Expenditure Switching/Reducing and the IS-LM Model  (revised 23.2.2004)

This set of notes extends the Keynesian model to incorporate the elasticities approach explicitly. It then further expands the model to incorporate the monetary sector, popularly known as the IS-LM model. It will then conclude by linking the monetary sector to the balance of payments. Note: The notation is changed somewhat from the previous set of notes and the textbook. Exports are now \( EX \), imports are \( IM \). Furthermore, \( M \) will now denote “money”.

### 1. The Expanded Model with Income and Price Sensitivities

#### 1.2 Derivation

<table>
<thead>
<tr>
<th>Eq.No.</th>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>( Y = AD )</td>
<td>Output equals aggregate demand – an equilibrium condition</td>
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<tr>
<td>(2)</td>
<td>( AD \equiv C + I + G + EX - IM )</td>
<td>Definition of aggregate demand</td>
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<tr>
<td>(3)</td>
<td>( C = CO + c(Y - T) )</td>
<td>Consumption function, ( c ) is the marginal propensity to consume</td>
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<tr>
<td>(4)</td>
<td>( T = TA + ty )</td>
<td>Tax function; ( TA ) is lump sum taxes, ( t ) is tax rate.</td>
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<tr>
<td>(5)</td>
<td>( I = IN )</td>
<td>Investment function</td>
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<tr>
<td>(6)</td>
<td>( G = GO )</td>
<td>Government spending on goods and services</td>
</tr>
<tr>
<td>(7)</td>
<td>( EX = EXP + vq )</td>
<td>Export spending</td>
</tr>
<tr>
<td>(8)</td>
<td>( IM = IMP + mY - nq )</td>
<td>Import spending</td>
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</table>

where \( v \) is the sensitivity of exports to the real exchange rate, and \(-n\) is the sensitivity of imports to the real exchange rate.

Substitute (3)-(8) into (2), and substitute (2) into (1):

\[
Y = AD = CO + c(Y - TA - ty ) + IN + GO + EXP - IMP - mY + (n + v)q
\]

Collect up terms:

\[
Y = A + EXP - IMP + (cY - ctY - mY) + (n + v)q \quad \text{where} \quad A \equiv CO - cTA + IN + GO
\]

Shift “\( Y \)” terms to the left hand side:

\[
Y - (cY - ctY - mY) = A + EXP - IMP + (n + v)q \Rightarrow
\]
\[ Y[1 - c(1 - t) + m] = \bar{A} + \bar{EXP} - \bar{IMP} + (n + v)q \]

Divide both sides by the term in the square bracket to obtain equilibrium income, \( Y_0 \):

\[
(12) \quad Y_0 = \left( \frac{1}{1 - c(1 - t) + m} \right) [\bar{A} + \bar{EXP} - \bar{IMP} + (n + v)q] \quad \text{let} \quad \bar{\alpha} = \left( \frac{1}{1 - c(1 - t) + m} \right)
\]

Interpretation of (12): equilibrium income is a multiple of the amounts of “autonomous” spending and the real exchange rate. The first component is familiar -- the higher the level of autonomous spending, the higher the equilibrium level of income. Notice also that lump sum taxes enter in negatively, so the higher lump sum taxes, the lower equilibrium income is. The second component is new – changes in real exchange rates (which are the same as the change in the nominal exchange rate when the price levels are fixed) affect the international components of aggregate demand, namely exports and imports.

To think about how changes in autonomous spending or exchange rates affect equilibrium income, think about a change of income (\( \Delta Y \)) as being attributable to changes in each of those autonomous spending components. Take equation (12):

\[
(13) \quad \Delta Y = \bar{\alpha} [\Delta A + \Delta EXP - \Delta IMP + (n + v)\Delta q]
\]

So if, for instance, the only autonomous spending component that changes is government spending (so \( \Delta A = \Delta GO \), and \( \Delta EXP = 0 = \Delta IMP \)) and the real exchange rate is constant (\( \Delta q = 0 \)), then:

\[
(14) \quad \Delta Y = \bar{\alpha} \Delta GO \Rightarrow \Delta Y / \Delta GO = \bar{\alpha}
\]

Notice that by this reasoning, all the multipliers can be derived. Now consider changes in the real exchange rate. Then:

\[
(15) \quad \Delta Y = \bar{\alpha} (n + v)\Delta q \Rightarrow \Delta Y / \Delta q = \bar{\alpha} (n + v)
\]

