Lecture 8: Fiscal Policy in the Growth Model

Economics 714, Spring 2016

1 Fiscal Policy in the Growth Model

Focus on deterministic neoclassical growth model with fiscal policy, date-0 sequence formulation

A government policy is a sequence of spending \( \{G_t\} \) and taxes \( \{T_t\} \) that satisfy the government budget constraint:

\[
\sum_{t=0}^{\infty} q_t G_t = \sum_{t=0}^{\infty} q_t T_t
\]

Government spending not valued, will consider different tax instruments to raise revenue \( T_t \).

Households face budget constraint:

\[
\sum_{t=0}^{\infty} q_t [C_t + I_t] = \sum_{t=0}^{\infty} q_t [r_t K_t + w_t N_t - T_t]
\]

Capital law of motion:

\[
K_{t+1} = (1 - \delta) K_t + I_t
\]

Goods market/ aggregate feasibility:

\[
C_t + I_t + G_t = F(K_t, N_t)
\]

A competitive equilibrium is a price system \( \{q_t, r_t, w_t\} \), an allocation \( \{C_t, K_t, N_t\} \), and a government policy \( \{G_t, T_t\} \) s.t. (i) households optimize, (ii) firms optimize, (iii) markets clear
Ricardian equivalence: If $T_t$ is lump sum (i.e. independent of household choices) then timing of taxes is irrelevant, all that matters is date-0 present value.

Level of taxes and spending (even if lump sum) matter because of wealth effects.

Assume $U(C_t, N_t) = U(C_t)$, so $N_t \equiv 1$, define $F(K, 1) = f(K)$.

Equilibrium conditions:

$$ u'(C_t) = \beta u'(C_{t+1})[r_{t+1} + 1 - \delta] $$

$$ = \beta u'(C_{t+1})[f'(K_{t+1}) + 1 - \delta] $$

$$ K_{t+1} = f(K_t) + 1 - \delta K_t - C_t - G_t $$

Dynamics:

$$ \Delta c = 0 \implies f'(K) = \rho + \delta $$

$$ \Delta k = 0 \implies f(K) = C + \delta K + G $$

1.1 Distorting Taxes

Now consider linear tax $\tau^N_t$ on labor income, $\tau^K_t$ on capital income:

$$ T_t = \tau^N_t w_t N_t + \tau^K_t (r_t - \delta) K_t $$

Government budget constraint as before with this definition of revenue.

Define after-tax gross return $R^K_t = 1 + (1 - \tau^K_t)(r_t - \delta)$.

Household budget constraint, using capital law of motion:

$$ \sum_{t=0}^{\infty} q_t [C_t + K_{t+1}] = \sum_{t=0}^{\infty} q_t [(1 - \tau^N_t) w_t N_t + R^K_t K_t] $$
Firm problem unaffected: \( w_t = F_N, r_t = F_K \).

Households: now allow elastic labor supply, so solve
\[
\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \quad \text{s.t. BC}
\]

Equilibrium conditions:
\[
\begin{align*}
  u_C(C_t, 1 - N_t) &= \beta u_C(C_{t+1}, 1 - N_{t+1})[1 + (1 - \tau_{t+1}^K)(F_K(K_{t+1}, N_{t+1}) - \delta)] \\
  u_L(C_t, 1 - N_t) &= (1 - \tau_t^N)F_N(K_t, N_t) \\
  K_{t+1} &= F(K_t, N_t) + 1 - \delta K_t - C_t - G_t
\end{align*}
\]

2 Ramsey Optimal Taxation

Look for linear taxes that fund given \( \{G_t\} \) and maximize household welfare

First, find implementability constraint summarizing equilibria. Rewrite HH BC:
\[
\sum_{t=0}^{\infty} q_t[C_t - (1 - \tau_t^N)w_tN_t] = \sum_{t=0}^{\infty} q_t[R_t^K K_t - K_{t+1}] = q_0 R_0^K K_0
\]

Then use HH first order conditions to substitute out for prices
\[
\sum_{t=0}^{\infty} \beta^t [U_C(C_t, 1-N_t)C_t - U_L(C_t, 1-N_t)N_t] = U_C(C_0, 1-N_0)K_0[1+(1-\tau_0^K)(F_K(K_0, N_0) - \delta)]
\]

Solve directly for optimal allocation, back out supporting tax rates from equilibrium conditions
\[
\tau_0^K \text{ only affects period zero: initial capital is inelastic, tax it as much as possible. To make problem interesting, restrict } \tau_0^K \leq \tau^K.
\]

Ramsey problem: maximize HH utility subject to feasibility, implementability.
Define \( U(C, N) = U(C, 1 - N) \) so \( U_N = -U_L \). Also define:

\[
W(C_t, N_t; \lambda) = U(C_t, N_t) + \lambda[U_C(C, N)C + U_N(C, 1 - N)N]
\]

Then Ramsey problem can be written:

\[
\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t W(C_t, N_t; \lambda) - \lambda U_C(C_0, N_0)K_0[1 + (1 - \tau^K_0)(F_K(K_0, N_0) - \delta)]
\]

subject to:

\[
C_t + G_t + K_{t+1} = F(K_t, N_t) + 1 - \delta K_t
\]

First order conditions for \( t \geq 1 \) give:

\[
W_C(C_t, N_t) = \beta W_C(C_{t+1}, N_{t+1})[1 + F_K(K_{t+1}, N_{t+1} - \delta)]
\]

\[
-\frac{W_N(C_t, N_t)}{W_C(C_t, N_t)} = F_N(K_t, N_t)
\]

Implications: zero long-run capital tax, smoothing of tax rates