THE OPPOSING FORCES DIFFUSION MODEL: THE INITIATION AND 
REPRESSION OF COLLECTIVE VIOLENCE

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Abstract

This paper re-evaluates an important deterministic model of collective violence diffusion and demonstrates a series of shortcomings in it. In response, a new model, the Opposing Forces Diffusion Model, is introduced. The Opposing Forces model treats observed event cycles as the result of two underlying logistic diffusion processes, one for provocation of events and one for repression. The result is a model that is more flexible, more straightforwardly interpreted, and considerably more accurate than its predecessors. Furthermore, because the new model treats provocation and repression as two distinct processes, they can be disentangled and subjected to lower-level scrutiny.
THE OPPOSING FORCES DIFFUSION MODEL: THE INITIATION AND REPRESSION OF COLLECTIVE VIOLENCE

It has long been recognized that social turmoil comes in clusters or waves. There are “rebellious centuries” (Tilly, Tilly, and Tilly 1975) and quieter ones. There are waves of race riots within countries, global wave of protest movements, and so forth. Although some of this clustering is due to common external circumstances, particularly economic cycles (Frank and Fuentes 1994), there is substantial evidence that “common causes” are insufficient explanators of these waves. In addition, waves of collective action are shaped by endogenous processes in which actions affect other actions (Oliver 1989) or, in standard diffusion language, the occurrence of an event changes the likelihood that a similar event will happen in the future (Rogers 1995, Strang 1991). Protests or riots in one locale serve as models for potential rioters and protesters in other locales (Myers 1996; 2000). Tactical innovations such as sit-ins spread from one city to another through mechanisms like telephone calls (Morris 1984) and energized successive waves of civil rights protest (McAdam 1983). Tactics spread through learning processes: successful tactics are emulated, while unsuccessful ones are avoided. Actions affect the prospects for subsequent action in a variety of other ways, as well, including altering the responsiveness of elites, creating new social bonds, or disrupting social order and social control mechanisms (Oliver 1989).

Although scholars of protest almost universally acknowledge the fact of the diffusion of collective action, there has been very little penetration of formal diffusion theory into theorizing or research about social movements and collective violence. Most substantive discussions remain at the qualitative descriptive level, noting the spread of particular forms through time and
space or the connections between events, for example telephone calls spreading the word about sit-ins (Morris 1984), without linking these descriptions to formal concepts of diffusion.

One reason for this lack is the continuing reaction against Le Bonian (1895) “contagion theory” and the connotations of irrationality in pre-1970s collective behavior theory in which the spread of collective behavior was likened to the spread of a disease. But, of course, reasonable choices by people can cause behaviors to diffuse in mathematical patterns that are analytically similar to disease contagion patterns, without in any way implying that the diffusing behavior is a disease. People do “imitate,” but this does not mean that their imitation is mindless or irrational. New communication technologies like the telephone and e-mail spread as a function of their utility and the number of others who had previously adopted the mode of communication. Sit-ins and other protest tactics spread because they were producing successes in breaking down segregation (McAdam 1983).

Another reason for the failure of diffusion theory to impact social movements theory has been the restrictions of existing diffusion models that prevent reasonable application to collective action. In particular, they have been inappropriate for behaviors that come and go, rather than the "once and forever" adoptions of innovations that have been the traditional province of diffusion theory (Rogers 1995; Majahan and Peterson 1985). However, this weakness is being redressed in recent work that is generating new ways to control for or measure diffusion processes in empirical studies of waves of protest, collective violence, and the founding of movement organizations (e.g., Hedstrom 1994; Hedstrom et al. 2000, Olzak 1992, Soule 1997; Myers 1996; 2000; Strang and Soule 1998).

In the context of this resurgence and redevelopment of diffusion models for the empirical analysis of collective events, it is important to develop adequate theoretical models for diffusion
processes in collective action. This paper develops an Opposing Forces Diffusion (OFD) model of collective action, in which the trajectory of action is shaped by the net effect of two diffusion processes, a provocative force which tends to spread action, and a repressive force which tends to curtail it. This model is derived from reasonable theoretical premises and assumptions, fits empirical data well, yields parameters which are substantively meaningful and exhibit plausible patterns, and provides a sound basis for future theoretical refinement and elaboration.

We begin our work by revisiting the last major model of for the diffusion of collective violence, that of Pitcher, Hamblin, and Miller (1978), (hereafter "PHM"). We do this for two reasons. First, the PHM model is a well-known formal diffusion model specifically created for collective violence and it has not been superceded since it was published (see Bohstedt 1994). Despite its prominence and the seemingly near-perfect fit between the PHM model and various sets of data on collective violence, however, it has stood unused and largely unexamined for twenty years. Second, in reviewing the PHM model, we are able to identify issues that must be addressed in creating better models of collective violence. We show that PHM’s emphasis on the dual processes of initiation and inhibition is essential for understanding the diffusion of collective action as a result of interplay between protest and repression. However, a more careful examination of the fit of the PHM model to empirical data and the substantive interpretation of the model’s parameters reveals substantial weaknesses. The first part of this paper reengages the fundamental theoretical premises and empirical fit of the PHM model and the work that preceded and inspired it. In the process, we show why simple goodness of fit is inadequate for evaluating models of this type. The second part of this paper develops an alternative, the opposing forces diffusion (OFD) model, which captures the same theoretical
insights that motivated the PHM model, but fits the data much better and generates model parameters which have straightforward substantive meaning.

THE PITCHER, HAMBLIN, AND MILLER DIFFUSION MODEL

Original Derivation and Test
The core notion in diffusion of innovations work is that adopters, by their own decisions to adopt, exert an influence on those who have not yet adopted. This simple seed has created a massive body of research over the last century (see reviews by Mahajan and Peterson 1985; Rogers 1995). The earliest efforts in this regard simply documented that adoption acts tend to accumulate in sigmoid or S-shaped curves (Tarde 1903). Subsequent work attempted to produce mathematical models accurately reflecting these curves, including Pemberton’s (1936) fit of the integrated normal curve to the adoption of the postage stamp, Griliches’ (1957) fit of the logistic model to the diffusion of hybrid corn, and Dodd’s (1953; 1955) information-trading experiments used to establish the theoretical basis of the logistic diffusion model. In general, this line of work led to a conclusion that a logistic model was the best empirical and theoretical fit to processes of innovation diffusion. In the logistic model, the change in the rate of adoption is the product of the number who have already adopted, the number who have not adopted, and a scalar imitation index.