### 1.2 Expenditure reduction versus expenditure switching

To figure out what happens to the trade balance in response to changes in government spending, take the definition of the trade balance:

\[
(16) \quad TB \equiv EX - IM = (\bar{EXP} + vq) - (\bar{IMP} + mY - nq)
\]

Break up the changes in the trade balance in the changes in the constituent parts,

\[
(17) \quad \Delta TB = \Delta EXP - \Delta IMP - m\Delta Y + (n + v)\Delta q
\]
If the only thing that changes is government spending, then substitute (14) into (17), and setting \( \Delta EXP = 0 = \Delta IMP \) and \( \Delta q = 0 \):

\[
\Delta TB = -m[\bar{\alpha}GO] < 0
\]

In other words, the effect of an increase in government spending is a deterioration in the trade balance, holding everything else constant. Thus, decreasing government spending would improve the trade balance. This is the “expenditure reduction” channel.

On the other hand, holding GO constant, one can substitute (15) into (17) to obtain:

\[
\Delta TB = (n + v)\Delta q > 0
\]

In words, the effect of a real exchange rate depreciation (\( \Delta q > 0 \)) is an improvement in the trade balance. This occurs because exports are now cheaper, so demand for exports rises, while imports become more expensive for home residents, so imports fall. This is the “expenditure switching” channel.

How does this relate to the “Swan Diagram” (Figure 18.3) on page 343 of the textbook? Take equation (17),

\[
\Delta TB = \Delta EXP - \Delta IMP - m\Delta Y + (n + v)\Delta q
\]

substitute in (14) and (15):

\[
\Delta TB = \Delta EXP - \Delta IMP - m(\bar{\alpha}GO + \bar{\alpha}(n + v)\Delta q)) + (n + v)\Delta q
\]

Starting from initially balanced trade, setting autonomous exports and imports constant, and requiring the change in the trade balance to be zero, yields:

\[
0 = -m\bar{\alpha}GO - m\bar{\alpha}(n + v)\Delta q) + (n + v)\Delta q
\]

Solving for the change in \( q \) (which is the same as the change in \( E \)):

\[
\Delta E = \frac{m\bar{\alpha}}{(1 - m\bar{\alpha})(n + v)}\Delta GO \quad \Rightarrow \quad \frac{\Delta E}{\Delta GO} = \left|_{TB \text{ constant}} \right. = \frac{m\bar{\alpha}}{(1 - m\bar{\alpha})(n + v)} > 0
\]

Notice that the slope of the BB curve is the change in \( E \) for a change in \( GO \), holding \( TB \) constant, or in other words, the expression in (22.b).
Notice the YY curve in 18.5 can be derived by taking equation (13) with the change in income set to zero, setting $\Delta EXP = 0 = \Delta IMP$. Then

\[(23) \quad 0 = \Delta Y = \alpha [\Delta GO + (n + v)\Delta q] \]

Once again letting $\Delta q = \Delta E$, and solving:

\[(24) \quad \left. \frac{\Delta E}{\Delta GO} \right|_{\text{const}} = \frac{-1}{n + v} < 0 \]

That is, the YY curve has slope $-1/(n+v)$. This means in words, for higher levels of government spending, a lower exchange rate (a stronger currency) is required to keep output constant.

2. IS-LM (or “Adding in Money”)

2.1 Derivation

To allow for a role for money, let’s first modify the model. On the real side of the economy, everything is the same, except for equation (5), for investment.

\[\begin{align*}
(1) \quad Y &= AD \\
(2) \quad AD &= C + I + G + EX - IM \\
(3) \quad C &= \overline{C}O + c(Y - T) \\
(4) \quad T &= \overline{TA} + tY \\
(5') \quad I &= \overline{IN} - bi \\
(6) \quad G &= \overline{GO} \\
(7) \quad EX &= \overline{EXP} + vq \\
(8) \quad IM &= \overline{IMP} + mY - nq
\end{align*}\]

The only essential difference is that investment spending now depends on the interest rate. The coefficient $b$ is the interest sensitivity of investment. Since income now depends on interest rates, which is endogenous, then solving equations (1)-(8) yields an equation of a line.

\[(25) \quad Y = \alpha [\overline{A} + \overline{EXP} - \overline{IMP} + (n + v)q - bi] \quad <\text{IS curve}>\]

\[(25') \quad i = \frac{\overline{A} + \overline{EXP} - \overline{IMP} + (n + v)q}{b} - \left(1 - c(1 - t) + m\right)Y \quad <\text{IS curve}>\]

This expression means that for lower levels of interest rates, investment, a component of aggregate demand, is higher, and thus income is also higher.
We introduce the monetary sector by setting out money supply and money demand.

