}{g_1} = n(t)
$$

(A3)
The left-hand-side of (A3) is increasing in $t$ for $t \in [0, t_1]$ (i.e., $t_1$ could not be on the “wrong” side of the Laffer curve if $E_0$ is an equilibrium). When $t = t_1$, the left-hand-side equals $n_1$. This, $n(0) > 0$, $n(t_1) < n_1$, and continuity of $n(t)$ imply that there is a solution to (A3) and any solution satisfies $t < t_1$. Let a solution be denoted $t^*$ and the associated zoning constraint $h_m = h_m(t^*)$. Thus, $p_h$ rises, and it then follows from equation (3) that the price of land rises. All voters (i.e., all original residents of $C_1$) prefer this proposal to $E_0$. All obtain a capital gain if this proposal is adopted while having access to a $(g,p)$ pair that is as attractive as that in $E_0$.

Our equilibrium concept requires that any proposal be the ideal point of some voter. Given that the proposal with zoning is unanimously preferred by residents of $C_1$, the initial allocation could not be any voter’s ideal point. Hence, the initial allocation cannot be an equilibrium.

Remarks on the Proof of Proposition 2: 1. The argument above makes no assumption about how the population is distributed between the two communities under $E_0$. The populations can exhibit arbitrary income mixing or complete income stratification. If the latter, the proof presumes the poor community adopts zoning. Residents of the poor community unanimously adopt a zoning constraint that requires most of them to leave—and yields anticipated capital gains for all of them.

2. As noted in the Proof, those that reside in Community 1 after imposition of zoning are actually indifferent to residing there (while those that reside in Community 2 have a strict preference due to the zoning constraint). Any perturbation of $h_m$ from $h_m(t^*)$ that increases housing demand would permit an increase in $g_1$ with the same $p_1$, while preserving the capital gain to initial residents of Community 1. Those that choose
Community 1 would then have a strict preference for it, while preserving the unanimity of preference for the policy with zoning.

Proof of Proposition 3b: In equilibrium, no capital gains arise and residential choice maximizes \( V_i \) over \( i \in \{1, 2, ..., J\} \) with \( Y = y \) (refer to Equation (10)). Consider any two communities, \( i \neq j \). If \( v(g_j)w(p_j) = v(g_i)w(p_i) \), then either equilibrium is homogeneous or everyone prefers one community (i.e., that with substantially lower \( h_m \)).

Suppose, then, that \( v(g_j)w(p_j) > v(g_i)w(p_i) \). It is straightforward to show that \( v(g) = g^\beta \) and \( w(p) = (1 - \alpha)^\alpha (\alpha/p)^\alpha \) for Cobb-Douglas utility function. Taking community \( i \) as the example, we have:

\[
\frac{dV_i}{dy} = \begin{cases} 
  g_i^\beta h_m^\alpha (1 - \alpha)(y - p_i h_m)^{-\alpha} & \text{if } y \leq Y_{mi} \\
  g_i^\beta (1 - \alpha)^{-\alpha} (\alpha/p)^\alpha & \text{if } y \geq Y_{mi} 
\end{cases}
\]

(A4)

Refer to Figure A-1 that shows \( V_j \) and \( V_i \) as functions of \( y \). Stratification is implied if \( V_j \) and \( V_i \) cross at most once, which we show. In the Cobb-Douglas case, \( h_d = \alpha y/p \), implying \( Y_{mi} = p_i h_m/\alpha \). There are two cases, the first with \( p_j h_{mj} \geq p_i h_{mi} \). This case has \( Y_{mj} \geq Y_{mi} \) and is depicted in Figure A-1. It is clear by inspection of Figure A-1 that double crossing of \( V_i \) and \( V_j \) would require that \( V_j \) is flatter at one crossing where \( y < Y_{mi} \). (The latter is not shown in the Figure as it cannot occur.) But the upper line of the second equality in (A4) precludes this, since \( v(g_j)w(p_j) > v(g_i)w(p_i) \) and \( p_j h_{mj} \geq p_i h_{mi} \).