Hamblin, Miller and colleagues developed one subset of this work. Their 1973 A Mathematical Theory of Social Change developed a wide variety of models for the diffusion of various kinds of ideas or behaviors, all built upon either exponential (power) or logistic

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2 Diffusion models which focus on external engines of influence rather than inter-actor influence also exist, but because the current paper is focused on diffusion as a process of influence within a population, these models are not discussed. Exemplars and discussions of external diffusion models and mixed models (incorporating external and internal influences) may be found in Coleman, Katz, and Menzel (1966), Mahajan and Peterson (1985), Bass (1969), and Lawton and Lawton (1979).
distributions. This work builds on Bandura’s (1973, 1977) social learning theory. It assumes that individuals learn vicariously by observing the behavior of others and then judge what worked and what did not. In one chapter, Hamblin et al. used logistic models to analyze airline hijackings and political violence in Latin America as innovations. Their discussion noted that these models failed to include terms for the counter-actions of the authorities that are also diffusing as innovations, and speculated that two synchronous logistic processes resolve to a power function.

These initial ideas were developed and elaborated in Pitcher, Hamblin, and Miller’s (1978) “The Diffusion of Collective Violence.” PHM began with an empirical critique of logistic models of collective violence diffusion because the logistic is necessarily symmetric while empirical collective violence waves tend to be asymmetric. They also identified two fundamental theoretical problems. First, disruptive forms of action never evolve simply of their own logic, but are always countered by forces of social control. Second, the logistic model assumes “once and forever” adoption, while actors in collective violence waves often act multiple times.

Building on their past work, PHM assumed that individuals gather information about past collective violence via the mass media. A learning process produces both imitation and inhibition effects as actors evaluate the outcomes of prior actions. Derivation of the PHM model begins by assuming that the change in the number of events (\(dV\)) is a function of the number of prior events (\(V\)) and a scalar factor \(c\) which is interpreted as the net rate of imitation, so that \(dV=\dot{c}V\). However, this net rate of imitation has two components: \(p\), the positive imitation effect from successful actions, and an inhibition effect from failures or repression. Inhibition is specified to operate through increasing \(i\), the number of potential actors who are inhibited from
action, which, in turn is specified to be inversely related to the rate of change in V. The rate of violence over time, then, is:

$$\frac{dV}{dt} = \frac{p}{i} V$$ \hspace{1cm} (1)

This equation says that the rate of change in the adoption of violence is a function of the ratio of the product of number of prior events and the positive imitation rate (i.e. \(pV\)) to the number of actors who have previously become inhibited (\(i\)). Note that the units of \(p\) and \(i\) are different: \(p\) is a rate, while \(i\) is numbers of actors. The rate at which actors become inhibited from violence is a product of a scalar \(q\) and the number previously inhibited:

$$\frac{di}{dt} = qi$$ \hspace{1cm} (2)

Using these two equations, PHM derive a single expression relating the rate of violent events as a function of two parameters, \(c\) and \(q\). Integrating (2), they find \(i = i_0 e^{qt}\), where \(i_0\) is the initial level of \(i\). Substituting for \(i\) in (1) and letting \(c = p/i_0\), yields:

$$\frac{dV}{dt} = ce^{-qt}V$$ \hspace{1cm} (3)

The parameter \(c\), then, is the net rate at which units are instigated to imitate, scaled by the number initially inhibited. Solving for \(V\) by integration produces:

$$V = V_0 e^{(c/q)} e^{(-c/q)e^{-qt}}$$ \hspace{1cm} (4)

where \(V_0\) is the accumulated number of events at \(t = 0\). This equation has the form of a Gompertz growth curve.

PHM fit the model in equation (4) to twenty-five "collective violence" data sets using non-linear least squares. In each case, the parameters estimated by the fit procedure were \(c\), interpreted as the net rate of imitation instigation, and \(q\), the inhibition rate. Because PHM’s goal
was to develop a general model, they used a very broad definition of collective violence that encompassed individuals and large groups, a wide range of organization and spontaneity, and differing levels of violence. Despite this expanse of data, they reported what appeared to be an extraordinarily close fit between their model and the empirical data. The median $r^2$ between the model and data was .995 with only one data set (1967 civil disorders in the U.S.) producing an $r^2$ of less than .98.  

**Merits of the Model**

The PHM model represented several important advances in the modeling of collective violence diffusion, each of which should be carried forward into any model intended to supersede it. First, PHM propose that imitation processes underlying diffusion are based in rational decision-making rather than in unconscious or irrational processes. While they were not the initial pioneers of this view, their emphasis on the rational bears reiteration because of the unfortunate history of the contagion notions of crowd and collective violence scholarship (see McPhail 1991 for a detailed account).

Secondly, PHM recognize the significance of a fundamental difference between the diffusion of collective action and the diffusion of cultural innovations: actors can exhibit the behavior more than once. PHM therefore shift to a focus on adoption acts instead of adopters as the key units in the diffusion of collective violence. This shift is enormously important, as it is essential to any plausible model of the diffusion collective violence.

Finally, PHM recognize the critical significance of de-mobilizing processes in the trajectory of collective violence. Breaking with prior diffusion models which assumed that only an imitative push underlies the pattern of action, PHM explicitly recognize that forces working to

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3 See Pitcher, Hamblin, and Miller (1978, p. 28, Table 1) for details.
quell the wave of action also contribute to its overall trajectory. Even though we shall argue that the PHM model has insufficiently captured them, our work builds on their central insight that the trajectory of collective violence (and, we will argue, of collective protest more broadly) arises from the conjunction of two forces, one leading to the spread of action, and the other to its inhibition or repression.

**Problems of the Model**

**Model Fit.** The fit statistics PHM report appear to prohibit any criticism of their model. However, the usual expectations for $r^2$ are inappropriate in this and similar contexts. Cumulative event counts for waves are necessarily highly constrained in the shape they can take: if the frequency of an event rises and then falls over time, the cumulative frequency must approximate a standard sigmoid S-shape and therefore even simplistic models can produce very high $r^2$ values. Marquette (1981) demonstrated that even a simple linear model produced $r^2$ values ranging from .85 to .95 for several sigmoid cumulative count curves. Any curve that has roughly the right S-shape can produce an $r^2$ greater than .95. Therefore, any $r^2$ less than .99 is an indication of suspect fit and an $r^2$ of less than .98 usually indicates a very poor fit.\(^4\) Instead of relying solely on $r^2$, comparisons of the predictive value of competing theoretical models need to examine patterns of residuals and the plausibility of a model’s parameters in a variety of circumstances.

