The Keynesian Model of Equilibrium and the Trade Balance

This set of notes outlines the Keynesian model of national income determination and trade balance ("net export") determination. It then shows how to solve for multipliers.

1. An Expanded Model

<table>
<thead>
<tr>
<th>Eq.No.</th>
<th>Equation</th>
<th>Description</th>
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<tbody>
<tr>
<td>(1)</td>
<td>$Y = AD$</td>
<td>Output equals aggregate demand – an equilibrium condition</td>
</tr>
<tr>
<td>(2)</td>
<td>$AD = C + I + G + X - M$</td>
<td>Definition of aggregate demand</td>
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<td>(3)</td>
<td>$C = \overline{C} + c(Y - T)$</td>
<td>Consumption function, $c$ is the marginal propensity to consume</td>
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<td>(4)</td>
<td>$T = \overline{TA} + tY$</td>
<td>Tax function; $\overline{TA}$ is lump sum taxes, $t$ is tax rate.</td>
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<td>(5)</td>
<td>$I = \overline{I}$</td>
<td>Investment function</td>
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<tr>
<td>(6)</td>
<td>$G = \overline{G}$</td>
<td>Government spending on goods and services</td>
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<td>(7)</td>
<td>$X = \overline{X}$</td>
<td>Exports, simplification of $X = X_d(E,Y^<em>)$ where $E$, $Y^</em>$ fixed</td>
</tr>
<tr>
<td>(8)</td>
<td>$M = \overline{M} + mY$</td>
<td>Import spending, simplification of $M = M_d(E,Y)$ where $E$ fixed</td>
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Substitute (3)-(8) into (2), and substitute (2) into (1):

$Y = AD = \overline{C} + c(Y - \overline{TA} - tY) + \overline{I} + \overline{G} + \overline{X} - \overline{M} - mY$

Collect up terms:

$Y = \overline{A} + \overline{X} - \overline{M} + (cY - ctY - mY)$ where $\overline{A} \equiv \overline{C} - c\overline{TA} + \overline{I} + \overline{G}$

Shift "$Y$" terms to the left hand side:

$Y - (cY - ctY - mY) = \overline{A} + \overline{X} - \overline{M} \Rightarrow Y[1 - c(1 - t) + m] = \overline{A} + \overline{X} - \overline{M}$

Divide both sides by the term in the square bracket to obtain equilibrium income, $Y_0$:

$Y_0 = \frac{1}{1 - c(1 - t) + m} [\overline{A} + \overline{X} - \overline{M}]$ let $\overline{\alpha} \equiv \frac{1}{1 - c(1 - t) + m}$

Notice that if there are no taxes, $t=0$ and $\overline{TA} = 0$, so $1-c = s$ and (12) becomes identical to (17.7) in the textbook.
Y_0 = \left( \frac{1}{s + m} \right) [\bar{A} + \bar{X} - \bar{M}] \text{ where } \bar{A} \equiv \bar{C} + \bar{I} + \bar{G}

Interpretation of (12): equilibrium income is a multiple of the amounts of “autonomous” spending. The higher the level of autonomous spending, the higher the equilibrium level of income. Notice also that lump sum taxes enter in negatively, so the higher lump sum taxes, the lower equilibrium income is.

2. Effects of changes in autonomous spending

To think about how changes in autonomous spending – government spending (\bar{G}), investment spending (\bar{I}), export spending (\bar{X}) and import spending (\bar{M}) – affect equilibrium income, think about a change of income (\Delta Y) as being attributable to changes in each of those autonomous spending components. Take equation (12):

\[ \Delta Y = \alpha [\Delta A + \Delta X - \Delta M] \]

So if, for instance, the only autonomous spending component that changes is government spending (so \Delta A = \Delta G, and \Delta X = 0 = \Delta M), then:

\[ \Delta Y = \alpha \Delta G \]

To figure out what happens to the trade balance in response to changes in autonomous spending (let’s use government spending again), take (17.6) from the textbook:

\[ TB \equiv X - M = \bar{X} - (\bar{M} + mY) \]

Break up the changes in the trade balance in the changes in the constituent parts,

\[ \Delta TB = \Delta X - \Delta M - m\Delta Y \]

If, once again, the only thing that changes is government spending, then substitute (14) into (15), and setting \Delta X = 0 = \Delta M :

\[ \Delta TB = -m[\bar{A}\Delta G] < 0 \]

In other words, the effect of an increase in government spending is a deterioration in the trade balance, holding everything else constant.