Policy in the New Keynesian Model

Noah Williams

University of Wisconsin-Madison
Solving the model for the rational expectations equilibrium

- Recall the 2 equation system:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) \left( \hat{i}_t - E_t \pi_{t+1} - r^n_t \right) \]

- Suppose \( i_t = r^n_t + \delta \pi_t \).

- Write system as

\[
\begin{bmatrix}
\beta & 0 \\
\frac{1}{\sigma} & 1
\end{bmatrix}
\begin{bmatrix}
E_t \pi_{t+1} \\
E_t x_{t+1}
\end{bmatrix}
= \begin{bmatrix}
1 & -\kappa \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\frac{1}{\sigma}
\end{bmatrix}
\delta \pi_t
\]

- or

\[
\begin{bmatrix}
E_t \pi_{t+1} \\
E_t x_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\beta} & -\frac{\kappa}{\beta} \\
\frac{\beta \delta - 1}{\sigma \beta} & 1 + \frac{\kappa}{\sigma \beta}
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t
\end{bmatrix}
\]

- Two eigenvalues outside the unit circle if and only if

\[ \delta > 1 \]
The Taylor Principle

- Policy must respond sufficiently strongly to inflation.

**Definition**

The condition that the nominal interest rate respond more than one-for-one to inflation is called the Taylor Principle.
Lessons

- Policy based on responding to exogenous disturbances does not ensure a unique equilibrium.
- Policy must respond to endogenous variables.
- In particular, the Taylor Principle needs to be satisfied.
Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

\[ E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right] . \tag{1} \]

\( x_t \) is the gap between output and the output level that would arise under flexible prices, and \( x^* \) is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.
Comparison to a standard loss function

- This looks a lot like the standard quadratic loss function. There are, however, two critical differences.
  1. The output gap is measured relative to the rate of output under flexible prices.
  2. Inflation variability enters because, with price rigidity, higher inflation results in an inefficient dispersion of output among the individual producers.

  ★ Because prices are sticky, higher inflation results in an increase in overall price dispersion.
Policy weights

- Theory says something about the weights in the loss function:

\[
E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right],
\]

where

\[
\Omega = \frac{1}{2} \bar{\Upsilon}_c \left[ \frac{\omega}{(1 - \omega)(1 - \omega \beta)} \right] (\theta^{-1} + \eta) \theta^2
\]

and

\[
\lambda = \left[ \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \right] \frac{(\sigma + \eta)}{(1 + \eta \theta) \theta}.
\]

- Greater nominal rigidity (larger \(\omega\)) reduces \(\lambda\).
- Loss function is endogenous.
- Calvo specification implies \(\lambda\) is small – Taylor specification leads to larger weight on output gap.
The basic new Keynesian inflation adjustment equation took the form

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \]

That is, there is no additional disturbance term.

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \]

The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.

Just set \( x_{t+i} = 0 \) for all \( i \); keeps inflation equal to zero.
Thus, the key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.

No policy conflicts.

When prices are sticky but wages are flexible, the nominal wage can adjust to ensure labor market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable.
Policy implications of price stickiness

- Models that combine optimizing agents and sticky prices have very strong policy implications.
- When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.
  1. The relative price of firms who have not set their prices for a while falls. They experience in increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.
  2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.
Cost shocks

- Assume

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]

where \( e \) represents an inflation or cost shock.

- Then

\[ \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i} \]

- Cannot keep both \( x \) and \( \pi \) equal to zero.

- Trade-offs must be made.
A common approach to “optimal” policy is in terms of simple rules. The most famous of such instrument rules is the Taylor Rule (Taylor 1993):

$$i_t = \pi_t + 0.5x_t + 0.5 \left( \pi_t - \pi^T \right) + r^*,$$

where $\pi^T$ was the target level of average inflation (Taylor assumed it to be 2%) and $r^*$ was the equilibrium real rate of interest (Taylor assumed this too was equal to 2%).

