MATH NECESSARY

VARIABLES ARE FUNCTIONS OF TIME.

EX: \( y(t) \), \( x(t) \)

\[
\frac{\text{Growth Rate}}{\text{of } y(t)} = \frac{\Delta y(t)}{\Delta t} = \frac{dy}{dt} = \frac{d}{dt} \log y(t)
\]

\( \frac{dy}{dt} = 0 \) \( \Rightarrow \) \( \log y(t) = 0 \)

\( \text{constant growth equal to zero } \Rightarrow \) y(t) does not change

MAT ALGEBRA

\[
\begin{align*}
(x + y) &= x + y \\
(x - y) &= x - y \\
(-x) &= -x
\end{align*}
\]

STEADY STATE: SITUATION WHERE THE VALUE OF A VARIABLE (OR SYSTEM) does NOT CHANGE (i.e. is steady)

EX: \( y = 0 \)
Balanced Growth Path:

(OF SYSTEM OF VARIABLE)

SITUATION WHERE ALL VARIABLES GROW AT CONSTANT RATES.

i.e.,

\[ y = \text{constant} \]
\[ \dot{y} = n \]

NOT NECESSARILY THE SAME CONSTANT

Diagrams help with

PATHS OF VARIABLES

AND GROWTH RATES

Ex: IF \( \dot{y} > 0 \) \( \Rightarrow \) \( y \uparrow \)
(i.e., \( y \uparrow \))

IF \( \dot{y} = 0 \) \( \Rightarrow \) \( y \) CONSTANT

IF \( \dot{y} < 0 \) \( \Rightarrow \) \( y \downarrow \)
TECHNOLOGY / PROD FUNCTION REVIEW

- PROD. FUNCTION
  - HANG. PRODUCTS
  - KENNY TO SPACE

PROD. FUNCTION: 2 INPUT (K, L)

\[ Y = F(K, L) = A K^a L^b \]

\[ a > 0 \quad b > 0 \quad L > 0 \]

\[ MPL = \text{MARGINAL PRODUCT LABOR} = \frac{\Delta Y}{\Delta L} \bigg|_{\text{MARRIAGE}} \]

\[ MPK = \frac{\Delta Y}{\Delta K} \bigg|_{\text{MARRIAGE}} \]

\[ \text{IN THIS CASE MPL \& AS L \uparrow \text{T.I.C. MPK \& AS K \uparrow} \text{MARRIAGE PRODUCT}} \]

KENNY TO SPACE

ProD. FUNCTION HAS CONSTANT KENNY TO SPACE (CKT) I.E.
- IF K \& L CHANGES IN THE SAME PROPORTION, Y CHANGES IN THE SAME PROPORTION

**EXAMPLE**: ASSUME K \& L \uparrow BY 30% \( \Rightarrow \)

\[ K_1 = 1.30 K_0 \]

\[ L_1 = 1.30 L_0 \]
Then \[ Y_1 = P(k_1, \xi) = A (1.30 \xi)^{\alpha} (1.30 \xi_0)^{-\alpha} \]
\[ = A (1.30)^{\alpha+1-\alpha} \xi_0^{-\alpha} \xi_0^{1-\alpha} \]
\[ = 1.30 \frac{A \xi_0^{1-\alpha}}{\xi_0} = 1.30 Y_0 \]
Change of variable

\[ y = F(k, L) \] (3 variables)

\[ y = k^a L^b \]

Difficult to graph, etc.

We will make a change of variable to have only two

EASY DIAGRAM.

New variables: per person/worker

We can do this.

Because our function has CCS.

Steps:

Divide both sides by \( L \)

\[ \frac{y}{L} = \frac{k}{L^a} \]

Work with right-hand side.

\[ \frac{y}{L} = k^{\frac{1}{a}} \]

\[ L^{\frac{1}{a}} \]

\[ y = A + L^{\frac{1}{a}} \]

Let our new variables: \( A,k \).

\[ x = L^{\frac{1}{a}} \]

Output per person/worker.

\[ \text{Capital per worker} \]

\[ x = \frac{k}{L} \]

\[ x \leq 0 \rightarrow x \rightarrow + \]

But at \( + \rightarrow 0 \)