The second case has \( p_j h_{mj} < p_i h_{mi} \) and thus \( Y_{mj} < Y_{mi} \). It is easy to see in a figure like Figure A-1 (not drawn) that a double crossing could only occur at values \( y < Y_{mj} \).
where households \( y \) would be constrained in both communities. But \( V_i = V_j \), given here by 
\[ g_i h_{m_i}^a (y - p_i h_{m_i})^{-a} = g_j h_{m_j}^a (y - p_j h_{m_j})^{-a}, \]
has at most one positive solution for \( y \). \( \square \)

Properties of Voting Preferences. Here we provide conditions such that a voter prefers a zoning constraint that is self-binding. Simplifying notation, write the conditions describing the equilibrium a voter anticipates when voting as:

\[
n(p, g, h_m)g = \frac{tp}{(1 + t)} H_s(p/(1 + t)). \tag{A5}
\]

and

\[
H_{d}(p, g, h_m) = H_s(p/(1 + t)), \tag{A6}
\]

Conditions (A5) and (A6) are, respectively, (14) and (15) where we have: (a) substituted in (16); (b) suppressed the ‘\( j \)’ subscripts and ‘\( a \)’ superscripts (reintroducing them as needed below for clarification); (c) let \( H_{d} \) denote demand housing demand and \( n \) denote the number of community residents; and (d) suppressed that these conditions take as given the equilibrium values in other communities.\(^{35}\) Recall that these conditions determine anticipated \( p \) and \( g \) as functions of the policy pair \((t, h_m)\) that is voted on.

Equilibrium must be the ideal point of the pivotal voter in the community. It must be the point that maximizes indirect utility in (10) over \((t, h_m)\) subject to (A5) and (A6), with

\[ Y = y + (p^a (p^a / (1 + t)) - p^a \ell), \]

where \( \ell \) has been chosen in the first stage to equal the household’s efficient quantity of land. Let \( V(t, h_m) \) denote the pivotal voter’s indirect utility function, with the constraints (A5) and (A6) substituted in, and let \( t^* \) denote the voter’s ideal tax rate. Consider \( V_{h_m}(t^*, h_m) \) for \( h_m < h_d(Y, p) \), i.e., the partial derivative of the voter’s indirect utility with respect to the minimal housing over the range of \( h_m \) such that the voter would not constrain his own housing choice. Over this range,

\[^{35}\text{We have also substituted out } p^a_j (p^h) \text{ using } p^h = p^a / (1 + t).\]
\[ V = v(g^a(h_m, t^*))u(h_d(\cdot), y + (p_f^{fa}(p^a(h_m, t^*)/(1 + t^*)) - p^f)\ell - p^a(h_m, t^*)h_d(\cdot)), \]  
(A7)

where \( h_d \) is evaluated at \((Y, p) = (y + (p_f^{fa}(p^a(h_m, t^*)/(1 + t^*)) - p^f)\ell, p^a(h_m, t^*)).\) Using the Envelope Theorem:

\[ V_{h_m} = v' u \frac{\partial g^a}{\partial h_m} + v u \left( \frac{dp^a}{dp} - \frac{1}{1 + t^*} \ell - h_d \right) \frac{\partial p^a}{\partial h_m} = v' u \frac{\partial g^a}{\partial h_m} - v u \frac{th_d}{1 + t} \frac{\partial p^a}{\partial h_m}. \]  
(A8)

The second equality follows using that competitive provision of housing with constant returns implies \( dp^f/dp = h/\ell \) with efficient housing production and the initial purchase of land is efficient. We have:

**Proposition 1A:** If \( V(t^*, h_m) \) is single-peaked in \( h_m \) and \( V_{h_m}(t^*, h_d(\cdot)piv,j) > 0 \), then the pivotal voter chooses a zoning constraint that is self binding (where \( y_{piv,j} \) is the pivotal voter’s income).\(^{36}\)

