Problem 1 (Monopoly and Price Discrimination) (points b,c and d are optional)

Consider Microsoft Company from Problem Set 10 question 4. Microsoft’s inverse demand for the
new operating system is given by

\[ p(y) = 100 - y \]

a) (1st-degree price discrimination) Suppose that Microsoft can perfectly price discriminate among
the consumers. What is Microsoft’s profit if in that case? Is the allocation pareto efficient? What is
consumer surplus?

Let’s get more realistic and assume that Microsoft can charge different prices on two segments of the
market: individual buyers and firms. The demands on two segments are:

\[ y^I(p^I) = 50 - (4/5)p^I \]
\[ y^F(p^F) = 50 - (1/5)p^F \]

b) Show that if Microsoft does not price discriminate, the aggregate inverse demand is as in part a).

c) (3rd-degree price discrimination) Find the level of sales, price, profit and elasticity of demand for
each segment of the market.

d) Compare producer’s and consumer’s surplus in the three cases: uniform price (PS#9 Q4), perfect
discrimination (PS#10 Q1 part a) and 3rd-degree price discrimination (PS#10 Q1 part c).

Problem 2 (Demand elasticity)

Suppose the demand facing the monopolistic firm is given by

\[ y(p) = 1 - p \]

a) Plot the inverse demand function, indicating the corner points.

b) Calculate the value of the demand elasticity for \( y = 0, y = 0.5 \) and \( y = 1 \).

c) Suppose the total cost is given by \( TC(y) = cy \) where \( c \) is some positive number \( (c > 0) \). Show
formally that the monopolist will choose a point on the elastic portion of the demand. (i.e. elasticity =
\( \varepsilon \in (-\infty, -1) \)).

d) Find the markup over a marginal cost \( c \).

Problem 3 (Oligopolistic Industry)

Below you can find market shares of major beer producers in the USA in 2000.
Bud Light 36.8%
Coors Light 19.1%
Miller Lite 18.5%
Natural Light 9.2%
Busch Light 6.1%
Michelob Light 3.3%
Keystone Light 2.6%
Milwaukee's Best Light 2.3%
Old Milwaukee Light 0.8%
Miller Genuine Draft Light 0.7%

The total light beer sales in 2000 amounted to 87 million barrels.
a) Find the concentration ratio (a "big four" index) for the beer industry in the USA.
b) Is this industry perfectly competitive or oligopolistic (concentrated)? Why?
c) In your opinion how would Trade Commission react to a merger proposal by Bud Light and Coors Light? Why?

Problem 4 (Aircraft Industry)

The jet aircraft industry is dominated by two major competitors: Airbus (A - based in Europe) and Boeing (B - based in the USA). Both companies have similar technology allowing each firm to produce a jet at a cost of $20 (in millions). Accordingly, their costs functions are given by:

\[ TC(y_A) = 20y_A \]
\[ TC(y_B) = 20y_B \]

In order to simplify our analysis, we assume there are no fixed costs. The inverse demand function for jets by major airlines is estimated to be

\[ p(y) = 200 - y \]

a) Find analytically profit function \( \pi_B(y_B) \) for Boeing, given that the production of Airbus amounts to \( y_A = 100 \) jets. In a graph with \( y_B \) on the horizontal axis and \( \pi \) on the vertical one, plot the profit function.
b) Is the production \( y_B = 100 \) jets Boeing’s best response to \( y_A = 100 \)? Why or why not? Find the optimal level of production, given Airbus produces \( y_A = 100 \), \( y_A = 50 \), and \( y_A = 0 \)? Mark the three points in space \( (y_A, y_B) \).
c) Find analytically the best response function for Boeing \( R_B(y_A) \) and plot it in the graph from point b).
d) Find analytically the best response function for Airbus, \( R_A(y_B) \) and add it to your graph from point b)
e) Find analytically the market price of an aircraft, the level of individual and aggregate production in a Cournot-Nash equilibrium. Also find the level of profit of each individual firm. Show the equilibrium in your graph from b.
f) What is the deadweight loss (DWL) associated with oligopolistic trading by the two firms?
g) Suppose the two firms A and B form a cartel. What is the aggregate level of production, and profit per firm given collusion? Does collusion benefit the two producers?

h) Find a deadweight loss (DWL) given collusion, and compare it to the one from f). Which loss is greater, why?

i) Is the considered cartel sustainable if the interactions, as described above, are only in the short run? Why? How about if the market interactions are repeated? Why?

Problem 5 (Accounting & Audit services in the USA)

There are \( N > 2 \) auditing firms in the USA (\( N \) is a parameter). "Production" \( y^i \) of a firm \( i \) is measured in auditors' hours and a cost function is given by

\[
TC(y^i) = 10y^i
\]

You can think of $10 as an hourly wage paid to an auditor. Again we assume no fixed cost. The inverse demand for auditing in the USA is

\[
p(y) = 1000 - y
\]

where \( y \) is an aggregate supply.

a) Find the level of aggregate production and market price in two extreme cases: monopoly (\( N = 1 \)) and perfect competition (Hint: recall that in the case of perfect competition the secret of happiness is \( p = MC \)).

b) Plot the inverse demand function and mark the two points located on it - one for competitive interactions and one for monopoly.

c) Find analytically the level of production \( y^i \) supplied by each auditing firm and aggregate \( y \) number of hours, market price for one hour, \( p \), the level of profit and the deadweight loss in the industry, all in the Cournot-Nash equilibrium. Find all variables as functions of \( N \).

d) Find the values of aggregate production \( y \) and \( p \) for \( N=2, 5 \) and 10. Mark the corresponding values on the graph from b).

e) In the graph with \( N \) on the horizontal axis and \( p \) on the vertical one, plot the equilibrium price. What can you say about the price limit, as \( N \) goes to infinity?

f) In the graph with \( N \) on the horizontal axis and \( y \) on the vertical one plot the equilibrium aggregate production. What can you say about the limit of aggregate production, as \( N \) goes to infinity?

g) In the graph with \( N \) on horizontal and \( DWL \) on vertical one plot the equilibrium \( DWL \). What can you say about the limit of aggregate production, as \( N \) goes to infinity?