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<td>(26)</td>
<td>( \frac{M^d}{P} = \frac{M^s}{P} )</td>
<td>Equilibrium condition</td>
</tr>
<tr>
<td>(27)</td>
<td>( \frac{M^s}{P} = \frac{M}{P} )</td>
<td>Money supply</td>
</tr>
<tr>
<td>(28)</td>
<td>( \frac{M^d}{P} = kY - hi )</td>
<td>Money demand</td>
</tr>
</tbody>
</table>

Substitute (27) and (28) into (26), to obtain:

(29) \( \frac{M}{P} = kY - hi \)  

\(-h (k)\) is the interest (income) sensitivity of real money demand

Solving for the interest rate yields:

(30) \[ i = -\left( \frac{1}{h} \right) \left( \frac{M}{P} \right) \left( \frac{k}{h} \right) Y \]  

<LM curve>

There are two unknowns, and two equations. To figure out equilibrium income and equilibrium interest rates, one would need to solve the system. This is shown in the appendix. For now, I’l merely show the answer, and relate it to the graphical depiction.

(31) \[ Y_0 = \hat{\alpha} \left[ A + EXP - IMP + (n + v)q + \left( \frac{b}{h} \right) \left( \frac{M}{P} \right) \right] \]  

where \( \hat{\alpha} = \frac{1}{1 - c(1-t) + m + \frac{bk}{h}} \)

Notice that equilibrium income now depends on the level of autonomous spending, the real exchange rate, and the money stock (in real terms). The equilibrium interest rate is a complicated function of autonomous spending, real exchange rate and the money stock. To obtain this value, one would substitute (25) into (30). This is done in the appendix.

The equilibrium income level and interest rate is depicted in the figure below:
One can see that the IS curve has a slope that depends upon the parameters by solving (25) with $i$ on the left hand side, as in equation (25').

The position of the IS curve depends upon $A$, $IMP$, $EXP$, and $q$.

The position of the LM curve depends upon the real money stock, $(M/P)$.

Only at the combination of $i_0$ and $Y_0$ is it true that both the goods market and the money market are in equilibrium ($Y = AD$ and $M' = M^d$ respectively).

2.2 Policy in the IS-LM Model

Notice that if one increases $A$, $EXP$, or $q$, then the vertical intercept increases, which is the same as the IS curve shifting out rightward. If one increases $M$ when $P$ is constant, then $M/P$ rises, and the LM vertical intercept shifts down, which is the same as the LM curve shifting out rightward. In the former case, output increases, and interest rates rise. In the latter, interest rates fall, and output rises.

Notation: $\Delta A = A + \Delta A$

Notice that the increase in output $\alpha \Delta A$ is smaller than that which would have been implied by the simple Keynesian multiplier ($\bar{\alpha} \Delta A$).
The reason the increase in output is less in this system is because of “crowding out”. Higher output leads to higher money demand which, given a constant money supply, results in a higher equilibrating interest rate. The higher interest rate depresses investment, thus offsetting in part the increase in output.

Note: \( \bar{M} = \bar{M} + \Delta M \)

Interest rates fall, resulting in a higher level of investment, thus a higher level of aggregate demand and hence output.

How can one solve for the change in income resulting from changes in policy variables analytically? Take equation (31), and break it up into the constituent changes (i.e., take a total differential):

\[
(31) \quad Y_0 = \hat{\alpha} \left[ \bar{A} + \bar{EXP} - \bar{IMP} + (n + v)q + \left(\frac{b}{h}\right)\left(\frac{\bar{M}}{P}\right) \right]
\]

\[
(32) \quad \Delta Y = \hat{\alpha} \left[ \Delta A + \Delta EXP - \Delta IMP + (n + v)\Delta q + \left(\frac{b}{h}\right)\Delta \left(\frac{M}{P}\right) \right]
\]

For changes in government spending only:

\[
(33) \quad \Delta Y = \hat{\alpha}\Delta GO \rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\alpha}
\]

