I - Homogeneous Products, 2 Firms.

Typical Demand function: \( p = A - b (q_i + q_j) \).
Costs (identical or not): \( MC_1 = c_1, MC_2 = c_2 \)

1) Firms choose quantities simultaneously. We look for the NE in quantities or Cournot equilibrium.
   Step 1: Calculate reaction functions by choosing \( q_i \) to max. \( \Pi_i (q_i, q_j) \) assuming \( q_j \)
is fixed.
   Let \( R_i (q_i) \) be firm \( i \)'s reaction function.
   Step 2: Calculate the NE by finding the intersection of the two reaction functions
   (i.e. find the values of \( q_i \) and \( q_j \) that solve \( q_i = R_i (q_j) \) and \( q_j = R_j (q_i) \)).

2) Firms choose quantities sequentially (assume Firm 1 moves first). We look for the SPNE in quantities
   or Stackelberg equilibrium.
   Step 1: Calculate Firm 2's reaction function by choosing \( q_2 \) to max. \( \Pi_i (q_1, q_2) \) assuming
   \( q_1 \) is fixed.
   Let \( R_2 (q_1) \) be firm \( 2 \)'s reaction function.
   Step 2: Calculate the optimal \( q_1 \) by max. \( \Pi_i (q_1, q_2) \) assuming \( q_2, R_2 (q_1) \).
   Replacing the optimal \( q_1 \) into \( R_2 (q_1) \) we get the value of \( q_2 \) at the SPNE.

Remark: \( \Pi_1 > \Pi_2 \) so the firm that goes first has an advantage.

3) Firms choose prices simultaneously. We look for the NE in prices or Bertrand equilibrium with
   homogeneous products.
   No mathematical calculation needed. Just a clear argument to justify the NE prices.

4) Limit output: we assume entry costs (f) and that firm 2 acts according to its reaction function. The
   limit output for firm 1 is the amount of \( q_1 \) such that:
   \[ \Pi_i (q_1, R_2 (q_1)) = [A - b q_1 - b (R_2 (q_1))] q_1 - c_1 q_1 - f = 0 \]

II - Differentiated Products, 2 Firms.

Typical Demand functions:
   \( p_1 = a - b q_1 - c q_2 \)
   \( p_2 = d - e q_2 - g q_1 \) for firms choosing quantity
   or
   \( q_1 = h - m p_1 + n p_2 \)
   \( q_2 = r - s p_2 + u p_1 \) for firms choosing prices
Costs (identical or not): \( MC_1 = c_1, MC_2 = c_2 \); no fixed costs.

5) Firms choose prices simultaneously. We look for the NE in prices or Bertrand with differentiated
   products.
   Step 1: Calculate reaction functions by choosing \( p_i \) to max. \( \Pi_i (p_1, p_2) \) assuming \( p_j \)
is fixed.
   Let \( R_i (p_i) \) be firm \( i \)'s reaction function.
   Step 2: Calculate the NE by finding the intersection of the two reaction functions
(i.e. find the values of \( p_1 \) and \( p_2 \) that solve: \( p_1 = R_1(p_2) \) and \( p_2 = R_2(p_1) \)).

(6) Firms choose prices sequentially (assume Firm 1 moves first). We look for the SPNE in prices.

Step 1: Calculate Firm 2’s reaction function by choosing \( p_2 \) to max. \( \Pi_2(p_1, p_2) \) assuming \( p_1 \) is fixed.

Let \( R_2(p_1) \) be firm 2’s reaction function.

Step 2: Calculate the optimal \( p_1 \) by max. \( \Pi_1(p_1, p_2) \) assuming \( p_2, R_2(p_1) \).

Replacing the optimal \( p_1 \) into \( R_2(p_1) \) we get the value of \( p_2 \) at the SPNE.

Remark: \( \Pi_1 < \Pi_2 \) so the firm that goes first is at a disadvantage.

**Strategic Trade Policy : Third Market Sales**

(III) *Cournot Competition in Output market (homogeneous products)*

(7) *Brander-Spencer argument for an optimal export subsidy.*

Set up: Two firms (Firm 1 and Firm 2) from countries 1 and 2 respectively choose quantities simultaneously. All their output is exported to a third country.

This is a sequential game where country 1 moves first choosing an export subsidy/tax and then the two firms compete as indicated. The government’s objective is to maximize the country’s welfare. By choosing a subsidy/tax level the government is choosing firm 1’s marginal cost.

Theoretical Result: The optimal policy is an export subsidy. With the optimal subsidy, firm 1’s output coincides with the output level of a Stackelberg leader. Therefore, the subsidy shifts profits from firm 2 to firm 1. Country 1 is better off and country 2 worse off than without intervention.

Steps:
- Calculate NE quantities for an arbitrary level of subsidy.
- Choose the subsidy/tax that maximizes country 1’s welfare.

(8) *Both Governments can choose a Subsidy/Tax*

Set up: Two firms (Firm 1 and Firm 2) from countries 1 and 2 respectively choose quantities simultaneously. All their output is exported to a third country.

This is a sequential game where in:

Stage 1: both countries choose a subsidy/tax simultaneously.

Stage 2: after the subsidy/taxes are known, the two firms compete choosing their outputs simultaneously.

Each government maximizes its social welfare. By choosing a subsidy/tax level governments are choosing their firm’s marginal cost.

Steps:
- Calculate NE quantities for arbitrary levels of subsidies/taxes.
- Calculate the NE in subsidies/taxes assuming that the firms will use their NE quantities in Stage 2

Theoretical Result: At the NE both countries subsidize their firms. In that equilibrium both
countries are **worse off** than without intervention.

**(IV) Bertrand Competition in Output market (differentiated products)**

**(9) Eaton-Grossman result that an export tax is optimal**

Set up: Two firms (Firm 1 and Firm 2) from countries 1 and 2 respectively choose prices simultaneously. All their output is exported to a third country.

This is a sequential game where country 1 moves first choosing an export subsidy/tax and then the two firms compete as indicated.

The government’s objective is to maximize the country’s welfare. By choosing a subsidy/tax level the government is choosing firm 1’s marginal cost.

Theoretical Result: The optimal policy is an export tax. With the optimal tax profits for Firm 1 are obviously higher than without intervention. In this case Firm 2's profit also increase. Therefore both countries are better off than without intervention.

Steps:
- Calculate NE prices for an arbitrary level of subsidy.
- Choose the subsidy/tax that maximizes country 1’s welfare.

**(10) Both Governments choose a Subsidy/Tax**

Set up: Two firms (Firm 1 and Firm 2) from countries 1 and 2 respectively choose prices simultaneously. All their output is exported to a third country.

This is a sequential game where in:
- Stage 1: both countries choose a subsidy/tax simultaneously.
- Stage 2: after the subsidy/taxes are known, the two firms compete choosing their prices simultaneously.

Each government maximizes its social welfare. By choosing a subsidy/tax level governments are choosing their firm’s marginal cost.

Steps:
- Calculate NE prices s for arbitrary levels of subsidies/taxes.
- Calculate the NE in subsidies/taxes assuming that the firms will use their NE prices in Stage 2

Theoretical Result: At the NE both countries tax their firms. In that equilibrium both countries are **better off** than without intervention.