The Taylor Rule for general coefficients is

$$i_t = r^* + \pi^T + \delta_x x_t + \delta_\pi \left( \pi_t - \pi^T \right).$$
Taylor rules

- A larger literature has now developed that has estimated the Taylor Rule, or similar simple rules, for a variety of countries and time periods.
  - For example, Clarida, Galí, and Gertler (2000) do so for the Federal Reserve, the Bundesbank, and the Bank of Japan.
  - Estimates for the United States under different Federal Reserve Chairmen are reported by Judd and Rudebusch (2000).
  - In general, the basic Taylor Rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate.
  - Orphanides (2000), however, that when estimated using the data on the output gap and inflation actually available at the time policy actions were taken (i.e., using real-time data), the Taylor Rule does much more poorly in matching the U.S. funds rate.
Time Consistency and Commitment

- Now turn to analyzing optimal policy. With rational expectations, policy impact depends on ability of government to commit.
- Assume the loss function takes the form

\[ L_t = \left[ \pi_t^2 + \lambda (x_t - x^*)^2 \right]. \]  

(3)

- The economy is given by the Lucas model:

\[ x_t = \alpha (\pi_t - \pi_t^e) + e_t. \]

- Average inflation has no benefit (because it is expected) but increases loss.
Discretion: central bank takes expectations as given.

- First order condition is

\[ \pi_t + \alpha \lambda [\alpha (\pi_t - \pi^e_t) + e_t - x^*] = 0 \Rightarrow \pi_t = \frac{\alpha^2 \lambda \pi^e_t + \alpha \lambda (x^* - e_t)}{1 + \alpha^2 \lambda}. \]

- Rational expectations imply

\[ \pi^e_t = \frac{\alpha^2 \lambda \pi^e_t + \alpha \lambda x^*}{1 + \alpha^2 \lambda} \Rightarrow \pi^e_t = \alpha \lambda x^*. \]
Equilibrium in the Barro-Gordon model

- Equilibrium inflation rate is
  \[
  \pi_t = \frac{\alpha^2 \lambda (\alpha \lambda x^*) + \alpha \lambda (x^* - e_t)}{1 + \alpha^2 \lambda} = \alpha \lambda x^* - \left( \frac{\alpha \lambda}{1 + \alpha^2 \lambda} \right) e_t
  \]

- Output gap is
  \[
  x_t = \left( \frac{1}{1 + \alpha^2 \lambda} \right) e_t
  \]

- Inefficient average inflation bias equal to \(\alpha \lambda x^*\).
Eliminating the output distortion

- The efficiency distortion that leads to $x^*$ was used in the Barro-Gordon literature to motivate the presence of an overly ambitious output target in the central bank’s objective function. As a consequence, the presence of $x^* > 0$ implies that a central bank acting under discretion to maximize welfare would produce an average inflation bias.

- However, with average rates of inflation in the major industrialized economies remaining low during the 1990s, many authors simply assume $x^* = 0$.

- In this case, the central bank is concerned with stabilizing the output gap $x_t$, and no average inflation bias arises.
Discretion versus commitment

- If $x^* = 0$, is there any difference between discretion and commitment?
- In forward-looking models, the answer is yes.
- Discretion leads to a stabilization bias.
Basic model

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.

- Minimize
  \[ -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda x_{t+i}^2 \right] \]
  subject to
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t. \]

- Notice the Euler/IS equation imposes no constraint – use it to solve for \( i_t \) once optimal \( \pi_t \) and \( x_t \) have been determined.
Basic model – eliminating the steady-state distortion

- Note that $x^*$ has been set equal to zero in loss function

$$-\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda x_{t+i}^2 \right].$$

- Fiscal subsidy to offset distortion from monopolistic competition.
- If $x^* \neq 0$, can’t use first order approximations to structural equations to obtain a correct second order approximation to the representative agent’s welfare.
When the central bank operates with discretion, it acts each period to minimize the loss function subject to the inflation adjustment equation.

Because the decisions of the central bank at date $t$ do not bind it at any future dates, the central bank is unable to affect the private sector’s expectations about future inflation.

Thus, the decision problem of the central bank becomes the single period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to the inflation adjustment equation.
Central bank problem is to pick $\pi_t$ and $x_t$ to minimize

$$\pi^2_t + \lambda x^2_t + \psi_t \left( \pi_t - \beta \pi_{t+1} - \kappa x_t - e_t \right)$$

taking $E_t \pi_{t+1}$ as given.

