1 Ramsey Optimal Taxation

1.1 Setup

Then use HH first order conditions to substitute out for prices

$$\sum_{t=0}^{\infty} \beta^t [U_C(C_t, 1-N_t)C_t - U_L(C_t, 1-N_t)N_t] = U_C(C_0, 1-N_0)K_0[1+(1-\tau^K_0)(F_K(K_0, N_0) - \delta)]$$

Ramsey problem: maximize HH utility subject to feasibility, implementability.

Primal approach: solve for allocation first, back out supporting taxes from equilibrium conditions:

$$\left(\frac{u_C(C_t, 1-N_t)}{u_C(C_{t+1}, 1-N_{t+1})} - 1\right) \frac{1}{F_K(K_{t+1}, N_{t+1}) - \delta} = 1 - \tau^K_{t+1}$$

$$\frac{u_L(C_t, 1-N_t)}{u_C(C_t, 1-N_t)F_N(K_t, N_t)} = 1 - \tau^N_t$$

$$\tau^K_0$$ only affects period zero: initial capital is inelastic, tax it as much as possible. To make problem interesting, restrict $$\tau^K_0 \leq \bar{\tau}^K$$.

Define $$U(C, N) = U(C, 1-N)$$ so $$U_N = -U_L$$. Also define:

$$W(C_t, N_t; \lambda) = U(C_t, N_t) + \lambda[U_C(C, N)C + U_N(C, 1-N)N]$$

Then Ramsey problem can be written:

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t W(C_t, N_t; \lambda) - \lambda U_C(C_0, N_0)K_0[1+(1-\tau^K_0)(F_K(K_0, N_0) - \delta)]$$
subject to (multiplier $\mu_t$):

$$C_t + G_t + K_{t+1} = F(K_t, N_t) + 1 - \delta K_t$$

1.2 Characterization

First order conditions for $t \geq 1$:

$$W_C(t) = \mu_t$$

$$W_N(t) = -\mu t F_N(t)$$

$$\mu_t = \beta \mu_{t+1} (F_K(t + 1) + 1 - \delta)$$

These imply modified Euler equation and intra-temporal optimality condition:

$$W_C(C_t, N_t) = \beta W_C(C_{t+1}, N_{t+1}) [1 + F_K(K_{t+1}, N_{t+1} - \delta)]$$

$$-\frac{W_N(C_t, N_t)}{W_C(C_t, N_t)} = F_N(K_t, N_t)$$

Implications:

- zero long-run capital tax: if $G_t \rightarrow \bar{G}$, allocation converges to steady state, then $\tau^K_t \rightarrow 0$.

- smoothing of tax rates: allocation smooths tax response to changes in $G_t$.

Example: $U(C, N) = \frac{C^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}$

Then $W_C = U_C(1 + \lambda - \lambda \gamma)$, $W_N = U_N(1 + \lambda - \lambda \phi)$.

Implies $\tau^K_t = 0$ for $t \geq 2$, and $\tau^N_t$ constant for $t \geq 1$. 

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