To demonstrate the problems with the PHM model (and the comparative advantages of the opposing forces model), we fit the PHM model to a variety of data on collective violence and collective protest around race relations in the US. These include Black victims of lynchings

\(^4\) These high $r^2$ values do not mean, however, that variation in the dependent variable has been lost. Period-to-period variation in event counts is reflected just as accurately in the cumulative event count as in the non-cumulative frequency distribution.
in the US 1882-1955 (McAdam 1982), race-related civil disorders 1964-1971 (Carter 1983, 1986) and civil rights activity in the 1960s (McAdam 1982). The lynching and riot series are different data on the same event types as PHM examine. PHM did not study civil rights activities, but they did examine other varied forms of disruptive collective actions.\(^5\)

Parameter estimates for the PHM model for these data were produced by fitting the model (Equation 4) to each data series using non-linear least squares.\(^6\) The results from these fit procedures are presented in Table 1. Generally, the fit of the PHM model to these data is much worse than was achieved in PHM’s original analysis. The overall fit of the PHM model varies greatly across data sets, varying from a strong fit with \(r^2 = .999\) for lynching to considerably weaker fit for riots with \(r^2 < .99\) for the whole series and a serious lack of fit \(r^2\) for some years and regions. The fit for civil rights activities is generally acceptable, but varies across issues and is weak for some. There appears to be no meaningful pattern with respect to the fit coefficients across the different data sets. We can conclude, however, that the PHM performs as well with non-violent action as it does with violent action and that there does not appear to be any particular reason to limit the application of the model to collective violence alone.

| Table 1 Here |

Asymmetry, Peaks, and Inflection Points. The peak of activity in an action wave translates into the inflection point in a cumulative event count. This is the point at which the acceleration of activity stops and the wave begins to die down. PHM criticize the logistic

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\(^5\) Similar analyses to those reported in this paper have been conducted for a wide variety of other kinds of collective events including funding, organizational, regional activity, and tactical innovation in the Civil Rights Movement (McAdam 1982; 1983), protest activity in Germany (Koopmans 1993), and collective violence in France (Tilly 1978). The additional analyses support the claims made in this paper.

\(^6\) Fitting models like PHM requires care to ensure that convergence of the non-linear model is global instead of local. Details on the procedures used to achieve this aim in the current analysis are available in Myers (1997).
diffusion model because it is symmetric with respect to its inflection point, while most collective violence waves are asymmetric. While Gompertz functions like the PHM model are asymmetric, the asymmetry is fixed—occurring at $1/e$ or approximately .37 of the total number of adoptions. However, collective violence wave do not all peak when 37% of their events have occurred anymore than when 50% of the events occur. In fact, the various empirical series we have examined have a wide variety of inflection points: Some rise rapidly and die down slowly, others rise more slowly and die down rapidly, and still others are fairly symmetric.

Figure 1 provides one example using the McAdam (1982) lynching data and the estimates of the PHM model parameters given in Table 1. This example is particularly instructive because the $r^2$ is quite high, implying good fit, the residuals do not have the systematic problem described below, and the distribution rises more rapidly than it dies down, as a Gompertz assumes. Nevertheless, the inflection points do not match well. Figure 1 plots the cumulative event count and the PHM model fit to it. The inflection point of the empirical distribution occurs at approximately event 850 in 1892, when about 25% of the lynchings had occurred. However, the PHM model predicts that inflection point or peak of the diffusion process should have occurred when 37% of the lynchings had happened, at approximately event 1251. In addition, PHM predicts that the inflection point would have occurred six years later than it actually did. The inflexibility of the PHM model in this respect is one reason for its poor fit to many collective violence waves, and even when the fit statistics appear good (as they do here), the model often gives incorrect information about the timing of the peak of the wave.

**Residuals.** The most important source of fit problems for the PHM model is that it is unable to track the kinds of peaked distributions that are common in collective violence waves, a
pattern that can be seen in its residuals from the cumulative event count data. In many instances of poor fit (and even in some cases where fit is quite good), the PHM model predicts too many events in the early portion of the collective violence cycle, too few in the middle portion, and too many again at the end. Figure 2 illustrates this problem with civil rights actions concerned with Black political power. The first panel of the figure plots the cumulative event count as predicted by the PHM model and the cumulative event count as actually observed; the lower left panel shows the residuals. Translating the cumulative counts into predicted and actual yearly counts in the second column makes the source of the problem clearer: The PHM model cannot rise and fall as rapidly as the empirical wave. This problem arose in three quarters of the event series we examined, and we found no consistent factors that predicted which kinds of events would exhibit the problem. In short, most empirical waves of collective action and collective violence are more peaked than a Gompertz function like the PHM model can track. Substantively, this means that the PHM model is generally wrong about the patterns of inter-actor influence. Compared to the PHM model, actors form a critical mass more slowly at first and then accelerate more rapidly in their influence on each other, and the repressive forces then bring the wave back down more quickly, than PHM predicts.

One explanation for this residual pattern might be that a discontinuity in the process occurred which caused a spike of activity during the usual diffusion process. This was the explanation PHM gave for poor fit of their model to the 1967 riot wave. They speculated that the Newark riot created inordinate press coverage and thereby violated the assumption of continuous derivatives. However, detailed examination refutes this explanation. The empirical and PHM-predicted patterns for 1967 are given in the upper panel of Figure 3. There is a steep increase in
riots in this series, but the Newark riot occurs toward the end of the spike and obviously could not have caused it. Furthermore, the only potential discontinuity in the wave occurred sometime after the Newark riot, and appears as a decelerating force flattening out the curve, rather than an inordinate push.

Further evidence against the discontinuity conjecture emerges when 1967 rioting is compared to 1968 rioting. In 1968, a massive discontinuity (unequivocally the most extreme discontinuity in the entire riot era) occurred when Martin Luther King, Jr. was assassinated in April. The spike in riot activity is extremely clear in the cumulative riot count plot in the lower panel of Figure 3, and it is followed by an abrupt decline in the rate of rioting. Despite the magnitude of this spike, the PHM model fits the 1968 data far better than the 1967 data ($r^2$ for 1968 = .986, $r^2$ for 1967 = .938). Examining Figure 3 closely indicates that the poor fit for 1967 is not due, then, to the “discontinuity” produced by Newark, but rather to the general inability of the model to track highly peaked distributions.

Finally, the pervasiveness of the particular residual pattern across so many different types of events and across many different time periods is further evidence against ad hoc hypotheses of discontinuity in the series. We conclude that the inability of the PHM model to track highly peaked waves is endemic to the model and a serious challenge to its empirical validity.

The Estimated Parameters. For theorizing, it is not enough to fit a curve to a distribution. In fact, if fit were the only criterion, simply increasing the order of the polynomial would

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7 The discontinuity introduced by the King assassination is so extreme that applying a sigmoid diffusion model is likely an inappropriate tactic altogether. One possible adjustment is to simply drop the King riots from the data and assume that they were a complete aberration—neither a product of prior riots or a contributor to future ones. The assumption is arguable, but does, of course, allow for improved fit of the model to the data. The results of this procedure are reported in Table 1 in the model labeled “1968b.”
produce better and better fit to any distribution. Theorizing is advanced only if the model’s parameters are grounded in theory and have substantive meaning. While the PHM model's parameters are interpretable in a general sense, the substantive meaning of each is imprecise and difficult to disentangle. The model has two parameters for the diffusion process: $c$, “the net rate at which units are instigated to imitate,” and $q$, “the rate at which they [units] are inhibited” (p.26). Higher values of $c$ increase the adoption rate and cause events to accumulate more quickly, while higher values of $q$ increase the rate at which inhibition grows and thus slows the overall process, thus reducing the net rate of increase, the cumulative count, and the overall upper limit of the process. The third parameter, $V_0$, is interpretable as the number of actions that have accumulated at time zero: it controls the “size” of the wave as a multiplier so that, for example, an increase in $V_0$ from 1 to 2 doubles the maximum number of actions produced in the wave. For a given $V_0$, the predicted upper limit is determined by the ratio $c/q$, while the peakedness of the distribution is determined by the overall magnitude of $c$ and $q$, with higher values leading to a process that terminates more quickly. However, these terms are intertwined in the equations, cannot be pulled apart, and cannot be readily compared because they have different metrics.