The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (4)$$
$$\lambda x_t - \kappa \psi_t = 0 \quad (5)$$

Eliminating $\psi_t$, $\lambda x_t + \kappa \pi_t = 0$. 

\(x_t\) and \(\pi_t\) satisfy
\[
\lambda x_t + \kappa \pi_t = 0.
\]
\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.
\]

Then
\[
\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa^2}{\lambda} \pi_t + e_t \Rightarrow \pi_t = \frac{\lambda \beta E_t \pi_{t+1} + \lambda e_t}{\lambda + \kappa^2}.
\]
Discretion
Equilibrium

- Suppose
  \[ e_t = \rho e_{t-1} + \varepsilon_t. \]
  and
  \[ \pi_t = Ae_t. \]
- Then, \( E_t \pi_{t+1} = AE_t e_{t+1} = A\rho e_t \) and
  \[ \pi_t = \left( \frac{\lambda \beta A \rho + \lambda}{\lambda + \kappa^2} \right) e_t \Rightarrow A = \left( \frac{\lambda \beta A \rho + \lambda}{\lambda + \kappa^2} \right) = \frac{\lambda}{\lambda(1 - \beta \rho) + \kappa^2}. \]
- Zero average inflation bias.
Discretion

Behavior of the interest rate

- From the IS equation,

\[ i_t = E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t) + r^n_t. \]

- Using solution,

\[ i_t = \left[ A\rho - \sigma \left( \frac{\kappa}{\lambda} \right) (\rho - 1) \right] e_t + r^n_t = Be_t + r^n_t. \]

- Shifts in natural rate of interest \( r^n \) are fully offset.

- So optimal policy involves \( i \) responding to shocks, but adopting a rule of the form

\[ i_t = Be_t + r^n_t \]

does not ensure a unique rational expectations equilibrium.
Precommitment

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.
- Under optimal commitment, central bank at time $t$ chooses both current and expected future values of inflation and the output gap.
- Minimize
  \[-\Omega \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right]\]

  subject to

  \[\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t.\]
Optimal precommitment

- The central bank’s problem is to pick $\pi_{t+i}$ and $x_{t+i}$ to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda x_{t+i}^2 + \psi_{t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right].$$

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (6)$$

$$E_t (\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0 \quad i \geq 1 \quad (7)$$

$$E_t (\lambda x_{t+i} - \kappa \psi_{t+i}) = 0 \quad i \geq 0. \quad (8)$$

- Dynamic inconsistency – at time $t$, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = - (E_t \psi_{t+1} - \psi_t)$. When $t+1$ arrives, a central bank that reoptimizes will again obtains $\pi_{t+1} = -\psi_{t+1}$ – the first order condition (6) updated to $t+1$ will reappear.
Timeless precommitment

- An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (7) and (8) for all periods, including the current period so that

\[
\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \quad i \geq 0
\]

\[
\lambda x_{t+i} - \kappa \psi_{t+i} = 0 \quad i \geq 0.
\]

- Woodford (1999) has labeled this the “timeless perspective” approach to precommitment.
Timeless precommitment

- Under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

\[ \pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \] (9)

for all \( i \geq 0 \).

- Woodford (1999) has stressed that, even if \( \rho = 0 \), so that there is no natural source of persistence in the model itself, \( a > 0 \) and the precommitment policy introduces inertia into the output gap and inflation processes.

- This commitment to inertia implies that the central bank’s actions at date \( t \) allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap.
Discretion vs Commitment

Responses of to a 1% inflation shock under the optimal commitment (solid) and discretion (dashed) policies.

