Problem Set 1
Due in class on February 9.

1. Using a static general equilibrium model, consider a government that must fund a given level of spending $G$, but does not have access to lump-sum taxes.

(a) The government imposes a proportional income tax $\tau$ on the representative consumer, so after-tax income is $(1 - \tau)[wN + rK + \pi]$. What effect does this have on the competitive equilibrium, compared to the case of lump-sum taxes?

(b) The government instead imposes a consumption tax $t$, increasing the effective cost of consumption goods from 1 to $(1 + t)$. What effect does this have on the competitive equilibrium, compared to the case of lump-sum taxes?

(c) The government must decide whether to impose an income tax or a consumption tax. In either case, total tax receipts must equal $G$. Does the representative consumer prefer one tax to the other? Explain.

2. Suppose that there is a progressive tax on labor income. We model this by supposing that labor income $wN$ below a threshold $Y^*$ then the household faces a tax rate $\tau$, but if $wN > Y^*$ the tax rate increases to $\tau' > \tau$ for that portion of income above the threshold.

(a) Suppose a worker has unearned income $\pi$, has standard preferences over consumption and leisure, and faces this tax schedule. Characterize the worker’s optimal choice.

(b) Suppose that preferences ($MRS$) differ across workers but that they all face the same taxes and wage rates, and characterize the optimal choices for different individuals. Is there likely to be a mass of workers with labor income equal to $Y^*$?

(c) Now suppose that there is an increase in the top labor tax rate $\tau'$, but $\tau$ is unchanged. What happens to the labor supply choices of different workers?

(d) Now suppose that there is an increase in the lower labor tax rate $\tau$, but $\tau'$ is unchanged. What happens to the labor supply choices of different workers?

3. Consider the optimal growth model from class, but add government spending. That is, there is now a specified amount of government spending $G$ which must be funded every period via lump sum taxes.

(a) Compared to the case of no government spending, how does having $G > 0$ affect the optimal steady state levels of consumption and capital $c^*$ and $k^*$?
(b) How are the dynamics affected? That is, suppose the economy is initially in the steady state \((k^*_0, c^*_0)\) associated with \(G = 0\). Then there is announcement that there will be government spending \(G > 0\) for all future dates. How do consumption and capital respond, both immediately upon the announcement and then in the succeeding periods?

4. Suppose that instead of labor being supplied inelastically, households value leisure. That is, in the optimal growth model we now have preferences:
\[
\sum_{t=0}^{\infty} \beta^t [U(c_t) + v(1 - N_t)],
\]
where households have 1 unit of time each period, and \(N_t\) is labor, so \(1 - N_t\) is leisure. Both \(U\) and \(v\) are strictly increasing and strictly concave. Firms produce using a constant returns to scale production function \(F(k, N)\), so the aggregate feasibility condition is now:
\[
F(k_t, N_t) = c_t + k_{t+1} - (1 - \delta)k_t.
\]
Consider the social planner’s problem in this environment.

(a) Write down the Lagrangian and find the optimality conditions for the choices of \(c_t, k_{t+1}\) and \(N_t\) at any date \(t\).

(b) Taking ratios of your optimality conditions, find the household Euler equation relating the marginal utilities of consumption at dates \(t\) and \(t + 1\) to the marginal product of capital. Also find the equation relating the marginal rate of substitution between consumption and leisure to the marginal product of labor.

(c) Find the equations determining the steady state, and characterize the steady state as sharply as you can. How does having elastic labor supply \((v \neq 0)\) affect the steady state levels of consumption and capital?

(d) Now consider the special case (like in class) with \(\delta = 1\) and \(F(k, N) = Ak^\alpha N^{1-\alpha}\) and \(U(c) + v(1 - N) = \log c + \gamma \log(1 - N)\). Show that the optimal solution is to save a constant fraction \(s\) of output, \(c_t = (1 - s)F(k_t, N_t)\) and work a constant number of hours \(N_t = \bar{N}\). Find expressions for \(s\) and \(\bar{N}\).

(e) Continuing with the example from the previous part, suppose that there is an increase in productivity \(A\). How will that affect the optimal levels of consumption, labor, and capital, both on impact of the change and over time?