AN ALTERNATIVE: THE OPPOSING FORCES DIFFUSION MODEL

To summarize, four problems were documented in the above critique of the PHM model: (1) The model has a poor fit to many empirical data series, and there are no meaningful patterns in terms of which kinds of events will or will not fit the model. (2) The assumption of a fixed inflection point or peak is unrealistic. (3) A systematic pattern of residuals exists in both poor-fitting and good-fitting cases, indicating a consistent problem in the model arising from its lack of mathematical flexibility and inability to track peaked distributions. (4) The model parameters
lack meaningful substantive interpretations. In this section, we will derive an alternative collective behavior diffusion model, the *Opposing Forces Diffusion Model*, that addresses these shortcomings of the PHM model while retaining the core theoretical insights that motivated it.

Both OFD and PHM recognize the existence of two fundamental forces that shape the diffusion and expression of collective action—one that mobilizes action and one that seeks to de-mobilize it. In discussing the OFD model we will call these two forces *provocation* and *repression* to distinguish them from PHM's *instigation* and *inhibition* forces. Whether violent or non-violent, protest or any kind of contentious collective behavior is subject to both provocation and repression. PHM limited their claims to collective violence, but we believe a two-force diffusion process is characteristic of a wide variety of contentious collective actions. As we showed above, the PHM model fits non-violent series as well as violent, and we will show that our OFD model also fits a wide variety of action forms.

We view provocation and repression as unobserved ideologies\(^8\) that are diffusing, and see the trajectory of collective action events as the outcome of the diffusion of these two forces. The *provocation* force is the ideology or belief system supporting whatever specific protest action is in question. Action ideologies are more volatile and specific than the larger grievances and belief systems that lie behind them. For example, ideologies supporting and decrying the oppression of Blacks existed in the U.S. throughout U.S. history. However, widespread Black acceptance of the notion that *rioting* would be an effective and justifiable tactic in pursuit of racial equality was not present prior to the 1960s, and spread rapidly in that decade. The repression force should not be equated with the effectiveness of police repression of riots, although it was obviously influenced by it. Instead, the repression force is the ideology or belief

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\(^8\) We use “ideology” here in a loose, non-technical sense to refer to a system of beliefs about action and its consequences, not in the more specific sense discussed by Oliver and Johnston (2000).
system that says riots are an inappropriate or ineffective or self-defeating mode of action. (See Tomlinson 1968, Spilerman 1970, Feagin and Hahn 1973 and Gale 1996 for discussion of the spread of riot and anti-riot ideologies).

When individual actors become adopters of the provocation ideology, they believe that the protest tactic is a justifiable and reasonable act that will contribute to achieving the agenda of the movement. Adoption of the provocative ideology makes people potentially active. The provocation force is the sum of all the factors that lead people to adopt this ideology, including the successes of past actions, the symbolic or emotional value attached to protest, and the influence of friends and acquaintances. For example, when street riots began occurring during the early part of the 1960s, many Blacks living in urban ghettos regarded them as necessary and successful responses to economic and living conditions. Even in neighborhoods that were burned out by a riot, Black residents believed that the riots were a positive influence and that they would ultimately improve the lot of inner-city Blacks (Feagin and Hahn 1973; Fogelson 1969; Bobo 1988; Marx 1967). The positive opinions about rioting were bolstered by the increased attention focused on ghetto conditions and even some concrete action intended to address inner-city problems (e.g., Governor’s Commission 1966; Crump 1966; Button 1978; Issac and Kelly 1981). Personal responses to riot participation also fueled the acceptance of the provocation ideology. The majority of Blacks in the Watts area reported that they felt increased pride as Blacks as a result of the riot (Fogelson 1969). In testimony given before the McCone Commission (Hacker and Harmatz 1969), rioters and observers reported: "It felt good all over." … "We were whole again." … "We were whole people, not just servants." … "It was the metamorphosis of the Negroes of southeastern Los Angeles from victims--historical objects--to masters." …"Violence is an alternative to despair. Through violence you can rid yourself of a
torturing feeling of helplessness and nothingness." Blacks’ experiences with rioting enhanced the view of the riot as a justified, viable protest tactic and thereby furthered the diffusion of the supportive ideology (Fogelson 1971; McAdam 1983; Tomlinson 1968).

Similarly, the *repression* force in the OFD model refers to an ideology representing all those forces that oppose and attempt to de-mobilize a particular form of action. The "repression" force responds not only to police or military efforts to directly quell or prevent protest, but also to perceptions that past actions were ineffective or counter-productive, feelings that the costs of action were too high, and beliefs that the particular kinds of action was immoral or unjustified. In the case of rioting in the 1960s, factors affecting the repression force included increasing calls for action against the rioters such as the "Law and Order" plank of Nixon’s 1968 presidential campaign (Nixon 1966), increasing formal acts of repression such as the mobilization of National Guard troops (McAdam 1982), and white vigilantism like Tony Imperiale’s North Ward Citizen’s Committee in Newark (Goldberger 1968), but also less formal social sanctioning and disapproval from family, friends, and community, and a belief that such action was an ineffective means of accomplishing positive change for African Americans.

Although both the OFD and the PHM models include two forces, the way these two forces work in each model is different. In PHM, the number of prior events directly affects the spread of both instigation and inhibition. In OFD, prior events do not directly affect future events. Instead, it is the provocation \( (P) \) and repression \( (R) \) ideologies that are diffusing, and the *probability* of action at time \( t \) is a function of the size of the difference between \( P \) and \( R \) when \( P > R \).
Formalizing the Opposing Forces Diffusion Model