FIGURE 1

Economic Responses Under Commitment and Discretion

(a) interest rates

(b) output

(c) inflation

(d) expected inflation
Improved trade-off under commitment

- The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.
- Consider a positive inflation shock, $e > 0$.
- A given change in current inflation can be achieved with a smaller fall in $x$ if expected future inflation can be reduced:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

- Requires a commitment to future deflation.
- By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $E_t \pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.
Gains from Commitment

**FIGURE 2**

Output and Inflation Tradeoffs

- Standard deviation of output gap vs. standard deviation of inflation
- Discretion vs. commitment
The Taylor principle and liquidity traps

- Benhabib, Schmit-Grohé, and Uribe (2001, 2002) have argued that deflationary paths cannot be ruled out if policy satisfies the Taylor Principle.
- The argument is based on the observation that the nominal rate of interest cannot fall below zero.
- Explosive deflations would eventually force the nominal interest rate to zero, but the nominal rate is then prevented from falling further.
- They argue that simple and seemingly reasonable monetary policy rules that follow the Taylor Principle in changing the nominal interest rate more than one-for-one in response to changes in inflation can introduce the possibility the economy will be caught in a deflationary liquidity trap.
The Taylor principle and liquidity traps

A simple example

- Suppose we have a model displaying superneutrality. The “monetary” side of the model is summarized by

\[
i_t = r^n_t + E_t \pi_{t+1}
\]

\[
i_t = g(\pi_t)
\]

where \( r^n_t \) is exogenous with respect to inflation and the nominal interest rate and \( g(\pi) \) is the policy rule.

- For simplicity, let

\[
g(\pi_t) = r^n_t + \pi^* + \delta (\pi_t - \pi^*)
\]

\[
= r^n_t + (1 - \delta) \pi^* + \delta \pi_t
\]

- Then combining these equations,

\[
E_t \pi_{t+1} = (1 - \delta) \pi^* + \delta \pi_t
\]
The Taylor principle and liquidity traps

A simple example

- This is an expectational difference equation in the inflation rate. When does it imply a unique stable equilibrium?
- There is a unique stationary equilibrium inflation rate equal to $\pi^*$ if $\delta > 1$.
- However, if the time $t$ inflation rate is not equal to $\pi^*$, the inflation process is unstable.
The Taylor principle and liquidity traps
A simple example
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A simple example
The Taylor principle and liquidity traps

A simple example

- The zero bound on the nominal interest rate may give rise to a liquidity trap equilibrium.
- Note that there are now two possible steady-state equilibria.
  - And in one, the nominal rate of interest is zero.
In this analysis, the current rate of inflation is non-predetermined – it can jump.

Standard stability arguments in the presence for forward looking jump variables rely on notions of saddle-path stability in which the inflation rate would jump to put the economy on a stable path converging to the unique, stable stationary steady-state.

- That is, the only perfect foresight equilibrium in a neighborhood of $\pi^*$ is that associated with inflation equal to the target rate $\pi^*$. 
The Taylor principle and liquidity traps

- If inflation starts out just to the left of $\pi^*$, the central bank cuts the nominal rate in an attempt to lower the real rate and stimulate the economy. But instead it simply generates expectations of lower inflation, causing actual inflation to decline further.
  - Expressed in terms of money, the lower nominal rate increases the demand for real money balances, forcing a fall in the price level pushing the economy into a deflationary equilibrium.

- In a neighborhood around $\pi^{**}$, there are many equilibria paths consistent with a perfect foresight equilibrium. Any inflation rate originating in a neighborhood of $\pi^{**}$ is consistent with a perfect foresight equilibrium. Given this non-uniqueness or indeterminacy, sunspot equilibria are possible.
The Taylor principle and liquidity traps

- Benhabib, Schmitt-Grohé, and Uribe (2000) characterize this as a *real indeterminacy*. That is, the inflation rate is indeterminate and therefore so are real money balances.

- In general, optimal monetary policy in the absence of nominal rigidities requires that the nominal interest equal zero.
  - Rather than reflecting a bad outcome, converging to a zero nominal interest rate is optimal as it eliminates the wedge between the private and social opportunity costs of money.

- If, however, for reasons not specified in the simple model used here, the liquidity trap is a bad outcome, then assumption that the policy maker follows an *ad hoc*, non-optimal decision rule must be questioned.
Liquidity traps: solutions

- How can the economy get out of a liquidity trap?
- Fiscal policies.
- Monetary policies
  - In the liquidity trap, nominal interest rates are zero. The demand for real balances is indeterminate.
  - So variations in the nominal stock of money may not affect the price level – there is a real indeterminacy (of real money balances).
- Credible (key) commitment to future inflation.