1. Standard IS-LM

The IS schedule is given by:

\[ i = \left(1 - c(1-t)\right) \frac{1}{b} Y + \left(\frac{1}{b}\right) \left[ A_0 - bi \right] \quad <\text{IS curve}> \]

The parametric form of the linear LM schedule is given by

\[ i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right) \left(\frac{M_0}{P_0}\right) + \left(\frac{k}{h}\right) Y \quad <\text{LM curve}> \]

Solving for \( Y \) yields:

\[ Y_0 = \hat{\alpha} \left[ A_0 + \left(\frac{b}{h}\right) \left(\frac{M_0}{P_0}\right) - \left(\frac{b}{h}\right) \mu_0 \right] \]

where \( \hat{\alpha} \equiv \frac{1}{1-c(1-t)+(bk/h)} \)

Consider what is assumed in this case, so that one obtains this solution. Starting from an initial budget surplus of zero, when government spending is increased, the budget deficit widens. The resulting deficit has to be financed somehow. One way is via bond financing, that is the government sells bonds to finance the shortfall. If bond demand rises dollar for dollar with wealth \( \text{wealth}\) \(\frac{\text{wealth}}{P} = \text{wealth} = (M/P) + (B/P) \), then the following occurs in the money and bond markets, and to the LM schedule (remember the bond market is the mirror image of the money market). See Panel A.

2. Portfolio Crowding Out

If on the other hand, money demand rises with wealth, \( \text{viz.} \),

\[ \frac{M^d}{P} = \mu_0 + kY - hi + j \left(\frac{\text{wealth}}{P}\right) \]

Where \( j \) has the interpretation of \( \Delta(B^d/P)/\Delta\text{wealth} \) (and is equal to \( 1 - \Delta(M^d/P)/\Delta\text{wealth} \)), then a budget deficit that increases the stock of bonds outstanding will cause the following situation, illustrated in Panel B. Notice that now interest rates rise in response to the government
deficit that increases the stock of bonds outstanding, and the LM shifts in response to a budget deficit. This will occur whenever $\Delta(B^d/P)/\Delta\text{wealth} < 1$ (or equivalently $\Delta(M^d/P)/\Delta\text{wealth} > 0$).

Panel A

Panel B

Note: The LM’s position, with this new money demand function, depends upon the amount of wealth, and hence amount of bonds, outstanding. To see this, algebraically solve for the LM to obtain:

\[ i = \frac{\mu_0}{h} - \left( \frac{1}{h} \right) \left( \frac{M_0}{P_0} \right) + \left( \frac{j}{h} \right) \left( \frac{M_0}{P_0} + \frac{B_0}{P_0} \right) + \left( \frac{k}{h} \right) Y \]

You will observe that the vertical intercept depends upon stocks of both money and bonds. Substituting this revised LM curve into the IS curve yields:

\[ Y_0 = \hat{\alpha} \left[ A_0 + \left( \frac{b}{h} \right) \left( \frac{M_0}{P} \right) - \left( \frac{b}{h} \right) \mu_0 - \left( \frac{bj}{h} \right) \left( \frac{M_0}{P} + \frac{B_0}{P} \right) \right] \]
Notice that the stock of bonds now enters into the determination of equilibrium income.

The linkage to fiscal policy becomes obvious when one considers how budget deficits are financed.

(7) \[ BuS \equiv T - G \]

To simplify matters assume \( t=0 \), and work with the budget deficit, \( BuD \),

(8) \[ -BuS \equiv BuD \equiv G - T = GO_0 - TA_0 = \Delta(B / P) \]

This budget deficit has to be financed somehow; this constraint is reflected in the last term on the right hand side of (8). In other words, if the government spends more than it takes in in terms of revenue, then it must borrow by issuing new debt.

So if one considers an increase in government spending on goods and services, \( \Delta GO \), **starting from an initial budget deficit of zero**, then:

(9) \[ \Delta GO = \Delta(B / P) \]

Now consider the total differential of (6):

(10) \[ \Delta Y = \hat{a} \left[ \Delta A + \left( \frac{b}{h} \right) \Delta \left( \frac{M}{P} \right) - \left( \frac{b}{h} \right) \Delta \mu - \left( \frac{bj}{h} \right) \Delta \left( \frac{M}{P} + \frac{B}{P} \right) \right] \]

And “zero out” those terms that are constant when only government spending is changed:

\[ \Delta \left( \frac{M}{P} \right) = 0 \]

And substitute (9) in:

(11) \[ \frac{\Delta Y}{\Delta GO} = \hat{a} \left[ 1 - \frac{bj}{h} \right] = \hat{a} \equiv \frac{1}{1 - c + bk / h} \]

Thus the change in output for a change in government spending will be the same as in the standard case, as long as money demand does not depend upon wealth (\( j = 0 \)). The larger either \( b \) or \( j \), the smaller the multiplier.

This quantitative result should point the way to the economic intuition. The initial output increase is mitigated by the fact that when the government has to bond finance the resulting budget deficit, it has to offer higher interest rates to induce the public to hold the additional bonds.

This means that in this model, output can actually fall in response to an increase in government spending (that is, nothing rules out \( Y_1 \) ending up less than \( Y_0 \)). The difference between \( Y_1 \) and \( Y_0 \) is called “portfolio crowding out” of income, to differentiate it from “transactions crowding out” of income, which is the difference between \( Y'_0 \) and \( Y''_0 \). Transactions crowding out (what is discussed in the textbook) arises because higher income spurred by higher government spending raises money demand and, given the fixed money supply, higher equilibrium interest rates.
Portfolio crowding out arises because higher government spending is associated with higher bond sales, hence higher wealth, and hence higher demand for money which, given the fixed money supply, results in higher interest rates for all income levels.

4. Empirical Evidence for the United States

Regression results, in levels:

\[
\text{Spread} = 2.27 + 0.077(\text{debt}_\text{gdp})
\]

\[
(1.55) (0.030)**
\]

Adj. R-sq. = 0.17 (HAC std errors)

Regression results, in differences:

\[
\Delta \text{Spread} = 0.11 + 0.152\Delta(\text{debt}_\text{gdp})
\]

\[
(0.24) (0.046)***
\]

Adj. R-sq. = 0.30 (HAC std errors)