The key insight undergirding the OFD model is that the observed trajectory of action is the net effect of provocation and repression forces which each diffuse as independent logistic functions. The rate of adoption for each of the logistic functions is initially low because there are too few adopters to have much influence, but as their numbers grow, the rate of influence grows until there are too few non-adopters left to adopt, and the rate of adoption slows again. This pattern produces the usual S-shaped cumulative adoption curve. The theoretical rationale for the logistic is as follows. Suppose there exists a population of potential adopters and that all of these potential adopters will eventually adopt the diffusing behavior. Let the total number in this population be $A_\infty$ (the number of adopters at time $= \infty$, positive infinity). The rate of adoption at any time $t$ then, is dependent on both the number of prior adopters ($A(t)$) and the number who have not yet adopted ($A_\infty - A(t)$). If we assume a constant imitation index ($a$) for a given phenomenon, the logistic diffusion model results:

$$\frac{dA(t)}{dt} = aA(t) \left[ A_\infty - A(t) \right]. \quad (5)$$

In this model, both the increasing number of prior adopters and the decreasing number of non-adopters have a direct influence on the rate of adoption (see Hamblin, Jacobson, and Miller 1973 and Mahajan and Peterson 1985 for a detailed derivation). In its integrated form (Equation 6), the logistic model predicts the cumulative number of adoptions at any point in time, and when plotted against time produces the familiar sigmoid shape.

$$A(t) = \frac{A_\infty}{1 + \frac{A_\infty - A_0}{A_0} \exp[-a t A_\infty]} \quad (6)$$
When interpreting the logistic model, the key element is the parameter $a$, which conveys the influence a prior adoption has on future adoptions. When $a$ is relatively high, the influence of prior adoptions is much greater and causes the process to accelerate and become exhausted much more quickly. Lower values indicate a more gradual diffusion process.

The OFD model assumes that the observed action trajectory is the net effect of two independent diffusion processes and is constructed by using two logistic diffusion equations, one representing the provocation diffusion force ($P(t)$) and the second representing the repressive diffusion force ($R(t)$):

$$P(t) = \frac{P_\infty}{1 + \frac{P_\infty - P_0}{P_0} \exp[-atP_\infty]}, \quad R(t) = \frac{R_\infty}{1 + \frac{R_\infty - R_0}{R_0} \exp[-btR_\infty]}, \quad (7, 8)$$

where the notation parallels that used above. The parameter $a$ is the influence index for the provocation process and $b$ is the influence index for the repression process.

Rather than modeling raw adopter counts for $R(t)$ and $P(t)$, we simplify by modeling the proportion of a population which adopts each ideology. Letting $P^*$ and $R^*$ symbolize the proportion ($P^*(t) = P(t)/P_\infty$ and $R^*(t) = R(t)/R_\infty$, $P^*_\infty = R^*_\infty = 1.0$), while $P^*_0$ and $R^*_0$ represent the proportion who have adopted the provocation and repression ideologies at $t = 0$.\(^9\)

Substituting into equations 7 and 8 produces:

$$P^*(t) = \frac{1}{1 + \frac{1 - P^*_0}{P^*_0} \exp[-pt]}, \quad R^*(t) = \frac{1}{1 + \frac{1 - R^*_0}{R^*_0} \exp[-rt]}, \quad (9, 10)$$

\(^9\) When we force the upper limits of the provocation and repression diffusion functions to be equal by modeling proportions, we also restrict the application of the model to collective action cycles that eventually die out. In other words, the OFD model applies only to collective action waves where the repression forces eventually catch up with provocation forces and extinguish the action wave.
where \( p \) and \( r \) are the relevant imitation indexes.

Because \( P^*_{\infty} = R^*_{\infty} \), it can be shown by equations 9 and 10 that at \( t = 0 \), \( P^*(t) = R^*(t) \), (or \( P^*_0 = R^*_0 \)). Using \( N^*_0 \) as a generic reference (\( P^*_0 = R^*_0 = N^*_0 \)), equations 9 and 10 become:

\[
P^*(t) = \frac{1}{1 + \frac{1 - N^*_0}{N^*_0} \exp[-pt]}, \quad R^*(t) = \frac{1}{1 + \frac{1 - N^*_0}{N^*_0} \exp[-rt]}.
\] (11, 12)

At this point, we have two logistic diffusion curves with common upper limits and common values at \( t = 0 \) such as illustrated in the top panel of Figure 4. As the two diffusion processes progress, the difference in their relative progress toward their upper limits predicts the relative probability of an event occurring at any particular time. In other words, as the gap between the two curves grows, so will rate of action. As the curves converge toward the end of the cycle (i.e. as repression catches up with provocation), the rate of action will drop and eventually return to zero.\(^{10}\) Therefore, we assert that the probability of an event (\( V \)) at any time is a function of the differences between \( P^*(t) \) and \( R^*(t) \):

\[
\frac{dV(t)}{dt} = P^*(t) - R^*(t).
\] (13)

The cumulative event count, \( V(t) \), is thus given by:

\[
V(t) = \int P^*(t)dt - \int R^*(t)dt.
\] (14)

Substituting from equations 11 and 12 integrating, we obtain:

\[
V(t) = \frac{\ln(1 - N^*_0 + \exp[pt]N^*_0)}{p} - \frac{\ln(1 - N^*_0 + \exp[rt]N^*_0)}{r} + V_0,
\] (15)

\(^{10}\) Exactly what raw count of \( R(t) \) relative to \( P(t) \) is necessary to extinguish the wave remains unknown given that we are modeling the proportion of total adopters of each ideology, \( P^*(t) \) and \( R^*(t) \), instead of raw counts. In essence, modeling proportions frees us from the metric, meaning that we do not have to specify how many repression adoptions are required to counteract one provocation adoption, and so forth.
where \( V_0 \) is the number of observed events at the start of the cycle. The final model has three parameters to be estimated: \( N^*, a, \) and \( b \). Because \( V_0 \) is an additive baseline in this model, it can be assumed to be zero without loss of generality in understanding the dynamics of the OFD model.11

**Assumptions of the Logistic and the Opposing Forces Diffusion Models**

PHM rightly rejected a single logistic function as a model of collective violence diffusion for a number of reasons. Because the OFD is built on the logistic, we must review the relevance of their objections for OFD. First, although the assumption of “once and forever” adoption is inappropriate for collective action, in the OFD, this assumption is applied to the adoption of an underlying ideology, where the assumption is more reasonable. In fact, the OFD model does not require permanent adoption, only that the ideology is held until the end of the action wave. Secondly, PHM rejected the logistic because it is symmetric while most waves of violent events are asymmetric. In the OFD, however, it is the diffusion of underlying ideologies that is assumed symmetric; the distribution of action can be asymmetric. In fact, the OFD model has a flexible inflection point that allows not only asymmetry, but different degrees of asymmetry.

Finally, PHM also correctly observed that the logistic model falls short when explaining the waning of the adoption wave. Although they attribute this difficulty to lack of redundancy in communication among collective violence actors, it is better understood as a product of the actor’s ability to adopt repeatedly. The logistic model requires a dwindling of potential adopters

11 There are, of course, alternative ways of specifying the counteracting nature of the provocation and repression diffusion processes other than subtraction—for example, by taking a ratio of the two functions. Preliminary analyses revealed that such alternatives produced considerably more complex functions and hampered straightforward interpretation without achieving a compensatory increase in accuracy.
to slow the adoption wave, but when actors can adopt repeatedly, the body of adopters never decreases. The OFD model posits for each of the dual diffusion processes of provocation and repression that each actor can adopt only once, and thus the assumptions of the logistic model are not violated. Furthermore, in these underlying processes, the body of potential adopters is systematically exhausted, accounting for the decline of each constituent process. The body of actors that can exhibit a violent behavior event does not decrease, however, and the mechanism that slows the event cycle is the growing acceptance of the repression ideology.