The interpretation of Proposition 1A is straightforward. To assess the likelihood that \( V \) is increasing in \( h_m \) in this range, refer to (A8). Voting for higher zoning constraint has no direct effect on utility in the range where it will not bind the voter. The second term on the right-hand side of (A8) combines the effect of a change in anticipated housing price on the voter’s cost of purchasing housing and on the capital gain/loss on the initial land purchase. Land prices absorb changes in the net price of housing. Because the initial purchase of land equals the voter’s ultimate purchase, the capital gain/loss effect of the anticipated change in housing price is offset by the cost-of-purchase effect but for the fact that the net housing price is below the gross price of housing. An increase in the gross price of housing is bad for the consumer because the

\(^{36}\) It is straightforward to show that the indirect utility function is differentiable at \( h_m = h_d(y, p) \), implying that the pivotal voter strictly binds himself in case (ii).
land price rises more slowly as does the net price of housing. But this combined effect is then of order \( t/(1+t) \). We find for reasonable parameterizations that whether \( V \) rises with \( h_m \) (while not self-constraining) is driven by the effect on \( g \), captured by the first term on the right-hand side of (A8).

Whether \( g \) can be expected to increase with \( h_m \) depends on several effects. One result concerning this is:

**Proposition 2A:** If anticipated equilibrium housing demand rises with \( h_m \), then so too will \( g \) increase.

Proof of Proposition 2A: To form a contradiction, suppose that housing demand increases with \( h_m \) and \( g \) does not. By (A6), the gross price of housing must increase. Given an increase in \( h_m \), an increase in housing price, and no increase in \( g \), then \( N \) cannot increase. Then \( g \) must increase by (A5), a contradiction. \( \square \)

One can, of course, write out the total derivative of housing demand with respect to \( h_m \). This is sufficiently complicated that it is not particularly enlightening. Understanding, however, that an increase in housing demand with increased zoning restriction is sufficient for voter support of the latter provides intuition. One can also compute the derivative of \( g^a \) with respect to \( h_m \) from (A5) and (A6):

\[
\frac{\partial g^a}{\partial h_m} = \frac{[g(\frac{1}{1+t}H_s - \frac{\partial H_d}{\partial p} \frac{\partial N}{\partial h_m}) - \frac{t}{1+t} (H_s + \frac{p}{1+t}H_s) \frac{\partial H_d}{\partial h_m}]}{\frac{t}{1+t} \frac{\partial H_d}{\partial g} + \left( \frac{\partial H_d}{\partial p} - \frac{1}{1+t}H_s \right) \frac{\partial N}{\partial g}}.
\]  

(A9)

While complicated, one can see from (A9) that if the effects on housing demand are all small (i.e., suppose that \( \frac{\partial H_d}{\partial x} = 0 \) for all \( x \)), then \( \frac{\partial g^a}{\partial h_m} > 0 \), due to increased \( h_m \)
causing an exodus of relatively poor types from the community. The effects on demand and thus housing price complicate the total effect on $g$, but we know from Proposition 2A that if the net effect on demand is an increase, that $g$ must rise.

\[ \frac{\partial N}{\partial h_m} \]

37 Note that $\frac{\partial N}{\partial h_m}$ cannot be positive and will be negative if there are multiple communities.
Bibliography


________________ , “Tiebout and Inefficiency,” work in progress.


### Table 1

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Figure 1
Equilibrium with a Single Metropolitan Jurisdiction

- Gross-of-Tax Housing Price
- Net-of-Tax Housing Price

- Government Expenditure Per Capita

- Utility of Median-Income Voter

- Derivative of \( V(g(t), p(t), \hat{p}(t)) \) with Respect to \( t \)
Figure 2

Ideal Points as Function of Income and Vote Favoring Median Ideal Point

Vote Favoring Median Policy against Ideal Points of Other Community Residents

Government Spending
Zoning

Gross-of-Tax Price
Net-of-Tax Price
Tax Rate
Figure 3

Distributional Effects of Zoning
Compared to Benchmark Homogeneous Equilibrium
Figure A-1