For changes in money only (with prices constant):
\[ \Delta Y = \hat{\alpha} \left( \frac{b}{h} \right) \Delta \left( \frac{M}{P} \right) \Rightarrow \frac{\Delta Y}{\Delta \left( M/P \right)} = \hat{\alpha} \left( \frac{b}{h} \right) \]

### 2.3 The TB=0 curve

Recall from (16):

\[ TB \equiv EX - IM = (\overline{EXP} + nq) - (\overline{IMP} + mY - nq) \]

One can obtain the vertical TB=0 curve in figure 18.7 (page 347) by setting TB=0, and solving for \( Y \):

\[ 0 = (\overline{EXP} + vq) - (\overline{IMP} + mY - nq) \]

\[ mY = \overline{EXP} - \overline{IMP} + (v + n)q \]

\[ Y = \left( \frac{1}{m} \right) [\overline{EXP} - \overline{IMP} + (v + n)q] \]

Since none of the terms in (37) depend on the interest rate, then the TB=0 curve is just a vertical line at the value on the right hand side of (37). Assuming internal and external (TB=0) equilibrium, we have the figure below.

Any point to the right of the TB=0 line is associated with a trade deficit; to the left, a trade surplus.
2.4 Linking the trade deficit to the LM curve

We can link the IS-LM framework to the trade balance and the balance and payments under certain assumptions. If there are no private capital flows, and no factor payments, so:

\[ CA + KA + ORT \equiv 0 \rightarrow BP + ORT \equiv 0 \]  

(38)

\[ TB = BP \equiv -ORT \]  

so if \( BP > 0 \) then \( ORT < 0 \) (foreign exchange reserves are increasing).

Now, from Chapter 19,

(40) \[ MB \equiv Res + NDA \]

That is the money base \((MB)\) is the counterpart to foreign exchange reserves \((Res)\) and net domestic assets \((NDA)\), also called domestic credit. If the central bank does not undertake sterilization operations (i.e., the central bank does not manipulate net domestic assets to offset changes in foreign exchange reserves), then:

(41) \[ \Delta MB = \Delta Res = BP = -ORT \]

So when there is a balance of payment surplus (here, a trade surplus, since private capital flows are zero), foreign exchange reserves increase and hence so too does the money base. The money base is linked to the money supply – increases in the money base result in increases in the money supply.

To see how the IS-LM-TB=0 system works together, consider the scenario outlined in Figure 19.1 (page 368).

The economy starts at a position of initial trade balance \((EX=IM)\).

A monetary expansion shifts the LM out, thereby increasing output to \(Y_1\). As output rises, imports rise, so that a trade deficit is run. In the absence of private capital inflows, foreign exchange reserves are decumulated. As a consequence, the money base falls, and hence the money supply.
The reduction in the money supply shifts in the LM curve. As long as output exceeds the income at the TB=0 line, the LM shifts in.

This example demonstrates the linkage between the real side and the money side through the balance of payments.

Appendix
Solving the IS-LM System Algebraically

To solve for equilibrium income, substitute the equation (30) <LM> into (25) <IS>:

\[ (25) \quad Y = \alpha \left[ A + EXP - IMP + (n + v)q - bi \right] \quad \text{<IS Curve>} \]

\[ (30) \quad i = -\left( \frac{1}{h} \right) \left( \frac{M}{P} \right) + \left( \frac{k}{h} \right) Y \quad \text{<LM Curve>} \]

\[ (A1) \quad Y = \alpha \left[ A + EXP - IMP + (n + v)q - b \left( \frac{1}{h} \right) \left( \frac{M}{P} \right) + \left( \frac{k}{h} \right) Y \right] \]

Bring the multiplier and the Y term to the left hand side.

\[ (A2) \quad Y \left( 1 - c(1-t) + m + \frac{bk}{h} \right) = A + EXP - IMP + (n + v)q + \left( \frac{b}{h} \right) \left( \frac{M}{P} \right) \]

Divide both sides by the term in parentheses to obtain equation (31):

\[ (31) \quad Y_0 = \hat{\alpha} \left[ A + EXP - IMP + (n + v)q + \left( \frac{b}{h} \right) \left( \frac{M}{P} \right) \right] \quad \text{where} \quad \hat{\alpha} \equiv \frac{1}{1 - c(1-t) + m + \frac{bk}{h}} \]

To obtain the equilibrium interest rate, substitute (25) into (30). It will depend upon all the variables that output depended upon.