These aspects of the OFD model are made clear by a hypothetical plot of the model and its underlying functions. The first panel in Figure 4 shows the plots of two logistic diffusion processes, the first representing the provocation diffusion process and the second, the repression. At any point in time, the relative probability of observing a collective event is given by the difference between these two curves. Thus, the density function of the event cycle is given by subtracting the repression function from the provocation function resulting in the second panel. Conceptually, this means that the total amount of action and the duration of the wave arise from the gap between the $P^*(t)$ and $R^*(t)$ curves. The longer there is a gap and the wider it is, the more action there will be and the peak of activity occurs when the gap between $P^*(t)$ and $R^*(t)$ is largest. $P(t) > R(t)$. Eventually, repression catches up to provocation, and action declines. Integrating the rate function produces the expected sigmoid cumulative event count function in the final panel.

**Interpretation of Parameters**

One strength of the OFD model is the interpretability of its parameters. Fundamentally, $p$ and $r$ are imitation indices associated with the two underlying logistic diffusion models: They indicate the influence of prior ideological adoptions acts on potential adopters. As such,
comparisons of \( p \) and \( r \) lend additional insight into the model. The overall trajectory of an event cycle is largely determined by the relative size of \( p \) and \( r \). If \( r > p \), repression always keeps action in check and no action wave will develop. The larger \( p \) is relative to \( r \), the faster provocation diffuses relative to repression, and the more rapidly observed actions accumulate. Figure 5 shows this general pattern in the first two panels. In panel A, \( r \) is held constant and \( p \) varies while in Panel B, \( p \) is held constant while \( r \) varies. In both instances, when the values of \( p \) and \( r \) are closer, the total accumulation of events is lower (the upper portion of each panel) and the maximum rate of adoption is lower (the lower portion of each panel).

Several other important features of the model can be seen in Figure 5. First, changes in the relative size of \( p \) and \( r \) achieved by raising (or lowering) \( r \) have a different effect on the action wave than lowering (or raising) \( p \). For a given rate repression diffusion \( r \), lowering the provocation rate \( p \) lowers the peak of the action and makes it occur later, thus lowering the total amount of action that occurs in the wave but leaving the duration of the wave constant (Panel A). By contrast, for a given rate \( p \) of provocation diffusion, an increase in the rate \( r \) pulls the action down faster, lowering the peak and shortening the wave of action (Panel B). Thus, only the repression rate, and not the provocation rate, controls the duration of the protest cycle. Second, all curves with the same ratio of \( p \) to \( r \) have the same maximum rate of adoption (i.e. the same height of a density function). However, higher magnitudes of \( p \) and \( r \) lead to action curves which rise and fall more rapidly, generating less total action across time, while the diffusion process takes longer and generates more total action when the magnitudes of \( p \) and \( r \) are smaller (Panel C).
More substantively, when activists and authorities interact to produce provocation and repression, the character of the \( p \) and \( r \) forces imply strategies of action. When activists seek to increase levels of protest, the possible strategies dictated by the OFD framework are either to maximize the infectiousness of the provocation ideology or to minimize the infectiousness of the repression ideology.\(^\text{12}\) Looking at a process at its starting point, if activists wish to maximize the duration of the protest cycle, they would be well advised to concentrate on reducing the spread of a repression ideology (i.e., reducing \( r \)). If, on the other hand, the activists believe their cause would best be served by concentrated protest occurring as soon as possible (perhaps to achieve a quick victory), then they should concentrate on increasing provocation. Both strategies increase the amount of protest, but the former extends the duration of the protest cycle while the second hastens the peak of activity.

From the perspective of the state, authorities, or any body whose interests are served by quelling the protest wave, two opposite strategies are suggested by the OFD model: Reduce the diffusion of the provocation force or increase the diffusion of the repressive force. If ending the event cycle quickly is most important, the latter strategy is preferred. Increasing the diffusion of repression will compress the protest activity into a smaller time frame, but will also cause the wave to end more quickly. If the social system can tolerate conflict better if it is spread out over time, then reducing the infectiousness of the provocation wave may be a more useful strategy.

The \( N^*_{0} \) parameter captures how much of the ideological diffusion processes has occurred prior to the point at which \( R^*(t) \) first exceeds \( P^*(t) \) and produces the first action in the series. Higher \( N^*_{0} \) values mean that more diffusion of provocative and repressive ideologies has

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\(^{12}\) Although space prevents a review here, the social movements literature suggests a number of possible strategies for increasing infectiousness. For example, protest rhetoric that has high-frame consonance (Snow et al. 1986) might increase provocation infectiousness. Repression infectiousness might also be affected by framing activity or by attempts to placate authorities or elites (Jenkins and Eckert 1986).
occurred before the first action. This leaves fewer targets left to be influenced in the remainder of the processes causing repression to catch up to provocation more quickly. This makes the peak of the process occur earlier and reduces the total amount of action (Panel D of Figure 5).13

In other words, the more people are previously set in their opinions, the less diffusion will be a factor in the progression of a protest wave.

The sources of these patterns can be traced by examining the constituent functions underlying the models. In Figure 6, we plot the diffusion curves underlying two of the models in Figure 5 ($N^*_0 = .4$ and $N^*_0 = .02$). First, it is apparent that both diffusion processes were underway before $t = 0$. At this point, both diffusion processes have accumulated some fraction of their total adopters, $N^*_0$. This aspect of the model is very important because it recognizes that ideology precedes action and that a critical mass of adherents or converts must build prior to the outbreak of violence. It is also clear that prior to $t = 0$, $R^*(t)$ exceeds $P^*(t)$ and therefore no action will occur. To begin action, then, the provocation critical mass must exceed the accumulated level of repression.

Comparing the constituent curves of the two different models, we can see that when $N^*_0 = .4$, both the provocation and repression curves are to the left of their positions when $N^*_0 = .02$, but the repression curve has "moved" further than the provocation curve.14 This differential

13 The maximum of the OFD function is $V_0 + (1/r-1/p)(\ln[1/N^*_0])$ which makes it plain that the maximum number of events is inversely related to $N^*_0$.

14 Within a single OFD model, both constituent curves are subject to the same $N^*_0$ value. Because events can only be observed when $P^*(t) > R^*(t)$, then $P^*(t)$ must intersect $R^*(t)$ at approximately the time when events are first observed ($t = 0$), which in turn forces the provocation and repression processes to have elapsed equally at $t = 0$. 
causes the provocation and repression curves to be closer together for the $N^*_{0} = .4$ case at $t > 0$ and thus restricts the acceleration of the action wave.

Before moving on, it is important to note that the OFD model requires the estimation of one more parameter than the PHM model. It might be objected, therefore, that the increased fit of the OFD model is due merely to the mathematical flexibility derived from adding an additional parameter. While the OFD undoubtedly fits better because it has an additional degree of freedom, this remedy addresses exactly the problem with earlier models. As we have shown above, the Gompertz form of the PHM is not flexible enough to capture the dynamics of many protest waves—particularly those with extreme peaks. If a successful fit is to be obtained, an additional parameter must be added to the formulation. The situation is analogous to a regression model where one attempts to fit a straight line to an inherently curvilinear relationship. Transforming the independent variable to a polynomial form uses an additional degree of freedom and makes the function more flexible, but the transformation is required if the model is to be specified correctly. Again, what is most important is the additional parameters in the model have substantive meaning, as well as improving fit. In the case of collective violence waves, for example, even fifth-order polynomials have two to three more parameters but still fit the data less well than PHM or OFD, as well as lacking any theoretical meaning.15 It is not just the number of parameters that matters, but the specification of the particular mathematical form of a relationship to capture an underlying process.

15 We did not conduct a systematic study of polynomials, as they are theoretically meaningless, but a test of third-, fourth, and fifth-order polynomials on the 1960s riot series revealed that even a fifth-order polynomial yielded an $r^2$ of only .9849, a little less than PHM’s two-parameter model, and considerably worse than OFD’s three parameter model.
EMPIRICAL RESULTS USING THE OFD

Comparative Results versus PHM

Table 2 presents the results obtained by fitting the OFD model via non-linear least-squares to the same data used to test the PHM model.\(^{16}\) The first three columns give the parameter estimates for the model, the fourth (\(V_\infty\)) gives the maximum number of events estimated by the OFD model, and the fifth gives the \(r^2\) fit of the model to the data. The first notable difference between the two models is the improved fit achieved by the OFD model: All \(r^2\) values equal or exceed those from the PHM model. Furthermore, of the 27 data sets examined, OFD fits at \(r^2 > .99\) for all but three and the median fit is \(.997\). Nevertheless, \(r^2\) fit is only one criterion by which to judge the model and the while the fit gained by the OFD model is reliable and substantial, the other objections raised to the PHM must be addressed to recommend OFD.

| Table 2 Here |

One serious concern for PHM was the systematic pattern of residuals it produced. When the residuals produced by OFD are compared to those produced by PHM, the problems related to lack of flexibility are markedly reduced. As an example, Figure 7 plots the residuals of both models using the yearly riots counts data. The characteristic residuals of the PHM model are apparent, but the OFD model residuals are quite different: They are not only smaller, but also do not appear to exhibit a systematic pattern. The sources of improvement are clearer in the second column where yearly, rather than cumulative, counts are plotted. In short, the OFD model does a

\(^{16}\) A full investigation into the performance of the OFD model vis-à-vis alternatives like the PHM model is an involved undertaking that cannot be taken up here due to space limitations. Comprehensive tests are reported elsewhere (Myers 1997) and the subset of analyses reported herein serves mainly to illustrate the superior fit of the OFD model and provide an introduction to the interpretation of its parameters.
much better job accounting for the central spike of the event cycle and accommodates the very low levels of action in the tails of the cycle better than PHM.\textsuperscript{17}

One important limitation of PHM is its fixed inflection point, which makes it unable to track the peaks of waves. The OFD model, however, does not have this limitation and predicts a wide range of inflection points ranging from a low of .244 for rioting in 1969 to .673 for public transportation actions (column 6 of Table 2). Because empirical inflection points can only be estimated, it is difficult to precisely gauge the improvement offered by the OFD, but plots of the raw data are instructive.\textsuperscript{18} For example, in Figure 1, we showed that PHM predicted the inflection point at event number 1251 in 1898, while the real inflection point was closer to event number 850 in 1892. OFD’s flexible inflection point fits much better, predicting the inflection point at event 877 in early 1893.

Another element for comparison is the maximum number of events predicted by each model. For PHM, this value is given in Table 1 as $V_0[e^{(c/q)}]$ and for OFD in Table 2 as $V_\infty$. The observed values for each process are given in the final column of Table 1 and it is clear that OFD is almost always closer to the observed count than PHM. In fact, the event totals predicted by PHM are often unreasonably high, sometimes doubling the observed total. This occurs precisely because PHM cannot accurately track peaks of action: it essentially “assumes” that only 37% of

\textsuperscript{17} This simple demonstration does not prove that OFD lacks a systematic residual pattern and, in fact, our more extensive analyses indicate that even OFD is unable to track very extreme peaks of activity (Myers 1997). In all cases we have examined, however, OFD always tracks extreme peaks better than PHM.

\textsuperscript{18} We also tested for inflection point accuracy using simulated data with known inflection points. Using Euclidean distance, the median deviation of PHM inflection points from the true inflection point was 8.7 times that of the OFD (Myers 1997).
the events have occurred at the peak and thus models a total wave as lasting considerably longer than it actually did.

**Interpretation of Regional Differences**

Beyond indicators of fit, it also instructive to examine the parameter estimates to determine what they tell us about the diffusion processes. In the interests of brevity, we discuss here only the main details of the riot models by region; further detail is available elsewhere (Myers 1997). It is also useful in this context to examine the density functions estimated by OFD for the six regions (displayed in Figure 8).

As is apparent in the figure, the waves of rioting progressed quite differently in the different regions. Rioting peaked the earliest in the Agricultural Midwest and latest in the South. The riot wave was most extreme in the Industrial North, less so in the two southern regions, and least extreme in the West, Agricultural Midwest, and New England. But the graphs and the parameters tell us far more than this. For one, the ratio of the provocation parameter to the repression parameter tells how fast repression catches up with provocation and thus limits the expression of the provocation force. For example, both the Border South and the Industrial North have similar \( p \) values, and activity in both regions peaked around the same time. The wave was much more extreme in the Industrial North because the diffusion of repression was considerably slower.

Differences in the \( N^*o \) values are also evident in the data. The Agricultural Midwest and the West both have very similar \( p/r \) ratios which would indicate similar peak levels of action if \( N^*o \) values were the same. But the \( N^*o \) value for the Agricultural Midwest is more than twice the value for the West, which tempers the peak of the wave and causes it to occur earlier. A high
$N^*_{0}$ indicates that opinions about rioting were much more in place in the Agricultural Midwest at the start of the wave and therefore, diffusion had much less of an impact in that region than elsewhere.

Perhaps the most often cited regional difference with respect to this wave of rioting is the relatively depressed level of rioting in the South. First, it should be apparent that contrary to some claims, there was indeed a substantial amount of rioting in the South. As is apparent from Figure 8, however, the riot wave in the South was somewhat delayed compared to other regions. Influential early studies (e.g., Spilerman 1970; 1971) focused on the beginning of the riot wave, and thus did not detect the full southern riot wave. Nevertheless, there are differences between the South and other regions. For example, when comparing the South to the closely related Border South, we can see that the overall magnitude of the $p$ and $r$ parameters are considerably lower in the South, leading to a less peaked wave which takes longer to dissipate. Because the provocation effect is considerably less in the South, the wave takes longer to peak despite having a similar $p/r$ ratio (as in Panel C of Figure 5).

Given past arguments about why rioting seemed to occur less in the South, one might expect that repression should be relatively high in the South relative to provocation. But the OFD suggests something different was happening. In this model, the diffusion of the repression ideology (relative to provocation) was moderate in the South compared to other regions, but provocation was the lowest of all six regions. These findings support Oberschall’s (1978) argument that it was not repression that deterred rioting in the South. Rather, a great deal of protest energy was directed toward non-violent civil rights movement activity in the South and, as a result, less energy was channeled through violence.
DISCUSSION

Theorists increasingly recognize that waves of collective violence and protest are driven by the strategic interplay between disruptive actors and the regimes that seek to control them. This means that the spread of collective violence or collective protest necessarily involves the conjunction of two processes. In an important initial effort, Pitcher, Hamblin, and Miller (1978) developed a mathematical model to capture these two processes, and showed that it fit the data rather well. They assumed that some kind of rational calculus by actors underlay the diffusion of collective action, and recognized that any model of collective action had to account for multiple acts by the same actor. Perhaps this achievement seemed to be the last word on the subject, for no one ever revisited their model, either to apply it in a new context, or to criticize or extend it.

We think mathematical models of diffusion processes have an important role to play in advancing the theory of collective action. In this spirit, we have re-engaged the PHM work, not to diminish its contribution, but to build upon it, and move theory development forward. A careful statistical analysis revealed systematic weaknesses in the fit of the PHM model to empirical data. The Opposing Forces Diffusion model was developed as an alternate mathematical specification of the underlying insight of PHM that collective action waves are explained by the conjunction of two forces. Instead of specifying this insight as a kind of Gompertz distribution, we propose that it is better specified as the subtraction of two logistic diffusion processes. Our conception has theoretical plausibility, and it fits the data even better than the PHM model. It fits the data not just because it adds a parameter, but because the mathematical relationships the model specifies do a better job of tracking the way the provocative and repressive forces interact in a collective action trajectory. Polynomials with two or even three more parameters fit the data worse than either the PHM or OFD model do, with
their theoretically-specified relationships. We have shown that the two-parameter PHM model is not flexible enough to track the wide variation in “shapes” of empirical protest distributions, while the OFD model is more flexible.

The OFD model has advantages beyond simple fit, however. It is more directly tied to a theory of the underlying process. This leads to several consequences. First, the model parameters have straightforward interpretation. The terms for provocation and repression are in the same units, making it meaningful to compare their magnitudes. Second, the OFD model makes the notion of repression explicit within the model framework. Once the model is estimated, the provocation and repression effects can be disentangled and their individual forms can be viewed independently. As a result, the constituent components of the OFD diffusion process can be studied and tested on their own, given data appropriate to the task. This advantage stems from a different approach to events than was taken by PHM and others. Rather than events being the direct result and the direct cause of the diffusion process, events in the OFD model are treated only as outcomes of the underlying diffusion of provocation and repression ideologies. These counter-acting mobilizing and de-mobilizing forces produce a probability that an event will be observed at any given time. Higher event rates are seen as indicative of changes in the underlying forces but do not directly contribute to future rioting rates. This treatment of events makes theoretical sense. It means that events diffuse not from some sort of mindless “infection,” but mediated through the spread of ideas, in which it is perfectly plausible to imagine people seeking to influence each others’ opinions about whether a certain form of action is a good idea or not.

A detailed analysis of the theoretical implications of the OFD model reveals relationships not specifically recognized. In particular, it shows the greater efficacy of increasing or
decreasing repression relative to decreasing or increasing provocation as a strategy for affecting the total amount of action. It suggests that “fresh” diffusion processes that are beginning where few are already committed to either the provocation or repression ideologies will last longer and accumulate more actions than those building upon prior diffusion processes in which many have already adopted one or the other ideology.

Finally, the process of developing this new model opens the door to a fresh look at the modeling process. It points away from a narrow focus on the “fit” of a model to a serious examination of the model’s predictions, residuals, and interpretability as criteria for assessing its value. This work also points in important directions for future theorizing about collective action (for a more detailed treatment, see Oliver and Myers 1998; 2000). It shows that observed series of collective action can be best analyzed by explicitly modeling the dual processes of provocation and repression and showing how they interrelate. It also implies the value of future extensions in which more complex interactions can be modeled. While we do not expect to have the last word in modeling collective action diffusion any more than PHM did, we believe that OFD represents a significant step forward in theorizing collective action.
REFERENCES


Opposing Forces Diffusion Model


Table 1: PHM Parameter Estimates for Lynching, Rioting, and Civil Rights Actions

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<th></th>
<th>c</th>
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<td>171</td>
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<td>0.00117</td>
<td>0.996</td>
<td>147</td>
<td>129</td>
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</table>

\(^a\) Regions:
- Industrial North: NY, NJ, PA, OH, IN, IL, MI.
- New England: ME, NH, VT, MA, CT, RI.
- Agricultural Midwest: MN, IA, ND, SD, NE, KS, WI.
- South: LA, AL, MS, FL, GA, NC, SC, AR, TN.
- Border South: TX, OK, MO, VA, WV, KY, MD, DE, DC.
- West: CA, OR, WA, NV, MT, ID, CO, UT, WY, AZ, NM.

\(^b\) King assassination riots deleted
### Table 2: OFD Parameter Estimates for Lynching, Rioting, and Civil Rights Actions

<table>
<thead>
<tr>
<th>Category</th>
<th>p</th>
<th>r</th>
<th>$N^*_0$</th>
<th>$V_0$</th>
<th>$r^2$</th>
<th>Infl. Point</th>
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<td><strong>Black Lynching Victims 1882-1954</strong></td>
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<td>.00353</td>
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<td>By Region$^a$</td>
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<td>By Year</td>
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<td>.997</td>
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$^a$ See Table 1 for region definitions

$^b$ King assassination riots deleted
Figure 1: Inflection Points in the McAdam (1982) Lynching Data
Figure 2: Residuals from PHM models for Black Political Power Actions 1960-70
Figure 3: The PHM Model and Discontinuities in U.S. Rioting
Note: $p = .12$, $r = .08$, $N*o = .02$

Figure 4: A Example of the Opposing Forces Diffusion Model
Figure 5: Effects of Varying $p$, $r$, $p/r$ ratio, and $N^*_0$
Provocation and Repression

Figure 6: A Comparison of Two $N^*o$ values
Figure 7: Residual Comparison of PHM and OFD Models for Yearly Riot Counts
Figure 8: OFD Projected Riot Rates across Regions